Crumpling under an Ambient Pressure

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A pressure chamber is designed to study the crumpling process under an ambient force. The compression force and its resulting radius for the ball obey a power law with an exponent that is independent of the thickness and initial size of the sheet. However, the exponent is found to be material dependent and less than the universal value, 0.25, claimed by the previous simulations. The power-law behavior disappears at high pressure when the compressibility drops discontinuously, which is suggestive of a locked state.

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Crumpling is a common experience, be it on an aluminum can, a letter that fails our expectation, car wreckage from an accident, or the creation of mountains and canyons in plate tectonics. The energy focusing phenomenon into a two-dimensional sheet has attracted the interest of scientists since the last decade [1–8]. Furthermore, the origin of the extraordinary resistance of a crumpled structure is still an open question. For instance, how can a piece of crumpled paper offer such a large resistance when about 70% of its interior is still filled with air? This volume fraction is amazingly small when compared to the well-known disordered system, random closed packing. However, a crumpled structure is almost impossible to model since it involves not only the complicated collective behavior, but also the non-Markovian process caused by long-range correlations through the sheet, self avoidance, and the plasticity of the material. In addition to the intricacy of the interior structure, the evolution of sphere radius R with the compression force F is one relation that has been studied extensively in both experiments and theories [9–13]. Numerical simulations [11] showed that, when the thickness is much smaller than the initial radius R_0, the self-avoidance of the thin sheet played an important role in the crumpling process and

$$R \propto R_0^{\nu} F^{-\alpha} \tag{1}$$

where (ν, α) are expected to be universal at values $(3/8, 3/4)$ and $(1/4, 4/5)$, respectively, for membranes without and with the consideration that they cannot cross themselves. These exponents are believed to be independent of the material and its thickness.

Early experiments focused on the geometry [1,2] and the fractal dimension of hand-crumpled paper sheets [12,14,15]. Although the authors have tried to increase their accuracies by averaging over many samples, the grasping force after all varies from person to person and is impossible to calibrate and be used to verify the relationship of $R(F)$. Furthermore, different fingers tend to adjust their hold in order to make the ball spherical. This is against the natural tendency of the ball if crumpled by ambient pressure, as exemplified in Fig. 1. So the anisotropic force and its effect accumulated throughout the compression are another uncontrolled factor in the hand-crumpled measurements. In this Letter, we report a more rigorous setup which makes use of compressed air to provide an ambient pressure up to 1.1 MPa. To our knowledge, this is the first design which is capable of offering a direct and systematic confirmation of Eq. (1) and the relaxation behavior for a crumpled elastic thin sheet in three dimensions.

We use both aluminum foils and high-density PE films (HDPE) as our sample. The foil is of size $R_0 = 6.5$ to 22.5 cm and includes four different thicknesses, $h = 16$, 33, 63, and 96 μm. Compared to common papers, aluminum foil can be much thinner and moisture-proof. A photograph of the experimental setup is shown in Fig. 2. Ambient pressure provides the isotropic compression in a high-pressure transparent chamber with dimension

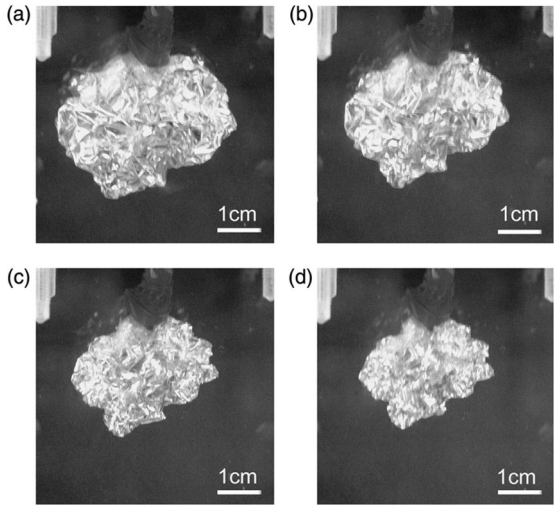

FIG. 1. The photo sequence of a 16 μm-thick aluminum foil crumpled by (a) 5.1, (b) 11.8, (c) 19.3, and (d) 32.5 kgw forces, respectively. They show that the ball has a tendency to deviate from a spherical shape and local structures intensify with further compressions.

