Mean-Field Derivation of the Interacting Boson Model Hamiltonian and Exotic Nuclei

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A novel way of determining the Hamiltonian of the interacting boson model (IBM) is proposed. Based on the fact that the potential energy surface of the mean-field model, e.g., the Skyrme model, can be simulated by that of the IBM, parameters of the IBM Hamiltonian are obtained. By this method, the multifermion dynamics of surface deformation can be mapped, in a good approximation, onto a boson system. The validity of this process is examined for Sm and Ba isotopes, and an application is presented to an unexplored territory of the nuclear chart, namely, the right lower corner of ^{208}Pb.

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The quadrupole collectivity is one of the most prominent features of the nuclear structure for both stable and exotic nuclei, and has been extensively studied in terms of the interacting boson model (IBM) [1] in addition to other approaches, e.g., [2,3]. In many IBM calculations the parameters of the Hamiltonian are adjusted to experiment. On the other hand, the model itself has a certain microscopic foundation [4], where a so-called proton (neutron) boson reflects a collective pair of valence protons (neutrons). As the number of valence protons (neutrons) is constant for a given nucleus, the number of proton (neutron) bosons, denoted, respectively, as n_π (n_ν), is set equal to half of the valence proton (neutron) numbers. While the IBM has been successful in reproducing experimental data, the microscopic derivation of the Hamiltonian has been done for limited realistic cases near spherical shapes [5–7], by using zero- and low-seniority states of the shell model [4,8]. In this Letter, we propose a novel way of determining the parameters of the Hamiltonian of the IBM for general cases, while the IBM stands as it has been.

The Skyrme models have been successful in performing mean-field studies on the atomic nucleus, including its size, surface deformation, etc [9,10]. One, however, has not been able to calculate levels and wave functions of excited states in general with the exact treatment of the angular momentum and the particle number [11], and thereby the Skyrme model appears to be rather insufficient for the purpose of nuclear spectroscopy. The IBM is also a model on the quadrupole collectivity. Thus, it should be very interesting to construct an IBM Hamiltonian based on Skyrme model. We shall first illustrate how this can be carried out with some examples as a *proof of principle*.

We first perform the constrained Skyrme Hartree-Fock + BCS (denoted by HF, for brevity) calculation in the usual way [3]. The constraint imposed here means the one with (mass) quadrupole moments including the triaxial degrees of freedom. Figure 1 shows the potential energy surfaces (PES's) in the β-γ plane, where β_{BM} and γ_{BM} imply geometrical deformation parameters [2].

We shall consider the energy range up to 2 MeV from the minimum, because the low-lying collective states are in this range. The Skyrme SLy4 [12] and SkM* [13] interactions are taken, while the following results do not depend too much on the choice of the Skyrme interaction as long as usual ones are taken. The EV8 code is used [14], with the

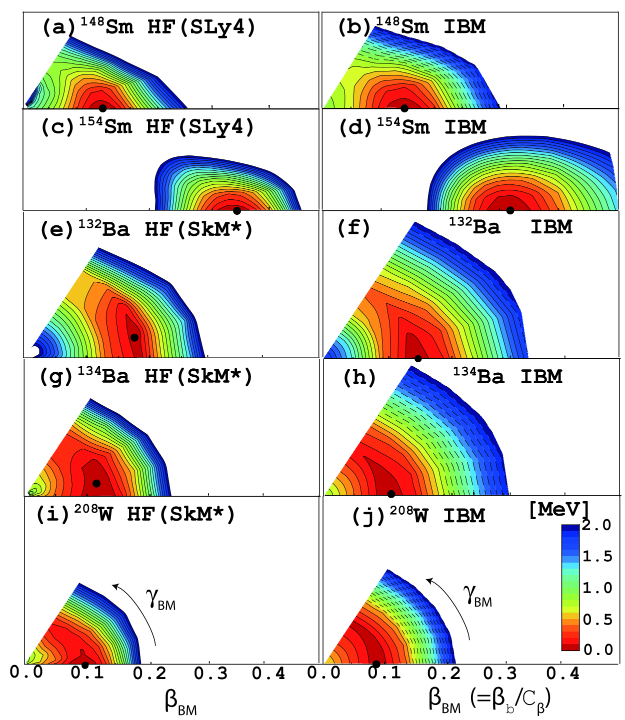

FIG. 1 (color online). Comparisons of PES's in the (β_{BM},γ_{BM}) planes for several nuclei calculated by HF (left) and IBM (right). Contour spacing is 0.1 MeV. Minima can be identified by solid circles.

pairing interaction of the δ-function type with the strength $V_0 = 1250$ MeV fm^3.

