Quantum Improvement of Time Transfer between Remote Clocks

Brahim Lamine,* Claude Fabre, and Nicolas Treps
Laboratoire Kastler Brossel, Université Pierre et Marie Curie-Paris 6, ENS, CNRS; 4 place Jussieu, 75252 Paris, France
(Received 7 April 2008; published 18 September 2008)

Exchanging light pulses to perform accurate space-time positioning is a paradigmatic issue of physics. It is ultimately limited by the quantum nature of light, which introduces fluctuations in the optical measurements and leads to the so-called standard quantum limit (SQL). We propose a new scheme combining homodyne detection and mode-locked femtosecond lasers that lead to a new SQL in time transfer, potentially reaching the yoctosecond range (10^{-21}–10^{-24} s). We demonstrate that this already very low SQL can be overcome using appropriately multimode squeezed light. Benefitting from the large number of photons and from the optimal choice of both the detection strategy and of the quantum resource, the proposed scheme represents a significant potential improvement in space-time positioning.

DOI: 10.1103/PhysRevLett.101.123601 PACS numbers: 42.50.Dv, 42.50.Lc, 42.62.Eh

Accurate spacetime positioning has become a crucial issue for future space experiments which require increasing resolution over large distances (see, for example, [1]). The position in space (by ranging to a reference) or time (by clock synchronization with a reference) between two observers A and B may be achieved through the Einstein protocol which consists in repeatedly exchanging light pulses [2].

The basic principle relies on the property that, in the absence of dispersion, each pulse carries along its propagation a mean light cone variable $u = t \pm x/c$ which remains constant so that the measurement of the time of arrival of each pulse allows either a determination of distance or clock synchronization. The generic situation considered in this Letter is the following (see Fig. 1): observer A regularly emits light pulses at a rate synchronized to its local clock; B receives these pulses and determines their times of arrival by measuring the difference between the arrival times of the incoming light pulses and light pulses delivered by a source located in B and synchronized to a reference clock in B. The accuracy of this measurement relies therefore on the precision of the clocks in A and B and on the sensitivity of the determination of the delay between two light pulses, that we will show how to optimize in the present Letter.

Such a delay can be measured by at least two ways: the first one consists in measuring the arrival time of the maximum of the pulse envelope. We will refer to this procedure as an incoherent time-of-flight (tof) measurement. The second method consists in using the information contained in the phase of the electric field oscillation by making an interference pattern between the pulses arriving from A and a local oscillator (LO) derived from the local clock in B. This pattern will give the desired information if the phase of the pulse coming from A and the phase of the LO in B are locked to their respective local clocks. This method will be referred to as a coherent phase (ph) measurement.

These measurement schemes suffer from quantum limits associated with the quantum nature of light [3]. For a coherent light pulse of central frequency ω_0 and frequency spread $\Delta\omega$, quantum fluctuations lead to the so-called standard quantum limit (SQL) of ranging for either time-of-flight [4] or phase [5,6] measurements. Those expressions are given by

$$(\Delta u)^{\mathrm{tof}}_{\mathrm{SQL}} = \frac{1}{2\Delta\omega\sqrt{N}}, \qquad (\Delta u)^{\mathrm{ph}}_{\mathrm{SQL}} = \frac{1}{2\omega_0\sqrt{N}}. \qquad (1)$$