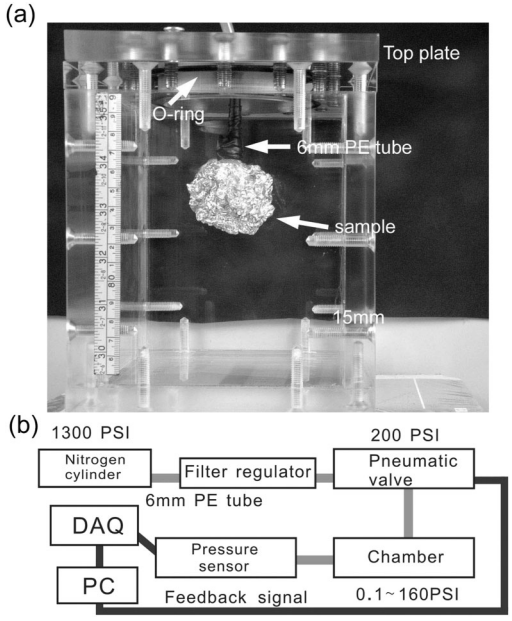

FIG. 2. (a) The sealed high-pressure chamber is made of acrylic plates of thickness 15 mm and its structure is enhanced by screws. An O-ring is placed between the top plate and the main body to keep the air from leaking. (b) The diagram of pressure control system. A regulator is placed in front of the pneumatic valve in order to protect the system from high pressure. The pressure in the chamber is well monitored by the pressure sensor, and a feedback signal is sent to the pneumatic valve through DAQ and PC.

10 cm $\times$ 10 cm $\times$ 15 cm. In order to compress the sample with high-pressure nitrogen, we use a piece of PVC wrap of thickness 11 μm to pack the sample and connect it to the outside of the chamber via a 6 mm-diameter PE tube. Since the size, thickness, and modulus of the PVC wrap are much smaller than those of the sample, we believe the work done on the wrap is insignificant compared with that on the sample. This was checked by adding more layers to the wrap, which did not affect our data.

The pressure applied on the crumpled material varies from 6.9×10^{-4} to 1.1 MPa. For instance, a 2.3 cm radius ball under 0.41 MPa pressure sustains about 280 kgw force which is about 10 times an average adult's strength of grasp. After precrumpling the thin sheet to a sphere by hand and putting it into the chamber for further compression, we use a CCD with resolution 2000 $\times$ 3008 pixels to monitor the size of the crumpled material.

The metastable states of crumpled material have been known for several years, and so it is important to have a protocol for the time point to measure its size during the compression. Two kinds of relaxation behavior have been reported [9,15]. Figure 3(a) shows that under a fixed pressure, the size of an isotropically crumpled aluminum foil decreases logarithmically in time for a period up to 10^4 seconds. Since the decreasing rate is much faster than that of the Mylar [9], it is permissible for us to use

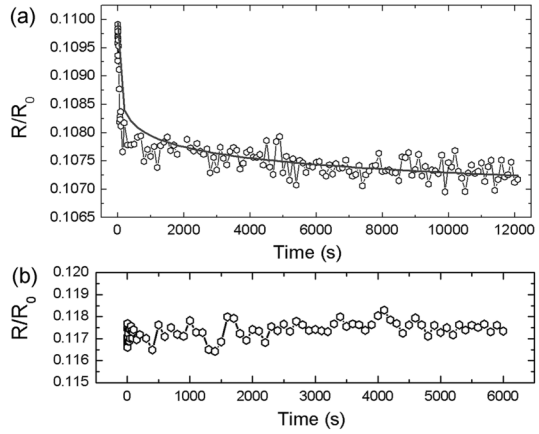

FIG. 3. (a) The compaction under 0.1 MPa is plotted against time. The solid line is fit with $R/R_0 = a - b\log(t)$, where (a, b) are determined to be (2.00, 0.005) in contrast to $\sim$(7.5, 0.15) in [11]. The value of our b being an order of magnitude smaller indicates a much faster relaxation rate. (b) Time evolution of the compaction when the same pressure is turned off. The lack of any significant swelling indicates that there is little elastic energy stored in the ball. The R_0 and h of the aluminum foils for both figures are 14.5 cm and 16 μm.

100 seconds (the same as in [9]) rather than weeks as our protocol for each measurement. However, the aluminum sheets did not exhibit the second relaxation, see Fig. 3(b); namely, it did not swell in size after we withdrew the pressure. Since we did not find the sheet springing open at any stage of unfolding it, we were sure that no elastic energy was stored. All the work done by the crumpling goes to folding the sheets and creating ridges and vertices.

Figure 1 already illustrates how the size and shape of the crumpled foil evolve under different pressure. It also shows that more local structures appear with increasing pressure. Since the ball is never a perfect sphere, we estimate its volume by measuring and averaging the cross sections from different angles. The compaction R/R_0 is plotted as a function of force in Fig. 4. Because the way to precrumple the sheet is somewhat arbitrary, it is not surprising to find deviations in this initial stage from the power law, which sets in after the compaction becomes smaller than 0.2. The exponent is found to be $\alpha = 0.195 \pm 0.008$, smaller than the universal value 0.25 predicted by the computer simulations [11]. The data were averaged from ten rounds of sampling under the same condition.

