Dynamically Assisted Schwinger Mechanism

Ralf Schützhold,[1,2] Holger Gies,[3,4] and Gerald Dunne[5]

[1]Fachbereich Physik, Universität Duisburg-Essen, D-47048 Duisburg, Germany
[2]Institut für Theoretische Physik, Technische Universität Dresden, D-01062 Dresden, Germany
[3]Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany
[4]Theoretisch-Physikalisches Institut, Universität Jena, D-07743 Jena, Germany
[5]Department of Physics, University of Connecticut, Storrs, Connecticut 06269, USA
(Received 24 July 2008; published 25 September 2008)

We study electron-positron pair creation from the Dirac vacuum induced by a strong and slowly varying electric field (Schwinger effect) which is superimposed by a weak and rapidly changing electromagnetic field (dynamical pair creation). In the subcritical regime where both mechanisms separately are strongly suppressed, their combined impact yields a pair creation rate which is dramatically enhanced. Intuitively speaking, the strong electric field lowers the threshold for dynamical particle creation—or, alternatively, the fast electromagnetic field generates additional seeds for the Schwinger mechanism. These findings could be relevant for planned ultrahigh intensity lasers.

DOI: 10.1103/PhysRevLett.101.130404 PACS numbers: 12.20.Ds, 11.15.Tk

As first realized by Dirac [1], a consistent relativistic quantum description of electrons necessarily involves negative energy levels, which—in the Dirac-sea picture—are filled up in the vacuum state. This entails the striking possibility of pulling an electron out of the vacuum by means of some external influence, such as a (classical) electromagnetic field [2], where the remaining hole in the Dirac sea is then associated with a positron. Of course, to create such an electron-positron pair out of the vacuum, one has to overcome the energy gap of $2mc^2$ between the filled and the empty levels. There are basically two main mechanisms for doing so: In a strong electric field E over a sufficiently long distance L, "virtual" electron-positron pair fluctuations may gain this energy when $qEL \geq 2mc^2$. This pair creation process is called the Schwinger mechanism [3,4] and can be understood as tunneling through the classically forbidden region (energy gap). Thus it is suppressed exponentially $O(\exp(-\pi E_S/E))$ for weak fields E, where $E_S = m^2c^3/(\hbar q)$ is the Schwinger critical field. For $E \simeq E_S$, the work done by separating the electron-positron pair over a Compton wavelength is of the order of the energy gap $2mc^2$. Alternatively, a classical time-dependent electromagnetic field will also create electron-positron pairs in general (dynamical pair creation). However, if the frequency ω of the external field is not large enough, $\hbar\omega < 2mc^2$, these nonadiabatic corrections correspond to higher-order (i.e., multiphoton) processes and are also suppressed exponentially $\exp[-O(1/\omega)]$ for small ω [5]. These pair-production processes are fundamental predictions of quantum electrodynamics, but only the multiphoton production process has so far been observed experimentally: the positron data taken at the SLAC E-144 experiment have convincingly been explained by n-photon production with $n \simeq 5$ [6]. However, a verification of the Schwinger mechanism has still remained an experimental challenge [7]. Since the Schwinger mechanism is nonperturbative in the field, its discovery would help in the exploration of the nonperturbative realm of quantum field theory in a controlled fashion. Here, we propose a new mechanism which can help to overcome the strong exponential suppression. The basic idea is similar in spirit to ideas in the study of atomic physics in strong fields, where new experimental and theoretical results show that controlled engineering of special electric field pulse shapes can enhance certain interesting physical processes, such as high-harmonic generation and above threshold ionization (for reviews, see [8]).

Many previous theoretical studies of pair production [5,9–12] have been motivated by the seminal work of Keldysh [13] on atomic ionization in time-dependent electric fields; in particular, the crossover between the two main mechanisms of pair creation due to strong constant electric fields and due to those with spatial or temporal variations has been of interest. It turns out that spatial variations tend to diminish the pair creation rate [10,12], whereas a time dependence typically increases the effect [11,14]. However, a realistic experimental situation is usually far more complex and may involve various frequency and amplitude scales over a wide range. This motivates us to study electron-positron pair creation in the presence of a strong and slow electric field plus weak and fast electromagnetic wiggles. We assume that the slow electric field E is strong but still far below the Schwinger limit E_S, and that the frequency of the weak electromagnetic wiggles is smaller than twice the electron mass. As explained before, the pair creation rate of each effect separately is strongly suppressed in this case. As we shall demonstrate below, however, their combined impact may be much stronger, i.e., yield an enhanced pair creation rate. These findings could be experimentally relevant in view of the next-generation light sources [15] aiming at approaching the Schwinger limit via high-harmonic focusing, which typically generates a high-frequency tail.