Where N is the total number of photons measured in the

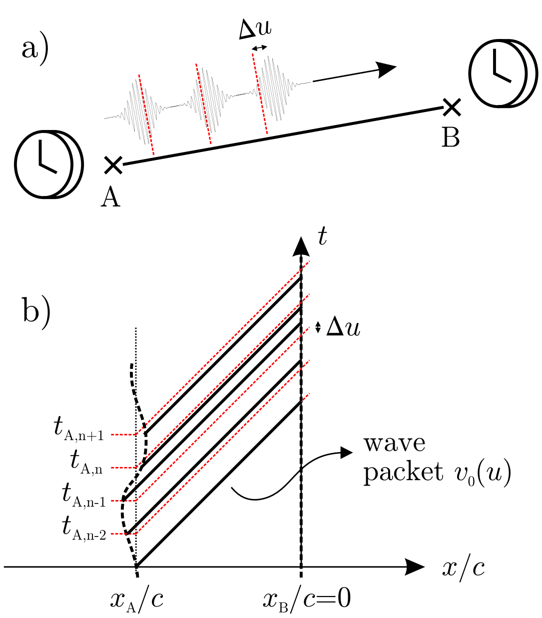

FIG. 1 (color online). (a) General scheme of a one way time transfer. (b) Spacetime representation in the reference frame of observer B ($x_B = 0$). A modification Δx_A of the position of the observer A leads to a modification $\Delta u = -\Delta x_A/c$ of the light cone variable that is emitted towards B and, consequently, leads to non regular time of arrival in B.

 © 2008 The American Physical Society

experiment during the detection time. Let us briefly discuss those two SQL. First, it is clear on these expressions that the SQL can be as small as needed if one can use intense enough light, but there are obvious practical limitations to the energy carried by the light pulses. In contrast, isolated photons give rise to very low photon fluxes, and the corresponding SQL is very quickly a limitation of experimental protocols using photon-counting techniques. The expressions also show that optical frequencies lead to much smaller SQL than microwave frequencies because of a larger ω_0 and $\Delta\omega$. Finally, as $\omega_0 > \Delta\omega$, the phase method has a better ultimate sensitivity than the time-of-flight technique but requires highly spatially and temporally coherent sources.

For the time being, the resolution in time transfer is limited by classical technical noises so that the previous SQL are not yet a limitation in time transfer. Nevertheless, with the recent developments in stabilization of frequency combs referenced to optical standard, it is getting closer and closer to these quantum limits [7,8]. Both for a fundamental point of view and for future experiments, it is therefore necessary to compute the ultimate sensitivity in time transfer with coherent pulses, situation epitomized by a mode-locked femtosecond laser. Indeed, it combines both a time-of-flight information in their envelope and a well-stabilized phase information inside the envelope.

In order to compute the SQL in timing, we write the positive frequency electric field operator $\hat{E}^{(+)}_{(0)}$ emitted by A in the absence of any perturbations as a decomposition in temporal modes:

$$\hat{E}^{(+)}_{(0)}(u) = \mathcal{E}\sum_n \hat{a}_n v_n(u), \qquad \mathcal{E} = i\sqrt{\frac{\hbar\omega_0}{2\varepsilon_0 cT}}, \tag{2}$$

where T is the measurement time. The orthonormal temporal modes $v_n(u)$ will be written as a (complex) time-varying amplitude $g_n(u)$ multiplied by a propagation phase factor of the form $e^{-i\omega_0 u}$:

$$v_n(u) = g_n(u)e^{-i\omega_0 u}. \tag{3}$$

The annihilation operator corresponding to those modes are noted $\hat{a}_n$. Without any loss of generality, we can appropriately choose the mode basis such that the mean value of the electric field operator $\hat{E}^{(+)}_{(0)}(u)$ is proportional to v_0, namely $\langle \hat{E}^{(+)}_{(0)}(u)\rangle = \mathcal{E}\sqrt{N}e^{i\theta}v_0(u)$, with N the mean number of photon and θ a global phase [9].

Now, any variation Δu of the mean light cone variable, caused, for example, by a distance change between A and B, leads to a modification of the field received in B which reads $\hat{E}^{(+)}(u) = \hat{E}^{(+)}_{(0)}(u - \Delta u)$ (see Fig. 1). The temporal mode corresponding to this field can be decomposed as follows if the perturbation Δu is small:

$$v_0(u - \Delta u) \approx v_0(u) - \Delta u \frac{dv_0(u)}{du}\bigg|_{u=0} = v_0(u) + \frac{\Delta u}{u_0} w_1(u). \tag{4}$$