We turn to the IBM description of the PES. The IBM is comprised of a scalar, s, boson and a quadrupole, d, boson. In this Letter, we discuss mainly the IBM-2 [4], consisting of proton s_π and d_π bosons and neutron s_ν and d_ν bosons, because of more direct link to microscopic structure. A coherent state has been introduced [15–17],

$$|\Phi\rangle \propto \prod_{\rho=\pi,\nu}\Big[s_\rho^\dagger + \sum_{\mu=0,\pm2}\alpha_{\rho\mu}d_{\rho\mu}^\dagger\Big]^{n_\rho}|0\rangle, \qquad (1)$$

where α's are amplitudes and $|0\rangle$ stands for the boson vacuum (i.e., inert core). The α's are expressed more transparently as $\alpha_{\rho 0} = \beta_\rho \cos\gamma_\rho$ and $\alpha_{\rho\pm2} = (1/\sqrt{2})\beta_\rho \sin\gamma_\rho$, where β_ρ and γ_ρ ($\rho = \pi$ or ν) are called intrinsic variables. The β's represent the relative d-boson probability over the s boson. As the s boson can create only spherical state and the description of the quadrupole deformation requires the d boson, the β's are parameters indicating the quadrupole deformation. The coherent state represents an intrinsic state, i.e., a state in the body-fixed frame. If the quadrupole deformation has an axial symmetry (the object is invariant under the rotation about the symmetry axis), one can choose the z axis to be the symmetry axis. In this case, the coherent state, $|\Phi\rangle$, must be invariant with respect to the rotation about the z axis. This means that the possible values of γ are 0 and 60 degrees. On the other hand, a different value of γ indicates a triaxial (i.e., nonaxially symmetric) deformation. Thus, we can describe the (intrinsic) shape of the nucleus in terms of β and γ. In principle, both β and γ can take different values for proton and neutron bosons. However, since protons and neutrons attract each other strongly, the proton and the neutron systems should have the same shape in the first approximation. We therefore assume that β and γ take the same values for proton and neutron, denoting them β_b and γ_b, respectively.

In the following, the expectation value of an operator $\hat{\mathcal{O}}$ with respect to $|\Phi\rangle$ is denoted by $\langle\hat{\mathcal{O}}\rangle \equiv \langle\Phi|\hat{\mathcal{O}}|\Phi\rangle/\langle\Phi|\Phi\rangle$. For the PES of the IBM, $\hat{\mathcal{O}}$ is the Hamiltonian. In this study, the standard IBM-2 Hamiltonian is taken

$$\hat{\mathcal{H}} = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa\hat{\mathcal{Q}}_\pi \cdot \hat{\mathcal{Q}}_\nu, \qquad (2)$$

where ϵ denotes the d boson energy relative to the s boson one. While ϵ can differ between proton and neutron, they are set to be equal for simplicity. The second term in Eq. (2) is the quadrupole-quadrupole interaction between proton and neutron bosons with the strength κ. The parameters $\chi_{\pi,\nu}$ appear as $\hat{\mathcal{Q}}_\rho = [s_\rho^\dagger\tilde{d}_\rho + d_\rho^\dagger\tilde{s}_\rho]^{(2)} + \chi_\rho[d_\rho^\dagger\tilde{d}_\rho]^{(2)}$ and determines the prolate or oblate shape of deformation, reflecting the structure of collective nucleon pairs as well as the numbers of valence nucleons [4,8].

