Magnetic Friction in Ising Spin Systems

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A new contribution to friction is predicted to occur in systems with magnetic correlations: Tangential relative motion of two Ising spin systems pumps energy into the magnetic degrees of freedom. This leads to a friction force proportional to the area of contact. The velocity and temperature dependence of this force are investigated. Magnetic friction is strongest near the critical temperature, below which the spin systems order spontaneously. Antiferromagnetic coupling leads to stronger friction than ferromagnetic coupling with the same exchange constant. The basic dissipation mechanism is explained. A surprising effect is observed in the ferromagnetically ordered phase: The relative motion can act like a heat pump cooling the spins in the vicinity of the friction surface.

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As friction is an intriguingly complex phenomenon of enormous practical importance, the progress in experimental techniques on the micro- and nanoscale [1,2] as well as the improved computational power for atomic simulations [3–5] has led to a renaissance of this old research field in recent years. Currently a large variety of microscopic models compete with one another [1,6,7]. Major complications are wear, plastic deformation at the contact, impurities, and lubricants. It is unlikely that in the general case only a single dissipation mechanism will be active. Defect motion, phononic and electronic excitations may be involved in a very complex blend. To reduce these complications and to focus on the elementary dissipation processes, increasing attention has been paid to noncontact friction: It can be measured as damping of an atomic force microscope tip which oscillates in front of a surface without touching it [8,9]. For this setup, too, phononic [10,11] as well as electronic dissipation mechanisms [12,13] have been discussed. Recently, a Heisenberg model with magnetic dipole-dipole interactions was studied at zero temperature as a model for magnetic force microscopy. In this case the moving tip excites spin waves, which dissipate part of the energy [14].

In this Letter a different mechanism is considered, by which the spin degrees of freedom of an Ising model contribute to friction. We imagine two magnetic materials with planar surfaces sliding on each other. Of course, if one of the materials is metallic, their relative motion will induce eddy currents [15]. The corresponding Joule heat is commonly associated with the term "magnetic friction," although the energy is not dissipated into the spin degrees of freedom, which can even be considered as frozen. By contrast, here we are interested in the case that both materials are nonmetallic (e.g., magnetite Fe_3O_4). To highlight the role of the spin degrees of freedom we do not take phononic and electronic excitations into account explicitly, but regard them as a heat bath of fixed temperature T to which all spins are coupled. Energy dissipation in Ising spin systems was studied previously [16,17], but there it was due to an oscillating magnetic field rather than the tangential relative motion of two lattices. The competition between the time scales for driving the system out of equilibrium and for its relaxation gave rise to hysteretic, and hence dissipative behavior. These time scales play also a role for magnetic friction, as we will show.

Specifically, we present Monte Carlo (MC) simulation results for a two-dimensional Ising square lattice with periodic boundary conditions. Each of the N lattice sites carries a classical spin variable S_i which can take the values ± 1. The Hamiltonian is $H = -J\sum_{\langle i,j\rangle} S_i S_j$, where $\langle i, j\rangle$ denotes nearest neighbors, and J is chosen as energy unit. Coupling to a heat bath of constant temperature T lets the spin configuration C relax towards thermal equilibrium. The relaxation kinetics are determined by the transition rate $w(C \to C')$ to a new configuration C', in which one randomly chosen spin is flipped. We consider fast relaxation with Metropolis rate [18]

$$w_M(C \to C') = t_0^{-1} \min(1, e^{-\beta\Delta E}) \tag{1}$$

and slow relaxation with Glauber rate [18]

$$w_G(C \to C') = w_M(C \to C')/(1 + e^{-\beta|\Delta E|}), \tag{2}$$

where $\beta = (k_B T)^{-1}$. The energy difference $\Delta E = E(C') - E(C)$ is received from ($\Delta E > 0$), respectively, transferred to ($\Delta E < 0$) the heat bath, when the spin is flipped. $t_0 \approx 10^{-8}$ s [19] is the typical time for relaxation of a spin into the direction of the local Weiss-field.

The system is constantly driven out of equilibrium in the following way: The lattice is cut parallel to an axis into an upper and a lower half. The former is displaced by one lattice constant $a \approx 10^{-10}$ m in regular time intervals a/v, where v is the sliding velocity (in the following given in natural units a/t_0). This means that N/v random sequential spin updates (i.e., $1/v$ Monte Carlo steps) are followed by a rigid translation of the upper half by one lattice constant parallel to the cut. $v = 1$ corresponds to 10^{-2} m/s. (Note that due to the periodic boundary conditions there is a second slip plane separating the upper half of the simulation cell from the periodic image of the lower half.) The exchange interaction J is the same, no matter whether the interacting spins are on the same or on different sides of the cut. This has the advantage that the relative velocity v and the temperature T (in natural units $|J|/k_B$) are the only parameters in the model. In the following we evaluate the accumulated energy (divided by two, because of the two equivalent slip planes) that has been exchanged with the heat bath during the time interval t, $\Delta E_{\text{bath}}(t)$, for different sliding velocities v and temperatures T. We first present our results for ferromagnetic coupling, $J > 0$. In the end we also discuss what is different for antiferromagnetic coupling, $J < 0$.