The constant u_0 ensures the normalization of the new mode $w_1(u)$. The latter one will be called the timing mode because it carries the timing signal Δu. For pulses of frequency spread $\Delta\omega$ [10], u_0 is given by $u_0 = 1/\sqrt{\omega_0^2 + \Delta\omega^2}$ and the expression of the timing mode is

$$w_1(u) = \frac{1}{\sqrt{\alpha^2 + 1}}(i\alpha v_0(u) + v_1(u)), \qquad \alpha = \frac{\omega_0}{\Delta\omega}. \tag{5}$$

α is roughly equal to the number of field oscillations within the pulse, which can be as small as a few units for femtosecond pulses. The timing mode w_1 contains two terms: the first one, namely iv_0, gives a contribution to the timing signal via a phase change (interferometric method of ranging). The second one, namely v_1, is normalized and orthogonal to v_0 so that it will be taken as the second mode of the basis $(v_n)_n$. It reads

$$v_1(u) = -\frac{1}{\Delta\omega}\frac{dg_0(u)}{du}e^{-i\omega_0 u}. \tag{6}$$

This mode gives a contribution to the timing signal via a time shift of the pulse envelope (time-of-flight technique). The latter mode is represented in Fig. 2 and is the temporal analog of the spatial TEM_{01} Gaussian mode when the emitted pulses are Gaussian.

The timing signal Δu can be retrieved by projecting $v_0(u - \Delta u)$ on the timing mode $w_1(u)$. This can be done using the balanced homodyne detection scheme represented in Fig. 2 where the input pulses are mixed with a LO put in the timing mode w_1, so that $\langle \hat{E}^{+}_{\mathrm{LO}}(u)\rangle = \mathcal{E}\sqrt{N_{\mathrm{LO}}}e^{i\theta_{\mathrm{LO}}}w_1(u)$, with N_{LO} the mean number of photon in the LO field and θ_{LO} its phase. Denoting $(\hat{b}_n)_n$ the annihilation operators for the LO, the homodyne signal reads $\hat{D} = |\mathcal{E}|^2\sum_n(\hat{a}_n^\dagger\hat{b}_n + \hat{b}_n^\dagger\hat{a}_n)$. The mean signal of the balanced homodyne detection when a timing offset Δu is present, then reads

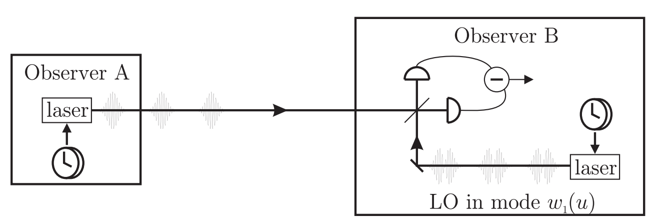

FIG. 2. Proposed balanced homodyne scheme to reach optimal detection in ranging measurement. The pulses synchronized on the clock in A are measured in B by homodyne detection with pulses synchronized on the local clock and in an adequate temporal mode (here is represented only the part v_1 of the LO for clarity).

$$\langle \hat{D} \rangle = 2|\mathcal{E}|^2\sqrt{NN_{\rm LO}}\left[\frac{\Delta u}{u_0}\cos(\theta - \theta_{\rm LO}) + \frac{\alpha}{\sqrt{\alpha^2+1}}\sin(\theta - \theta_{\rm LO})\right]. \quad (7)$$

We assume from now on that, as usual, the LO is much more intense than the input field. The general case can be treated without difficulty. In this situation, the variance of the balanced homodyne signal, taken for $\Delta u = 0$, is given by

$$\sigma_{\hat{D}}^2 \equiv \langle \delta \hat{D}^2 \rangle = \frac{|\mathcal{E}|^4 N_{\rm LO}}{1+\alpha^2}(\alpha^2\sigma_{\hat{P}_0}^2 + \sigma_{\hat{Q}_1}^2), \quad (8)$$

where $\sigma_{\hat{P}_0}^2$ and $\sigma_{\hat{Q}_1}^2$ are the variances of the quadrature operators $\hat{P}_0$ (phase operator of mode v_0) and $\hat{Q}_1$ (amplitude operator of mode v_1) of the input field

$$\hat{P}_0 = i(\hat{a}_0^\dagger e^{i\theta_{\rm LO}} - \hat{a}_0 e^{-i\theta_{\rm LO}}) \quad \text{and} \quad \hat{Q}_1 = \hat{a}_1^\dagger e^{i\theta_{\rm LO}} + \hat{a}_1 e^{-i\theta_{\rm LO}}. \quad (9)$$