Although we strengthened our acrylic chamber by steel bars, we decided not to push the maximum pressure above 1.1 MPa to avoid explosion. At about 180 kgw (0.8 MPa), we already observe the jamming transition reported in [13], beyond which the power law ceases to exist and the compressibility suddenly drops discontinuously. This is reminiscent of the locking phenomenon in applied mechanics. However, the body fraction at this locked state surprisingly is only 7%, which translates to 93% of void inside the ball. We contrast this to the grasping force, which turns out to be

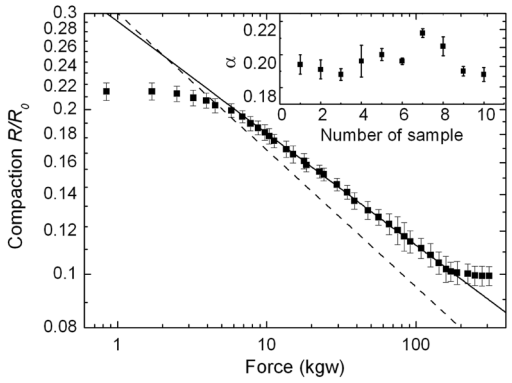

FIG. 4. The compaction R/R_0 is plotted against the applied force for a foil of $R_0 = 14.5$ cm and $h = 16$ μm. The solid line is a power-law fitting of the data, and the exponent is determined to be $\alpha = 0.195 \pm 0.008$. Although the magnitude of R/R_0 falls in the same range as the simulations by [11], the universal value $\alpha = 0.25$ deduced by the latter (dotted line) clearly lies outside of the error bars. Inset demonstrates the reproducibility of our data.

capable of compacting the ball to a void of 70% with a lesser force. We suspect it is due to the natural tendency for the ball to deviate from a sphere grows (see Fig. 1). Subconsciously, each finger then adjusts its hold or even rotates the ball to make it spherical at small compaction. This amounts to adding a fraction of unidirectional compression in selective points on top of an ambient pressure, which is more effective at compacting the ball and so can not be treated as a pure isotropic one.

We vary the thickness of the aluminum foil in Fig. 5. It shows that the exponent α is roughly independent of h, as concluded by [11]. It is hard to measure the locking point of thicker foils because their vertices will become too sharp for the wrap to withstand the puncture. To avoid unwanted

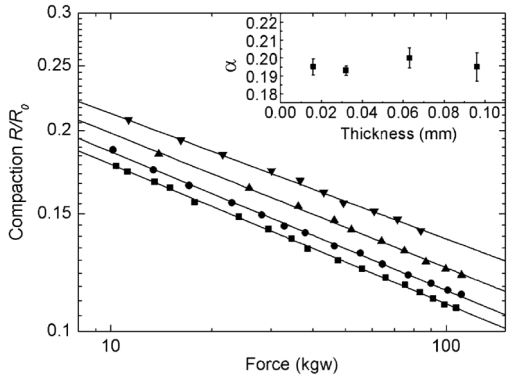

FIG. 5. Aluminum foils of $R_0 = 14.5$ cm and $h = 16$, 33, 63, and 96 μm are used to check whether the exponent α varies with thickness. The thicker the foil (the upper line), the harder to compress and a higher compaction is resulted under the same force. However, they share the same exponent, as shown in the inset.

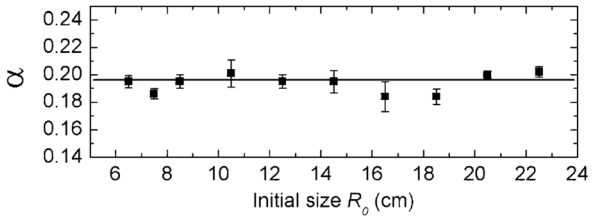

FIG. 6. The value of exponent α shows little dependence on the initial size, R_0, of the aluminum sheet with $h = 16$ μm. We also measure ν at about 0.83, which is close to the value 0.8 determined by computer simulations [7,11].

contribution to the resisting force, we are limited to the choice of more elastic or thicker wraps. We also checked the initial size of the foil. Figure 6 demonstrates that the power law persists and its exponent is indeed independent of R_0.

According to [11], the self-avoidance is the primary source of resistance against the crumpling force, which nature is shared by all elastic thin sheets, and so this value of exponent $\alpha = 0.25$ ought to be universal. To confirm that the deviation of α is not due to the plasticity of the aluminum foil, we repeat the measurement on HDPE. Figure 7 shows that, although the power law persists in HDPE, the value of $\alpha = 0.065 \pm 0.002$ is even smaller than 0.195 for the aluminum foil. This further deviation indicates that the nature of self-avoidance alone is not enough to determine the exponent of the power law. Different thicknesses of HDPE are tried, 33, 61, 68, and 79 μm. As in the case of aluminum foil, they do not affect the power law and its exponent. However, the critical force that leads to the locked state (see Fig. 7) does depend on the thickness as well as the material.