For a first estimate, let us treat the strong and slow electric field adiabatically and nonperturbatively (as our $\hat{H}_0$ problem) and the weak and fast electromagnetic wiggles nonadiabatically and perturbatively. In terms of the field operator $\hat{\Psi}$ and the Dirac matrices $\underline{\boldsymbol{\alpha}}$ and $\underline{\beta}$, the total Hamiltonian density reads ($\hbar = c = 1$)

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(i\underline{\boldsymbol{\alpha}} \cdot \boldsymbol{\nabla} + m\underline{\beta} + V)\hat{\Psi} + q\hat{\Psi}^\dagger\underline{\boldsymbol{\alpha}} \cdot \boldsymbol{A}(t,\boldsymbol{r})\hat{\Psi}, \tag{1}$$

where $V(\boldsymbol{r}) = qA_0$ encodes the (approximately static) electric field and $\boldsymbol{A}(t,\boldsymbol{r})$ is the vector potential of the fast electromagnetic wiggles (scale separation). After splitting $\hat{H}(t) = \hat{H}_0 + q\hat{H}_1(t)$, the $\hat{H}_0$ problem can be diagonalized via the usual normal-mode expansion $\hat{\Psi}(t,\boldsymbol{r}) = \sum_I \hat{a}_I u_I(\boldsymbol{r})e^{-i\omega_I t} + \hat{b}_I^\dagger v_I(\boldsymbol{r})e^{+i\omega_I t}$, where the positive or negative energy spinor solutions $u_I(\boldsymbol{r})$ and $v_I(\boldsymbol{r})$ depend on spin σ and wave number $\boldsymbol{k}$, which are combined into the index $I = \{\sigma, \boldsymbol{k}\}$. The electron (positron) creation (annihilation) operators $\hat{a}_I$, $\hat{a}_I^\dagger$ ($\hat{b}_I$, $\hat{b}_I^\dagger$) are time independent. Now, inserting this expansion into $\hat{H}_1(t)$, the pair creation amplitude $\mathfrak{A}_{IJ} = \langle e_I^+ e_J^- | \text{out}\rangle$ can be calculated in first-order perturbation theory

$$\mathfrak{A}_{IJ} = q\int d^4x u_I^\dagger(\boldsymbol{r})\underline{\boldsymbol{\alpha}} \cdot \boldsymbol{A}(t,\boldsymbol{r})v_J(\boldsymbol{r})e^{i\omega_I t + i\omega_J t}. \tag{2}$$

Within the present estimate, we are primarily interested in the size of the exponent governing the exponential suppression of the above amplitude. Therefore, we study a $(1 + 1)$-dimensional toy model, which should reproduce the exponent correctly, but disregard the subleading prefactor. Let us consider a constant electric field E over an interval of length L with vanishing field outside; see Fig. 1. For simplicity, we assume $qEL \gtrsim 2m$; i.e., we are just above the threshold for the Schwinger effect. In terms of the Klein paradox [16] language, the case lies near the border between the intermediate and strong potential regime.

For modes I, J whose frequency sum $\omega_I + \omega_J$ corresponds to the typical frequency of the perturbation $\boldsymbol{A}(t,\boldsymbol{r})$, the exponential suppression of the pair creation amplitude (2) is basically determined by the spatial overlap of the modes $u_I^\dagger(\boldsymbol{r})$ and $v_J(\boldsymbol{r})$. Assuming $\omega_I = \omega_J = \omega$ for simplicity (other distributions only induce a shift in x but lead to the same result as long as $\omega_I + \omega_J = 2\omega$), the spinor $u_I(\boldsymbol{r})$ describes electron modes which are incident from the right and totally reflected (due to $\omega < m$) by the strong field E, whereas $v_J(\boldsymbol{r})$ corresponds to positron modes which are incident from the left and also totally reflected. The classical turning points are given by $x_\pm = \pm(m - \omega)/(qE)$; see Fig. 1. As the electric field E is