The SQL is then obtained as the smallest Δu that can be measured using shot noise limited coherent light ($\sigma_{\hat{P}_0}^2 = \sigma_{\hat{Q}_1}^2 = 1$), assuming a signal to noise ratio equal to one ($\langle \hat{D} \rangle = \sigma_{\hat{D}}$). It is obtained for $\theta = \theta_{\rm LO}$ and is given by

$$(\Delta u)_{\rm SQL} = \frac{1}{2\sqrt{N}\sqrt{\omega_0^2 + \Delta\omega^2}}. \quad (10)$$

The expression (10) is one of the main results of this Letter and gives a new SQL in timing. The latter is lower than both the SQL in time-of-flight and phase measurements [see Eq. (1)], which obviously are special cases of our scheme when the LO is either in the iv_0 or v_1 mode. This means that the proposed balanced homodyne detection scheme has a better sensitivity than existing schemes based on either time-of-flight or interferometric measurement. The improvement comes from the fact that coherent pulses, in addition to their phase, carries a time-of-flight information in their time-varying envelope. Both pieces of information are read by the balanced homodyne detection if the LO is shaped in the mode w_1. Note that this shape can be obtained experimentally with presently available commercial pulse shaper [11]. Let us stress that such optimized measurements have already been successfully proposed and numerically tested for pure phase measurement [12] and even experimentally employed in the spatial domain to measure transverse beam displacement and tilt [13].

A natural question is to know whether it is possible to reach still better sensitivity on the same beam but by using another measurement strategy. An answer can be provided in the context of information theory with the help of the Cramer-Rao bound [14], which gives the smallest measurable delay Δu that can be achieved in the presence of a given distribution of noise. This bound has the property of being independent of the measurement strategy and depends only on the noise of the incoming signal. A calculation of the Cramer-Rao bound, analogous to the one detailed in [15,16] proves that using coherent light, this bound is precisely equal to the expression (10) of $(\Delta u)_{\rm SQL}$. We are therefore sure that no other measurement scheme will reach a better accuracy than the introduced balanced homodyne detection and in this sense this scheme is said to be efficient.

Obviously the SQL (10) is the fundamental limit when one restricts oneself to the use of classical states of light and coherent states, as proven with the previous standard Cramer-Rao bound. Nevertheless, it is well known that it can be beaten using quantum resources [17–19]. For example, the improvement of the sensitivity in interferometric measurements using squeezed light has been proposed [5,6], observed experimentally [20–22], and will be certainly practically implemented in the future generations of interferometric detectors of gravitational waves [23]. The use of an entangled photon source to improve time-of-flight ranging measurements in the photon-counting regime has been also proposed [4,24] and experimentally demonstrated [25] at a picosecond level of timing sensitivity. We propose here to improve the scheme introduced previously by using appropriately squeezed light.

Inspection of Eq. (8) immediately shows that in the case of a strong LO the signal to noise ratio is increased if the noise of the incoming mode w_1 is below the shot noise. This can be obtained if squeezing of the input field modes v_0 and v_1 is achieved along the quadratures $\hat{P}_0$ and $\hat{Q}_1$, respectively. If we assume that the squeezing coefficient is equal for the two states, namely $\sigma_{\hat{P}_0} = \sigma_{\hat{Q}_1} = e^{-r}$ ($r \geq 1$ being the squeezing parameter), then the new minimum measurable value of Δu is given by

$$(\Delta u)_{\rm squeezing} = \frac{1}{2\sqrt{N}\sqrt{\omega_0^2 + \Delta\omega^2}} e^{-r}. \quad (11)$$

This minimum resolvable Δu is thus reduced below the SQL (10) by the factor e^r. Note that the expression for the general case of different squeezing along $\hat{P}_0$ and $\hat{Q}_1$, as well as a LO not supposed strong, can be obtained straightforwardly from the equations given in the Letter.