Is there any energy dissipation within this simple model at all? To answer this question we first simulated a system consisting of 80×80 spins thermalized for 200 MC steps per spin at a temperature $T = 2.5$ above the critical temperature $T_C = 2/\ln(\sqrt{2} + 1)$ [20] (initial configuration). Figure 1 shows the energy exchange per spin with the heat bath for two cases: Without relative motion ($v = 0$) of the half-spaces ΔE_{bath} fluctuates around 0; i.e., no energy is dissipated. Switching on the relative motion with a velocity $v = 1$ leads to a linear increase of $\Delta E_{\text{bath}}(t)$. The total system energy E per spin stays constant at about the same value in both cases. This means that the sliding system quickly develops a steady state, where energy is transferred continuously to the heat bath. The slope in Fig. 1 is the constant dissipation rate $P = \Delta E_{\text{bath}}/\Delta t$. It is directly connected to the friction force F by $P = Fv$. We conclude that the Ising model gives rise to a truly magnetic friction force: The relative motion pumps energy into the spin degrees of freedom, which in the steady state is then transferred further into the heat bath.

The magnetic friction force turns out to be proportional to the length L of the periodic cell along the direction of the cut through the two-dimensional lattice. On the other hand, varying the system size perpendicular to the slip plane does not change the above results, as long as it remained larger than about 20 lattice constants. This shows that whatever energy the relative motion pumps locally into the spin degrees of freedom near the slip plane, gets transferred completely to the heat bath before it can drive more distant parts of the system out of equilibrium.

Figure 2 shows that the dissipation rate for small velocities starts out linearly, with a slight upward curvature, and saturates for large velocities. The saturation is expected when the velocity times the relaxation time τ becomes larger than the correlation length ξ [21], i.e., when $v > \xi/\tau$. Then the lower half-space is essentially confronted with uncorrelated configurations of the upper half-space, and a further increase of v does not change anything. For Glauber dynamics the relaxation time is larger by a factor of about 1.5 than for Metropolis dynamics. This explains the difference between the curves in Fig. 2: If one rescales time by this factor, i.e., multiplies velocity and dissipation rate by 1.5, the curve for Glauber dynamics is shifted such that it essentially coincides with the one for Metropolis dynamics. For small velocities the linear v dependence in Fig. 2 implies that the magnetic friction force approaches a constant, F_0. For $T = 2.5$ the velocity independent part of the magnetic frictional shear stress has the value $F_0/L = 0.114 \pm 0.004$. It is the same for Metropolis and Glauber dynamics.

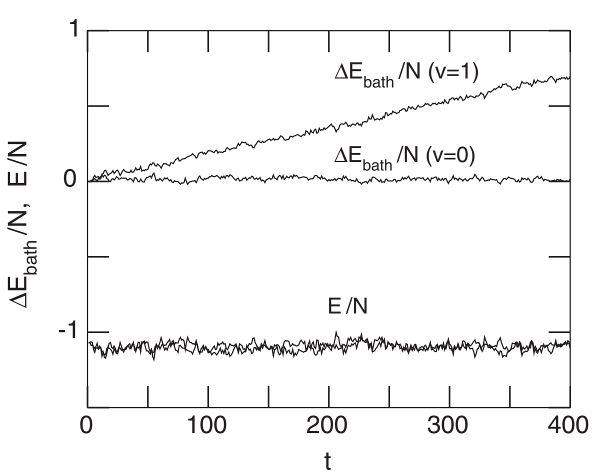

FIG. 1. Accumulated energy ΔE_{bath} per spin which is transferred to the heat bath during a time interval t, without motion ($v = 0$) and with motion ($v = 1$) of the two half-spaces. Simulation with Metropolis rates. The total energy per spin fluctuates around the exact value $E/N \approx -1.10608$ [20] in both cases.