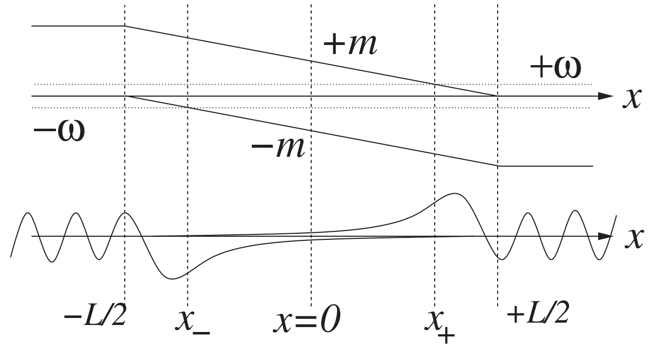

FIG. 1. Sketch (not to scale) of the level structure (top) and the mode functions $u_I^\dagger$ and v_J (bottom). The upper and lower surface of the Dirac sea at $\pm m$ are denoted by solid lines, which are distorted by the electric field E in the interval $-L/2 < x < +L/2$ with $qEL = 2m$ (top). The horizontal dotted lines at $\pm\omega$ represent the electron or positron levels $u_I^\dagger$ and v_J with the classical turning points at $x_\pm$.

assumed to lie far below the Schwinger limit, we may employ the WKB approximation and estimate the exponential suppression by the integral of the eikonal between the classical turning points

$$\int_{x_-}^{x_+} dx\sqrt{m^2 - (qEx - \omega)^2} = \frac{m^2}{4qE}f\left(\frac{\omega}{m}\right), \tag{3}$$

with $f(\chi) = \pi + 2\arcsin(1 - 2\chi) + 4\sqrt{\chi}(1 - 2\chi) \times \sqrt{1 - \chi}$, which can be approximated by $f(\chi) \approx 2\pi(1 - \chi)$ in the relevant interval $\chi \in [0, 1]$. In the limit $\chi = 0$, we exactly recover Schwinger's exponential factor $\exp\{-\pi E_S/E\}$ for the pair creation probability in a static field. For $\chi > 0$, however, the exponent is reduced. Intuitively speaking, the particles do not have to tunnel all the way from $-L/2$ to $+L/2$ because the frequency ω helps them to penetrate the strong field region up to $x_\pm$. In a dual picture, the particles tunnel through part of the gap, until they can be excited by frequency ω. At $\chi = 1/2$, we get exactly half the exponent, and hence the pair creation rate is drastically enhanced $\exp\{-\pi E_S/(2E)\}$. Note that, without the strong electric background field E, a single photon with $\omega = m/2$ would not have enough energy to create an electron-positron pair. Below threshold $\omega < m$, pair production requires multiphoton processes which occur at higher orders in the above expansion scheme. For $\chi = 1$, the exponent in Eq. (3) vanishes as expected, since the electromagnetic wiggles have enough energy for pair creation $\omega = m$.

The above approach based on the scenario in Fig. 1 has the advantage of allowing arbitrary wiggles $\boldsymbol{A}(t,\boldsymbol{r})$, but has the drawback that multiphoton processes require high-order calculations. Also, realistic backgrounds $E(t,x)$ are difficult to handle even though the tunneling exponent is expected to be universal for slowly varying fields. Time-dependent backgrounds as well as multiphoton physics can be dealt with by the worldline instanton technique [14] which we apply to the following specific example: we consider a strong and slow electric field pulse superimposed by a weak and fast pulse, both spatially homogeneous,

$$\boldsymbol{E}(t) = \frac{E}{\cosh^2(\Omega t)}\boldsymbol{e}_z + \frac{\varepsilon}{\cosh^2(\omega t)}\boldsymbol{e}_z \qquad (4)$$

with $E_S \gg E \gg \varepsilon > 0$ and $m \gg \omega \gg \Omega > 0$. With only one such pulse (say $\varepsilon = 0$), the corresponding pair creation rate can be computed analytically [10]. For the superimposed dual-pulse form in (4), we can compute the pair creation rate semiclassically using an analytic continuation to Euclidean time x_4 via