Improving the signal to noise ratio with nonclassical light therefore requires to generate an input beam with the proper squeezed quadratures: the phase of the mean field $v_0$and the amplitude of the v_1 vacuum mode. Such a multimode beam has already been performed in the spatial domain [13], and similar techniques could, in principle, be implemented in the time domain. For instance, one could use a synchronously pumped optical parametric oscillator (SPOPO) which naturally produces the desired squeezed temporal modes with a femtosecond pump beam [26]. Experimentally, squeezing of pulses have already been performed either with the Kerr medium [27] or using parametric down-conversion and mode-locked lasers [28,29]. Using the best present technology, the noise reduction factor can reach 10 dB [30,31], i.e., a factor of 10 improvement, even at low noise frequencies. The advantage of squeezing over the other proposed quantum techniques such as entanglement is that it can be used together with an intense beam for which the SQL is already very low. In addition the squeezed beam travels along with the signal beam and therefore they both share common noises. The main drawback of squeezing is its sensitivity to losses in the optical system and the detectors. This means that the technique could be used in situations where light propagates in vacuum, for example, between satellites in flying formation.

An experimental implementation of the scheme with the aim at reaching the SQL and then observe the quantum improvement suffers different technical challenges. Indeed, for a $P = 10$ mW mode-locked laser with $\lambda \simeq 810$ nm and a 10 fs pulse duration, the SQL is equal to $(\Delta u)_{\rm SQL} = 2 \times 10^{-23}$ s, i.e., a noise level of 2×10^{-23} s$/\sqrt{\rm Hz}$ (20 yoctoseconds for 1 s integration time). Reaching such a timing precision requires very stable laser repetition rate and phase stabilization. This can be eventually achieved with mode-locked femtosecond lasers which are already used for absolute and relative ranging in different measurement schemes [32–35]. The dominant source of noise in Eq. (8) is given by the noise $\sigma_{\hat{P}_0}$ of the phase of v_0. Self-referencing stabilization using a $f - 2f$ beat allows to keep this noise to a very low level, down to 3×10^{-6} rad$/\sqrt{\rm Hz}$ at 10^5 Hz with state-of-the-art stabilization techniques [36,37], corresponding to a timing noise of 10^{-21} s$/\sqrt{\rm Hz}$ at 10^5 Hz. Concerning the repetition rate $T_{\rm rep}$, the latter can be locked to an optical reference, and current technology leads to a time jitter noise level of 10^{-18} s$/\sqrt{\rm Hz}$ at 10^5 Hz [38–40].

We are grateful to Serge Reynaud and Vincent Delaubert for fruitful discussions. Laboratoire Kastler Brossel is unité mixte de recherche (UMR) No. 8552 of the CNRS.

*brahim.lamine@spectro.jussieu.fr

[1] P. Bender, J. Hall, J. Ye, and W. Klipstein, Space Sci. Rev. **108**, 377 (2003).
[2] M.-T. Jaekel and S. Reynaud, Phys. Lett. A **220**, 10 (1996).
[3] M. Jaekel and S. Reynaud, Phys. Rev. Lett. **76**, 2407 (1996).
[4] V. Giovannetti, S. Lloyd, and L. Maccone, Nature (London) **412**, 417 (2001).
[5] C. Caves, Phys. Rev. D **23**, 1693 (1981).
[6] M.-T. Jaekel and S. Reynaud, Europhys. Lett. **13**, 301 (1990).
[7] L.-S. Ma *et al.*, Science **303**, 1843 (2004).
[8] A. Ludlow *et al.*, Science **319**, 1805 (2008).
[9] N. Treps, V. Delaubert, A. Maître, J. Courty, and C. Fabre, Phys. Rev. A **71**, 013820 (2005).
[10] We use the standard definition of the statistical frequency width:

$$\Delta\omega^2 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 |\tilde{g}_0[\omega]|^2,$$

where $\tilde{g}_0[\omega]$ is the Fourier transform of the envelope $g_0(u)$.
[11] M. Sato *et al.*, Jpn. J. Appl. Phys. **41**, 3704 (2002).
[12] D. Berry and H. Wiseman, Phys. Rev. A **65**, 043803 (2002).
[13] V. Delaubert, N. Treps, C. Harb, P. Lam, and H.-A. Bachor, Opt. Lett. **31**, 1537 (2006).
[14] P. Réfrégier, *Noise Theory and Application to Physics* (Springer, New York, 2004).
[15] V. Delaubert, Ph.D. thesis, Université Pierre et Marie Curie, 2007.
[16] V. Delaubert, N. Treps, C. Fabre, H. Bachor, and P. Réfrégier, Europhys. Lett. **81**, 44001 (2008).
[17] V. Giovannetti, S. Lloyd, and L. Maccone, Science **306**, 1330 (2004).
[18] R. Jozsa, D. Abrams, J. Dowling, and C. Williams, Phys. Rev. Lett. **85**, 2010 (2000).
[19] M. de Burgh and S. Bartlett, Phys. Rev. A **72**, 042301 (2005).
[20] M. Xiao, L.-A. Wu, and H. Kimble, Phys. Rev. Lett. **59**, 278 (1987).
[21] P. Grangier, R. Slusher, B. Yurke, and A. LaPorta, Phys. Rev. Lett. **59**, 2153 (1987).
[22] S. Barnett, C. Fabre, and A. Maître, Eur. Phys. J. D **22**, 513 (2003).
[23] K. McKenzie, D. Shaddock, D. McClelland, B. Buchler, and P. Lam, Phys. Rev. Lett. **88**, 231102 (2002).
[24] V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. A **65**, 022309 (2002).
[25] A. Valencia, G. Scarcelli, and Y. Shih, Appl. Phys. Lett. **85**, 2655 (2004).
[26] G. J. de Valcárcel, G. Patera, N. Treps, and C. Fabre, Phys. Rev. A **74**, 061801(R) (2006).
[27] S. Schmitt, J. Ficker, M. Wolff, F. König, A. Sizmann, and G. Leuchs, Phys. Rev. Lett. **81**, 2446 (1998).
[28] R. Slusher, P. Grangier, A. LaPorta, B. Yurke, and M. J. Potasek, Phys. Rev. Lett. **59**, 2566 (1987).
[29] M. Rosenbluh and R. Shelby, Phys. Rev. Lett. **66**, 153 (1991).
[30] H. Vahlbruch *et al.*, Phys. Rev. Lett. **100**, 033602 (2008).
[31] Y. Takeno, M. Yukawa, H. Yonezawa, and A. Furusawa, Opt. Express **15**, 4321 (2007).
[32] J. Ye, Opt. Lett. **29**, 1153 (2004).
[33] K. Minoshima and H. Matsumoto, Appl. Opt. **39**, 5512 (2000).
[34] C. Towers *et al.*, Opt. Lett. **29**, 2722 (2004).
[35] K.-N. Joo and S.-W. Kim, Opt. Express **14**, 5954 (2006).
[36] T. Fortier *et al.*, Opt. Lett. **27**, 1436 (2002).
[37] A. Bartels, C. Oates, L. Hollberg, and S. Diddams, Opt. Lett. **29**, 1081 (2004).
[38] A. Bartels, S. Diddams, T. Ramond, and L. Hollberg, Opt. Lett. **28**, 663 (2003).
[39] R. Shelton *et al.*, Opt. Lett. **27**, 312 (2002).
[40] T. Schibli *et al.*, Opt. Lett. **28**, 947 (2003).