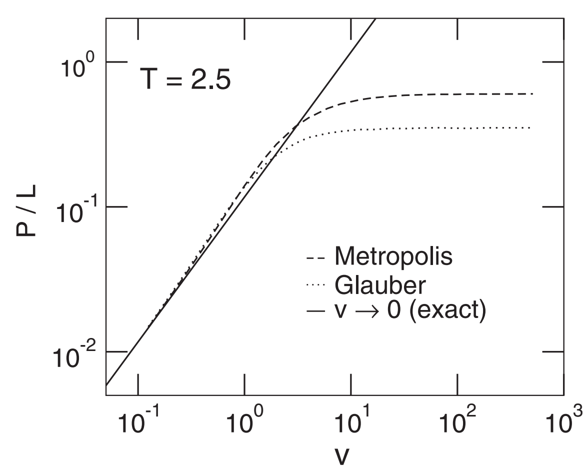

FIG. 2. Energy dissipation rate P per unit length as a function of the relative velocity v of the two half-spaces (averaged over 100 runs). Dashed line: Metropolis rates. Dotted line: Glauber rates. Solid line: Exact solution for the limit $v \to 0$.

F_0/L can be calculated analytically in the quasistatic limit, $v \to 0$, where the spin system has time enough to relax back into equilibrium after each displacement of the upper half. The energy of the spin configuration immediately after a displacement minus the equilibrium energy must be transferred to the heat bath during the time interval a/v. The rigid shift of all spins in the equilibrated upper half by one lattice constant places former next nearest neighbors in nearest neighbor positions on opposite sides of the slip plane. Thus the dissipated energy per unit length a (i.e., the friction force) can simply be expressed as JL times the nearest neighbor spin correlation function minus the next nearest neighbor spin correlation function. Both are known analytically; see, e.g., Eqs. (4.5) and (4.9) of [20]. At $T = 2.5$ this gives the value $F_0/L \approx 0.117$ in good agreement with the numerical result. For general temperature one obtains the solid curve in Fig. 3.

According to the picture of Bowden and Tabor [22] also Coulomb friction is independent of v and proportional to the real contact area, which due to surface roughness is smaller than the sliding surface macroscopically appears to be, and grows proportional to the normal load. Therefore, the velocity independent part of the magnetic friction force behaves like Coulomb friction. How does it compare to typical values for solid friction? The above results show that the magnetic shear stress $\sigma_t = F_0/L$ is of the order of 0.1 for the two-dimensional Ising model. The unit is J/a^2, the exchange constant divided by the lattice constant squared. If we regard the two-dimensional Ising model as a slice of thickness a of a three-dimensional system, then we may assume that the magnetic shear stress for a three-dimensional Ising model is of the order of $\sigma_{t,3d} \approx 0.1 J/a^3$. Inserting typical values ($J \approx 0.6 \times 10^{-20}$ Joule, $a \approx 3 \times 10^{-10}$ m) one gets the estimate $\sigma_{t,3d} \approx 20$ MPa. This is a surprisingly large value. Ordinary solid friction shear stresses are given by $\sigma_{t,\mathrm{Coulomb}} = \mu \sigma_c$ according to the Bowden-Tabor-theory, where a typical value for the friction coefficient is $\mu = 0.2$, and the yield stress σ_c at high temperatures is a few hundred to thousand MPa. We conclude that magnetic friction is probably not too weak compared with ordinary solid friction to be observable.

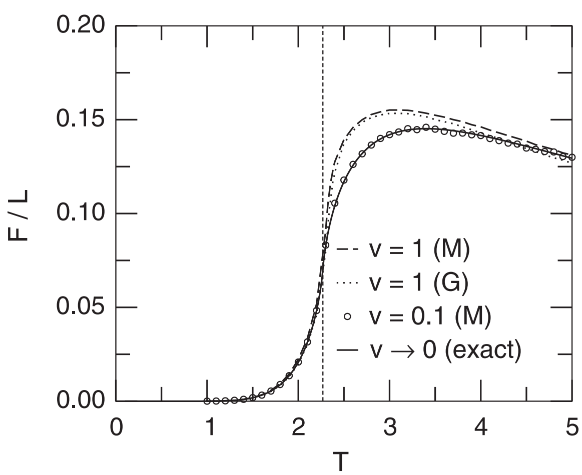

FIG. 3. Temperature dependence of the friction force per unit length, F/L. Solid line: Exact quasistatic limit $v \to 0$. Simulation results with Metropolis rates (circles) for $v = 0.1$ agree with the quasistatic limit. For $v = 1$ the friction forces for Metropolis rates (dashed line), respectively, Glauber rates (dotted line) are larger corresponding to the upward curvature in Fig. 2. All data are averaged over 100 runs. The critical temperature is indicated by the dashed vertical line.