The expectation value $\langle\hat{\mathcal{H}}\rangle$ is calculated as [18],

$$\langle\hat{\mathcal{H}}\rangle = \frac{\epsilon(n_\pi + n_\nu)\beta_b^2}{1+\beta_b^2} + n_\pi n_\nu \kappa \frac{\beta_b^2}{(1+\beta_b^2)^2} \times\Big[4 - 2\sqrt{\frac{2}{7}}(\chi_\pi + \chi_\nu)\beta_b\cos 3\gamma_b + \frac{2}{7}\chi_\pi\chi_\nu\beta_b^2\Big]. \qquad (3)$$

Once we obtain the boson PES as a function of β_b and γ_b, we map a point of it to an appropriate point of the HF PES. This is nothing but a mapping of (β_b, γ_b) onto $(\beta_{\rm BM}, \gamma_{\rm BM})$. We equate, for simplicity, γ_b to $\gamma_{\rm BM}$, as both of them should be limited to the 0 to 60° interval and have similar meanings. The other variable β_b is related to $\beta_{\rm BM}$. Along the line of $\gamma_b = 0$ (i.e., axially symmetric deformation), the intrinsic quadrupole moment can be defined as $Q_I = q\langle\hat{\mathcal{Q}}_\pi + \hat{\mathcal{Q}}_\nu\rangle$, where q is an overall scaling factor. Similarly to Eq. (3), one obtains $Q_I = q[2(n_\pi + n_\nu)\beta_b - \sqrt{2/7}(n_\pi\chi_\pi + n_\nu\chi_\nu)\beta_b^2]/(1+\beta_b^2)$. The actual range of β_b is $0 \le \beta_b \lesssim 1$, and also practically $|\chi_{\pi,\nu}| \lesssim 1$. Thus, the term $\propto \beta_b^2$ in Q_I becomes minor as compared to the rest, and can be neglected in the first approximation, leading us to $\beta_b \propto \beta_{\rm BM}$, because Q_I is proportional to $\beta_{\rm BM}$. We then assume hereafter, $\beta_b = C_\beta\beta_{\rm BM}$ with C_β being the scale factor with the maximum value of $5 \sim 6$.

We now sketch the procedure to determine values of ϵ, κ, $\chi_{\pi,\nu}$ and C_β, by taking ^{148,154}Sm as examples. Their HF PES's are shown in Fig. 1(a) and 1(c). The IBM parameters are adjusted so that the overall pattern of the HF PES up to 2 MeV from the energy minimum is reproduced, with certain attention to their gradual systematic changes from neighboring isotopes. The overall pattern reflects how nuclear force and Pauli principle work in determining the energy of collective state for the relevant range of shapes. Thus, by reproducing HF PES, the boson system is expected to simulate, to a good extent, effects of nuclear force, antisymmetrization, density dependences, etc., in a simple manner.

Figures 1(b) and 1(d) show IBM PES's, obtained from IBM parameters shown in Fig. 2(a) as functions of the neutron number, N. Figure 2(a) suggests that ϵ and χ_ν vary rather significantly, while κ and C_β change much less. A common value of χ_π is assumed for simplicity, being consistent with earlier works [4,8].

The spectra of ^{148}Sm look like a spherical vibrator or the U(5) limit of IBM. The HF PES somewhat differs from this picture, placing the energy minimum at $\beta_{\rm BM} \sim 0.15$. The IBM PES reproduces it, as well as the overall pattern of the HF PES. For ^{154}Sm, which is an example of the axially symmetric deformation, or the SU(3) limit of IBM, HF PES shows a pronounced sharp minimum, and IBM PES also exhibits a similar one. The minimum valley is, however, shallower for the IBM PES. This is a general trend and is probably due to the finite number and/or limited types of bosons. This tendency cannot be changed