$$A_3(x_4) = -i\frac{E}{\Omega}\tan(\Omega x_4) - i\frac{\varepsilon}{\omega}\tan(\omega x_4), \qquad (5)$$

with the tunneling exponent being related to the action of the worldline instanton [17]. Starting with the worldline representation of the path integral, we may use the electron mass m as a large parameter (assuming $qE \ll m^2$ and $\omega \ll m$) for the saddle point approximation. The saddle points corresponding to the tunneling events are worldline instantons $x_\mu = [0, 0, x_3(\lambda), x_4(\lambda)]$ which are closed loops in Euclidean space-time satisfying the equation of motion

$$\left(\frac{dx_4}{d\lambda}\right)^2 + q^2\left(\frac{E\tan(\Omega x_4)}{m\Omega} + \frac{\varepsilon\tan(\omega x_4)}{m\omega}\right)^2 = 1, \qquad (6)$$

where $d\lambda^2 = dx_3^2 + dx_4^2$ is the proper time. This equation describes the classical motion of a particle in a potential. For small ε, the second term $\tan(\omega x_4)$ acts as an infinitely high rectangular well potential and just reflects instanton trajectories $x_4(\lambda)$ at the walls $\omega x_4 = \pm\pi/2$. Between the walls, we have an approximately harmonic oscillation due to $\Omega \ll \omega$ and thus $\tan(\Omega x_4) \approx \Omega x_4$. The structure of the solution $x_4(\lambda)$ depends on the combined Keldysh adiabaticity parameter

$$\gamma = \frac{m\omega}{qE}. \qquad (7)$$

Note that the relevant Keldysh parameter in this multiscale problem is formed out of the dominant frequency ω of the fast pulse on the one hand and the dominant field strength E of the slow pulse on the other hand. For small $\gamma \ll 1$, we approach the pure Schwinger limit, whereas large γ do *not* correspond to a pure multiphoton regime [5] as measured in the SLAC E-144 experiment [6]; large γ still involve both multiphotons of frequency ω as well as a nonperturbative dependence on E.

For small Keldysh parameters $\gamma < \pi/2$, the instanton trajectories do not reach the walls and reflection does not occur; i.e., the $\tan(\omega x_4)$ term has no impact. In this case, the weak pulse is too slow to create pairs significantly, and we essentially reproduce Schwinger's result. Beyond this threshold, $\gamma > \pi/2$, however, the instanton trajectories $x_4(\lambda)$ change due to reflection at the walls, and the instanton action becomes modified

$$\mathcal{A}_{\text{inst}} = m\oint d\lambda\left(\frac{dx_4}{d\lambda}\right)^2 = 4m\int_0^{\lambda_*} d\lambda\left(\frac{dx_4}{d\lambda}\right)^2 \qquad (8)$$

with $\lambda_* = (m/[qE])\arcsin(\pi/[2\gamma])$ being the reflection points. Consequently, we obtain (for $\gamma \geq \pi/2$)

$$\mathcal{A}_{\text{inst}} \approx \frac{m^2}{qE}\left[2\arcsin\left(\frac{\pi}{2\gamma}\right) + \frac{\pi}{2\gamma^2}\sqrt{4\gamma^2 - \pi^2}\right]. \qquad (9)$$

At the threshold, $\gamma = \pi/2$, we reproduce the Schwinger value $\mathcal{A}_{\text{inst}} = \pi m^2/(qE) \gg 1$, as one would expect. Above the threshold, $\gamma > \pi/2$, the instanton action $\mathcal{A}_{\text{inst}}$ decreases significantly. For example, for $\gamma = \pi$, it is reduced by about 40%. For $\gamma \to \infty$, it decays as $1/\gamma$ in agreement with the expected multiphoton behavior [5]. For larger ε, the threshold behavior becomes smoother; see Fig. 2. Since the pair creation probability, determined by the imaginary part of the effective action $\Gamma[A_\mu] = -i\ln\langle\text{in}|\text{out}\rangle$, depends exponentially on the instanton action, i.e., the saddle point value,

$$\text{Im}(\Gamma[A_\mu]) \sim \exp\{-\mathcal{A}_{\text{inst}}\} \lll 1, \qquad (10)$$