There is one caveat, however: The exchange interaction is extremely short range, but in the simulation results presented here no reduced value was inserted for the interaction of spins on opposite sides of the slip plane. The above estimate should therefore only be applied if the surfaces are in close contact. As expected, simulations with a reduced magnetic exchange interaction across the slip plane lead to a smaller friction force.

Magnetic friction has characteristic features near the critical temperature, which should be useful to separate this contribution to solid friction from other ones. It is nearly zero at low temperatures, where the ferromagnetic ordering implies almost perfect translational invariance along the surface. As thermal fluctuations destroy the translational invariance, magnetic friction raises sharply to a maximum slightly above the critical temperature (Fig. 3). In the paramagnetic region the exact quasistatic limit shows that the friction force has the same $1/T$ asymptotics as JL times the nearest neighbor correlation function, because the next nearest neighbor correlation ($\propto 1/T^2$) becomes negligible.

What is the basic mechanism leading to magnetic friction in the Ising model? Obviously, shearing reduces the correlation length locally by disturbing the equilibrium correlations between spins on opposite sides of the slip plane. Above T_c this corresponds to an effective temperature increase, which explains the energy flow into the cooler heat bath. Since more neighbor pairs with antiparallel spin are present, the energy density is locally increased in the steady state, compared with its value in thermal equilibrium. As the correlation length vanishes for $T \to \infty$, this picture explains why magnetic friction vanishes in this limit.

Below T_c the correlation length can be associated with the diameter of thermally activated minority clusters of spins pointing into the direction opposite to the spontaneous magnetization. The relative motion distorts minority clusters, which extend across the slip plane, and possibly cuts them into two pieces. Again this reduces the effective correlation length. In thermal equilibrium a smaller correlation length indicates a better ordered magnetic state. Indeed we find an increased magnetization locally at the slip plane (Fig. 4). This effect is less pronounced for the Metropolis algorithm, where the spin configurations relax more quickly into thermal equilibrium.

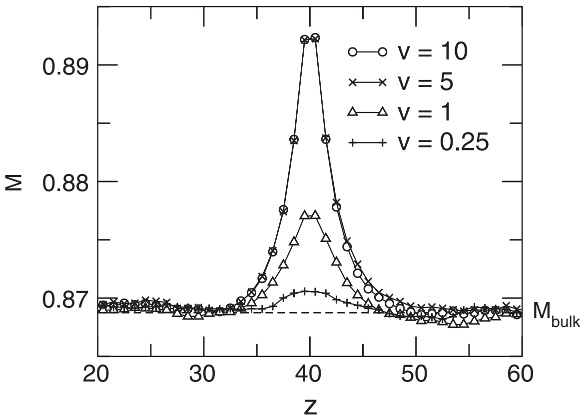

FIG. 4. Magnetization profile along the z axis perpendicular to the slip plane (at $z = 40$ in units of the lattice constant a) at $T = 2.1 < T_C$ for Glauber rates at different velocities. The local magnetization near the slip plane is enhanced. This effect becomes stronger for increasing velocity and saturates for the same reason as in Fig. 2. When using Metropolis rates this effect is less pronounced (not shown).

The local spin temperature in the vicinity of the slip plane drops due to the influence of shearing. The driven system acts like a "heat pump" cooling the spin degrees of freedom below the temperature of the heat bath. The shearing creates additional domain walls by deforming or fragmenting minority clusters. The system continuously tries to reduce these excess domain walls, thereby transferring domain wall energy to the heat bath. This is the dissipation mechanism.

Why does this "heat pump" work better for higher velocities, as shown by Fig. 4? Let us discuss first the case of sufficiently high velocities, where the magnetization near the slip plane saturates at a maximal value. Then correlations between the two half-spaces can be neglected. Instead, the spins in the lower half see an effective surface field corresponding to the average surface magnetization of the upper half. Hence minority spins near the slip plane flip more easily into the majority direction than in the bulk. For smaller velocities, however, minority clusters can be stabilized more and more because of correlations across the slip plane. Hence the surface magnetization decreases.

Analogous investigations for antiferromagnetic coupling ($J < 0$) were done, too. The dissipation rate turns out to be much higher than in the ferromagnetic case (with the same $|J|$). The friction maximum is more than 3 times larger for the Ising antiferromagnet than for the ferromagnet. The reason is that the local antiferromagnetic order across the slip plane is destroyed whenever the upper lattice is displaced by one lattice constant. This is a stronger perturbation than in the ferromagnetic case, where only the correlations of thermal disorder could be destroyed by the relative motion. In particular, magnetic friction does not vanish for $T \to 0$ in the antiferromagnetic case.

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