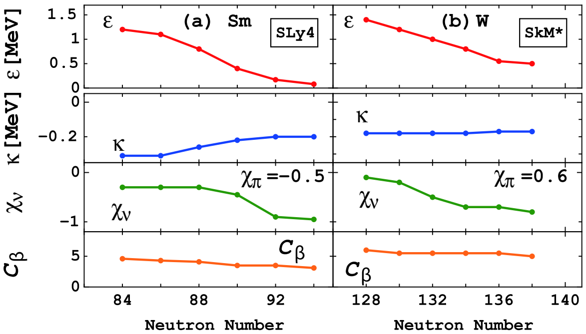

FIG. 2 (color online). Evolution of parameters in Eq. (3) with the neutron number. (a) Sm and (b) W isotopes are studied with SLy4 and SkM* forces, respectively. The parameter χ_π is kept constant as $\chi_\pi = -0.5$ (0.6) for Sm (W) isotopes.

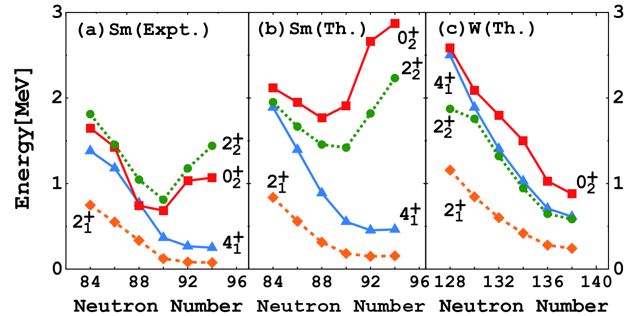

FIG. 3 (color online). (a) Experimental [27] and (b) calculated levels (IBM from SLy4) for Sm isotopes, and (c) calculated ones (IBM from SkM*) for W isotopes, as functions of the neutron number.

only by playing parameters, and its origin and improvement are interesting subjects. Note that χ^2-fitting may not make much sense as the present IBM Hamiltonian in Eq. (2) may be too simple to perform the complete fitting. The parameters shown in Fig. 2(a) are obtained by requesting, for individual nucleus, a good fit of HF PES by IBM PES similarly to ^{148,154}Sm. This results in some significant changes of the parameters as functions of N. For instance, Fig. 2(a) indicates notable changes in κ and χ_ν around $N = 86$ and $N = 90$, respectively. They seem to reflect structural evolutions, respectively, from spherical to transitional shapes and from transitional to deformed ones. The seniority prescription [4] gives the opposite dependence of χ_ν on N, while the present one appears to be consistent with a mapping method using deformed intrinsic states [19]. The gradual decrease of ϵ with N has been discussed as a consequence of stronger coupling between "unperturbed d boson" and other bosons, e.g., the one with spin 4 [4,20]. It is an interesting open question why this decrease occurs. As one will see, all these variations produce levels consistent with experimental tendencies, without adjustment to levels. We point out once again that the total energies of HF-BCS and the corresponding IBM states are compared in the present method. We diagonalize the boson Hamiltonian calibrated by this comparison. In some collective models, the PES is treated as an effective potential and a generalized kinetic energy (mass term) is introduced. In the present method, similarly to GCM, effects carried by the mass term are included in the diagonalization to a large extent. In the cases of strong deformation, however, the difference between the overlap of fermion wave functions and that of the corresponding boson wave functions may become large, leading us to additional terms, for instance, the so-called $L \cdot L$ term of the boson Hamiltonian introduced only phenomenologically.

Figures 3(a) and 3(b) exhibit the evolution of levels, computed by the NPBOS code [21], from $N = 84$ to 94. At $N = 84$, the low-lying spectra look like those of spherical vibrators [or U(5) limit]. As N increases, calculated levels come down consistently with experimental trends particularly for the yrast levels. There seems to be a critical point at $N = 88$ or 90, beyond which the 2_2^+ and 0_2^+ levels go up in both calculation and experiment. For $N \geq 92$, yrast spectra look like a rotational band.

The scale of the calculated levels is larger than that of experimental ones for $N \geq 90$. On the other hand, the ratios between levels are better reproduced with a clear signature of the spherical-deformed phase transition. This scale problem seems to be seen in many GCM results [22]. It should be investigated further, although it does not show up for moderately deformed cases.