such a reduction of $\mathcal{A}_{\text{inst}}$ implies a drastic enhancement of the pair creation probability $\text{Im}(\Gamma[A_\mu])$; e.g., a reduction of 40% in the exponent could make the difference between a suppression of 10^{-10} and 10^{-6}, which could mean a few electron-positron pairs per day, instead of one per year. Of course, one could also reduce $\mathcal{A}_{\text{inst}}$ by a factor of 2 via doubling the field E. However, such strong fields are at the edge of present experimental capabilities and focusing two ultrahigh intensity pulses into the same space-time region is much harder than superimposing the strong pulse and a

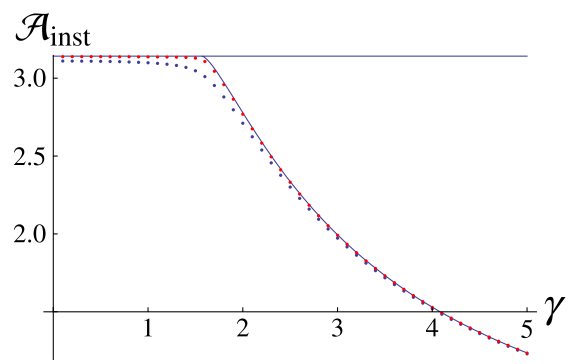

FIG. 2 (color online). Plots of the instanton action [in units of $m^2/(qE)$] for the electric field in (4), computed using the worldline instanton method, and plotted as a function of the combined Keldysh parameter γ defined in (7). The upper (red) dots correspond to $\omega = 100\ \Omega$ and $E = 100\varepsilon$, while the lower (blue) dots correspond to $\omega = 10\ \Omega$ and $E = 10\varepsilon$. The solid lines show the Schwinger value of π, estimated in the text to be valid for $\gamma < \pi/2$, and the expression (9), estimated in the text to be valid for $\gamma > \pi/2$. The numerical results agree very well with these estimates in the relevant limit where $E \gg \varepsilon$ and $\omega \gg \Omega$.

weak high-frequency pulse. Similarly, increasing the characteristic frequency of the ultrahigh intensity pulse is much harder than just adding a weaker field of high frequency. In fact, the envisioned generation mechanism (high-harmonic focusing) for the ultrahigh intensity pulses typically induces a high-frequency tail automatically (see below).

Because of the rather general nature of the arguments used above, they are not restricted to the specific pulse in Eq. (4); similar results are obtained for other localized pulse shapes [e.g., $(\omega x_4)^{2m+1}/(1-[\omega x_4]^{2n})^l$ with $l, n, m \in \mathbb{N}$ instead of $\tan(\omega x_4)$] which might capture more features of a realistic laser pulse. In order to understand the general result, it might be helpful to consider the following picture: The strong and slow pulse deforms the fermionic levels almost adiabatically. As a result, the expectation value of the *free* electron and positron number operator (valid for $A_\mu = 0$) would scale with E^2/m^4 (plus adiabatic corrections such as $\dot{E}/m^2$) and thus give a rather large result. However, this large number does not count *real* electrons or positrons (e.g., they do not annihilate) but mostly the instantaneous deformation of the ground state. The number of *real* electron-positron pairs, left over after the pulse, scales exponentially with $\exp[-O(1/E)]$, i.e., nonperturbatively in E [18]. On the other hand, the temporary deformation of levels during the strong pulse can be exploited by the small wiggles, which can turn the deformed ‘‘virtual’’ pairs into real electron-positron pairs.

The dramatic enhancement of the exponentially small pair creation probability should be relevant for the present experimental efforts aimed at the generation of light sources approaching the Schwinger limit [15]. One of the main envisioned mechanisms for the final amplification stage is coherent high-harmonic focusing: Let us imagine sending a laser pulse of ultrahigh intensity onto a curved metal surface. For a very high intensity, the Keldysh parameter of the laser is very small and hence electrons in the metal start to oscillate coherently and ultrarelativistically. Thereby, they effectively form a relativistically flying mirror, which reflects the incident light with a large Doppler shift and thereby generates high harmonics up to large order n. According to [19], the spectrum of these high harmonics is universal and the intensity of the nth harmonic scales with $n^{-8/3}$ up to some cutoff, which is proportional to the third power of the Lorentz boost factor of the mirror and thus depends on the incident laser intensity.