Figure 8 provides another interesting information on the evolution of inner structure during the compression. It is consistent with the intuition that the thin sheet begins by

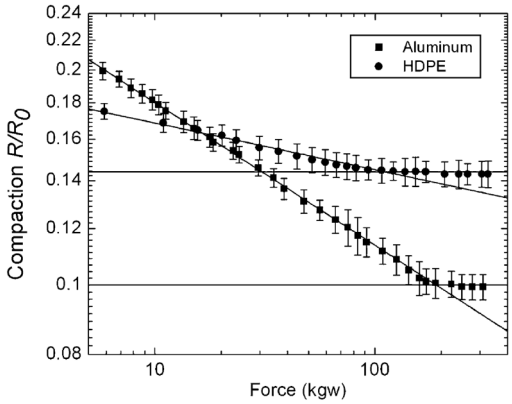

FIG. 7. The compaction-force relation for aluminum foil and HDPE with thickness 16 and 68 μm. Contradicting the prediction by [11] that the exponent is universal at value 0.25, they are measured to be 0.195 ± 0.008 and 0.065 ± 0.002, respectively.

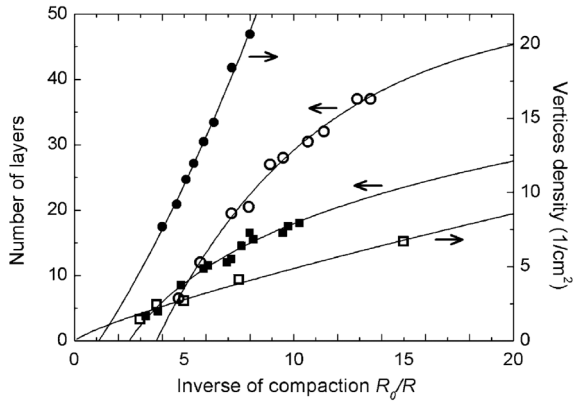

FIG. 8. The density of vertices and the averaged number of folded layers are plotted against the inverse of compaction for aluminum foil (solid dots) and HDPE (hollow dots) with the same $R_0 = 14.5$ cm and $h = 16$ μm. The layer number is an average determined by jabbing a long needle through the ball at ten different directions. Since the aluminum foil tends to shatter into pieces at small compaction when we try opening the ball to count the holes, the data were cross checked by the sound recording of each piercing of a layer makes.

folding itself freely in arbitrary directions to create new layers. When the compaction decreases, eventually there runs out of room for the layers to roam and fold on themselves. This is characterized by an decrease in the growth rate of the layer number. Further compression can only serve to buckle the existing layers and generate a surge in the ridges and vertices. What is interesting is that the power law is insensitive to this shifted nature of work until locking eventually kills it at high pressure.

It can be seen from Fig. 8 that if the sample contains more vertices, it has less number of layers, and vice versa. This property can be shown to be related by the fact that the total area must be conserved. When vertices consume a large part of the sheet due to the plasticity of the aluminum foil, the remaining area that creates layers is therefore reduced. This causes the average distance between neighboring layers in the aluminum foil wider than in HDPE under the same compaction. So the long and thin vertices in aluminum balls are less effective at supporting the structure than the short and thick ones in HDPE. This is why the magnitude of exponent is material-dependent and smaller for HDPE in Fig. 7. Note that this is opposite to their relative magnitude in the bulk form. From the point of view of energy, a thicker film requires more work for both elastic and plastic deformations. The fact that the exponent is independent of the thickness, as shown in Fig. 5, implies that these two works share the same enhancement by the thickness and it is their ratio that determines the compressibility in a crumpled structure.

In conclusion, a new experimental setup is proposed to study the three-dimensional isotropic crumpling. Our pressure chamber is capable of providing rigorous and reproducible reading of the compression force. The radius of the sample is found to vary with the compression force in a power-law fashion. And the exponent is independent of the thickness and initial size of the foil. These are consistent with the previous theory which assumed the self-avoidance of the sheet to be the dominant source of resisting force. However, the exponent is smaller than the predicted universal value of 0.25 and is material-dependent -0.195 and 0.065 for the aluminum foil and HDPE, respectively. The power-law behavior disappears at some critical pressure, magnitude of which depends on the material as well as its thickness. This locking transition is coincided by a discontinuous drop in compressibility, reminiscent of the jamming reported by [13].

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