Figures 3(a) and 3(b) show properties of the X(5) critical-point symmetry around $N = 90$ [23–25]. In fact, experimental, X(5) and present values of $R_{4/2}$ for ^{152}Sm are 3.01, 2.91, and 3.08, respectively, being close to each other. The HF PES is wider in the β direction in ^{152}Sm than in ^{154}Sm, but similarly sharp in the γ direction.

We shall now discuss the structure of ^{132,134}Ba. The former is an example of γ-unstable deformation, or O(6) limit of IBM, while the latter an example of γ-unstable E (5) critical-point symmetry [26]. In Figs. 1(e)–1(h) the IBM reproduces HF (SkM*) PES quite nicely, where both ^{132,134}Ba produce large flat areas in the HF PES. It is more spread for ^{134}Ba: ^{134}Ba is more like E(5), while ^{132}Ba is closer to O(6). While $R_{4/2} = 2.19$ in E(5), the experimental value for ^{134}Ba is 2.31, which agrees rather well with the present result, 2.49. Figures 4(a) and 4(b) present a comparison of levels for ^{134}Ba between the present calculation and experiment with a clear signature of γ-instability.

Having reasonable comparisons with experiments covering various situations, we try to describe unexplored nuclei with $A \gtrsim 200$ for W and Os isotopes, which are chosen because of no systematic theoretical work. Figures 1(i) and 1(j) show the HF (SkM*) and IBM PES's for ^{208}W. The PES has flat areas like ^{132,134}Ba, suggesting E(5) structure. The derived IBM parameters are shown in Fig. 2(b), where χ_π and χ_ν have opposite

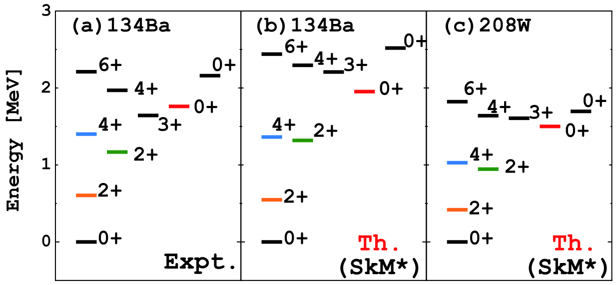

FIG. 4 (color online). Levels of ^{134}Ba and ^{208}W. (a) Experiment[27], (b) calculated levels of ^{134}Ba, and (c) calculated levels of ^{208}W. SkM* force is used.

signs with sizable magnitudes. In the IBM-2, this is the origin of the O(6)-E(5) pattern [4,8]. The level evolution for W isotopes is depicted in Fig. 3(c). It is of a considerable interest that the magnitude of deformation, represented by the lowering of $E(2_1^+)$, becomes larger as N, while the γ-unstable E(5)-O(6) level pattern is maintained all the way. Such sustained E(5)-O(6) structure has never been seen in stable nuclei, and may become one of the characteristic features of exotic nuclei. As an example, Fig. 4(c) shows predicted level scheme of ^{208}W, which is indeed similar to that of ^{134}Ba. A similar tendency is found in exotic Os nuclei.

In summary, we present a novel way of determining the IBM Hamiltonian based on the mean-field models, e.g., Skyrme models, many of which are good for drawing PES. The IBM and such mean-field models can be complementary, as the latter cannot give energy levels and wave functions precisely. In other words, we try to transport basic features of multinucleon systems, including effects of nuclear forces and Pauli principle, into a mathematically simpler boson model. The present method becomes almost the same as the previous OAI method [4], if the PES is drawn near spherical shapes and IBM Hamiltonian is derived from the comparison there. Standard Skyrme models are useful for bulk and surface properties with good calibration to experiment, and are suitable to start with, while a more realistic interaction can be taken in the future. Using the present method, unlike existing IBM studies, we gain a capability to predict levels and wave functions for experimentally unknown nuclei including those in unexplored territories on the nuclear chart. This can be a great advantage in the era of the third-generation rare-isotope accelerators producing many new *heavy* exotic nuclei.

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