For example, an incident optical laser intensity of order 10^{22} W/cm^2 reachable in the near future would correspond to $n = O(10^5)$, which may range up to a significant fraction of the electron mass [20]. Finally, the curvature of the metal surface allows us to focus the high harmonics into a small space-time region, such that the spatially and temporally compressed intensity might approach the Schwinger limit. In this scenario, the highest harmonics (near the cutoff) will not contribute to the peak intensity significantly—but they still can enhance the pair creation probability drastically (compared to what one would expect from the Schwinger mechanism alone).

R. S. and H. G. acknowledge support by the DFG under Grants No. SCHU 1557/1-2,3 (Emmy-Noether program), No. GI 328/5-1 (Heisenberg program), and the SFB-TR18. G. D. thanks the U.S. DOE for support through grant DE-FG02-92ER40716.

[1] P. A. M. Dirac, Proc. R. Soc. A **117**, 610 (1928); **118**, 351 (1928); **126**, 360 (1930).
[2] F. Sauter, Z. Phys. **69**, 742 (1931); **73**, 547 (1931).
[3] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936); V. Weisskopf, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. **XIV**, 6 (1936).
[4] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[5] E. Brézin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970).
[6] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[7] A. Ringwald, Phys. Lett. B **510**, 107 (2001); arXiv:hep-ph/0112254.
[8] W. Becker *et al.*, Adv. At. Mol. Opt. Phys. **48**, 35 (2002); T. Pfeifer, C. Spielmann, and G. Gerber, Rep. Prog. Phys. **69**, 443 (2006).
[9] A. I. Nikishov and V. I. Ritus, Sov. Phys. JETP **25**, 1135 (1967); F. V. Bunkin and I. I. Tugov, Sov. Phys. Dokl. **14**, 678 (1970); N. B. Narozhny, S. S. Bulanov, V. D. Mur, and V. S. Popov, Phys. Lett. A **330**, 1 (2004); JETP Lett. **80**, 382 (2004); S. P. Kim and D. N. Page, Phys. Rev. D **65**, 105002 (2002); **75**, 045013 (2007).
[10] N. B. Narozhnyi and A. I. Nikishov, Sov. J. Nucl. Phys. **11**, 596 (1970).
[11] V. S. Popov, JETP Lett. **13**, 185 (1971); Sov. Phys. JETP **34**, 709 (1972); JETP Lett. **18**, 255 (1973); Sov. J. Nucl. Phys. **19**, 584 (1974).
[12] H. Gies and K. Klingmuller, Phys. Rev. D **72**, 065001 (2005).
[13] L. V. Keldysh, Sov. Phys. JETP **20**, 1307 (1965).
[14] G. V. Dunne and C. Schubert, Phys. Rev. D **72**, 105004 (2005); G. V. Dunne *et al.*, *ibid.* **73**, 065028 (2006).
[15] See, e.g., the European ELI programme: http://www.extreme-light-infrastructure.eu/
[16] O. Klein, Z. Phys. **53**, 157 (1929).
[17] Since the leading rate is independent of the spin degree of freedom for $E = E(t)$ [14], the worldline spin factor will be ignored in the following. It may become important for inhomogeneous fields in space and time. Also the analogy to a particle in a potential in Eq. (6) works nicely for one-dimensional cases such as $E = E(t)$ only.
[18] F. Hebenstreit, R. Alkofer, and H. Gies, Phys. Rev. D **78**, 061701 (2008).
[19] S. Gordienko, A. Pukhov, O. Shorokhov, and T. Baeva, Phys. Rev. Lett. **93**, 115002 (2004).
[20] For example, for an (unfocused) intensity of 10^{22} W/cm^2, the threshold $\gamma = \pi/2$ is reached for $\omega = 120$ eV, corresponding to $n = 48$ for an optical laser at 2.5 eV; whereas a focused intensity of 10^{27} W/cm^2 requires $\omega \approx 38$ keV for reaching $\gamma = \pi/2$, which corresponds to $n = 1.5 \times 10^4$.