Probing Noise in Flux Qubits via Macroscopic Resonant Tunneling

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(Received 13 December 2007; published 10 September 2008)

Macroscopic resonant tunneling between the two lowest lying states of a bistable rf SQUID is used to characterize noise in a flux qubit. Measurements of the incoherent decay rate as a function of flux bias revealed a Gaussian-shaped profile that is not peaked at the resonance point but is shifted to a bias at which the initial well is higher than the target well. The rms amplitude of the noise, which is proportional to the dephasing rate $1/\tau_\varphi$, was observed to be weakly dependent on temperature below 70 mK. Analysis of these results indicates that the dominant source of low energy flux noise in this device is a quantum mechanical environment in thermal equilibrium.

DOI: 10.1103/PhysRevLett.101.117003 PACS numbers: 85.25.Dq, 03.67.Lx

The viability of any scalable quantum computing architecture is highly dependent upon its performance in the presence of noise. In the case of superconducting qubits, it has been shown that low frequency ($1/f$) flux noise is of particular concern [1]. Furthermore, evidence suggests that such devices generically couple to an ensemble of effective 2-level systems (TLSs) that may be materials defects [2]. A number of theories exist that attempt to correlate these two observations [3]; however, it is not certain whether the TLSs observed in spectroscopy experiments are *the* dominant source of low frequency noise in these devices [4]. The development of additional experimental probes of noise will prove critical in the quest to build reliable superconductor-based quantum computing hardware. In this Letter, we demonstrate a new experimental procedure for quantifying the broadband integrated flux noise in rf SQUID qubits. The procedure developed herein complements other approaches to using qubits as spectrometers for studying noise [5].

Macroscopic resonant tunneling (MRT) [6,7] is an important probe of quantum effects in Josephson junction-based devices. In an MRT experiment, flux tunnels between two wells of a double well potential when energy levels are aligned. The tunneling rate and width of the tunneling region are strongly influenced by the environment. The effect of flux noise on the two lowest energy levels can be described using an effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\tfrac{1}{2}[\epsilon\sigma_z + \Delta\sigma_x] - \tfrac{1}{2}Q_z\sigma_z, \tag{1}$$

where ϵ is the bias energy between the wells, Δ is the tunneling energy, Q_z is an operator that acts on the low energy modes of the environment, and $\sigma_{x(z)}$ are Pauli matrices. In general, there will also be transverse coupling to the environment, but it is believed to be subdominant to longitudinal coupling in flux qubits [8].

A theory of MRT for small Δ in the presence of low energy (non-Markovian) flux noise was reported in Ref. [9]. The transition rate Γ_{01} from state $|0\rangle$ to state $|1\rangle$ (eigenfunctions of σ_z) was found to be ($\hbar = k_B \equiv 1$)

$$\Gamma_{01}(\epsilon) = \sqrt{\frac{\pi}{8}}\frac{\Delta^2}{W}\exp\left[-\frac{(\epsilon-\epsilon_p)^2}{2W^2}\right],$$
$$\epsilon_p = \mathcal{P}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{S(\omega)}{\omega}, \quad W = \left[\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}S(\omega)\right]^{1/2}, \tag{2}$$

where $S(\omega) = \int dt e^{i\omega t}\langle Q_z(t)Q_z(0)\rangle$ is an unsymmetrized spectral density [10]. Γ_{10} is obtained by the substitution $\epsilon_p \to -\epsilon_p$. Equation (2) is valid if $S(\omega)$ is peaked at low ω and the integrals defining ϵ_p and W are finite [9]. The latter constraint may require low and high ω cutoffs. The quantities ϵ_p and W represent the energy shift and width, respectively, of a Gaussian-shaped peak. For a classical environment $S(\omega) \approx S(-\omega)$, and hence $\epsilon_p = 0$. For a quantum environment, $S(\omega)$ need not be symmetric and $\epsilon_p \neq 0$. In thermal equilibrium at temperature $T_{\rm eff}$, the fluctuation-dissipation theorem states that $S(\omega)$ can be written as a sum of symmetric and antisymmetric components $S(\omega) = S_s(\omega) + S_a(\omega)$, where $S_s(\omega) = S_a(\omega)\times\coth(\omega/2T_{\rm eff})$. Consequently,

$$W^2 \approx \int_{-\infty}^{\infty} d\omega S_a(\omega)\frac{2T_{\rm eff}}{\omega} = 2T_{\rm eff}\epsilon_p. \tag{3}$$

Note that Eqs. (2) and (3) satisfy the Einstein relation between the downward and upward transition rates: $\Gamma_{01}/\Gamma_{10} = e^{\epsilon/T_{\rm eff}}$.

The width W is an important parameter in adiabatic quantum computation [11] as it defines the precision to which a Hamiltonian can be specified in the logical basis defined by the eigenstates of σ_z. From the perspective of gate model quantum computation, W is related to the dephasing time τ_φ in the energy basis defined by the eigenstates of Eq. (1). For low frequency noise, the decay due to dephasing has the form $e^{-t^2/2\tau_\varphi}$, where $1/\tau_\varphi = W|\epsilon|/\sqrt{\epsilon^2 + \Delta^2}$.

To demonstrate the MRT method for characterizing flux noise, we employed an rf SQUID qubit [12,13]. Previous experimental observations of MRT in rf SQUIDs have been limited to tunneling into higher energy levels with and without the help of microwave activation [7,14]. While the theory of Ref. [9] predicts that higher order MRT peaks will be shifted by noise, this effect would have been difficult to resolve in the experiments of Refs. [7,14] in which it was practical to observe tunneling in only one direction for a given qubit bias. In this Letter, we focus upon MRT between the two lowest energy levels of an rf SQUID as one can readily measure both directional tunneling rates Γ_{01} and Γ_{10} as a function of bias and therefore unambiguously identify ϵ_p.

A schematic of a compound Josephson junction (CJJ) rf SQUID is shown in the inset in Fig. 1(a). It consists of a main loop and a CJJ loop subjected to external flux biases Φ_x^q and $\Phi_x^{\rm cjj}$, respectively. The CJJ loop is interrupted by two nominally identical Josephson junctions connected in parallel with total capacitance C and critical current I_c. The CJJ and main loop possess inductances $L^{\rm cjj}$ and L, respectively. If $L^{\rm cjj} \ll L$, then the rf SQUID Hamiltonian can be written as

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} + U(\Phi),$$
$$U(\Phi) = \frac{(\Phi - \Phi_x^q)^2}{2L} - E_J \cos\left[\frac{\pi\Phi_x^{\rm cjj}}{\Phi_0}\right]\cos\left[\frac{2\pi\Phi}{\Phi_0}\right], \quad (4)$$

where Φ represents the total flux threading the main loop, Q is the charge stored in the capacitance, $E_J \equiv \Phi_0 I_c/2\pi$, and $\Phi_0 = h/2e$ is the flux quantum. This device can be operated as a qubit for $\Phi_x^{\rm cjj} \in [0.5, 1]\Phi_0$ and $\Phi_x^q \approx 0$. Denoting the ground and the first excited state of $\mathcal{H}$ at $\Phi_x^q = 0$ by $|g\rangle$ and $|e\rangle$, respectively, the qubit states can be expressed as $|0\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ and $|1\rangle = (|g\rangle - |e\rangle)/\sqrt{2}$. The bias energy of Eq. (1) is given by $\epsilon = 2|I_p|\Phi_x^q$, where the persistent current $|I_p| \equiv |\langle 0|\Phi/L|0\rangle| = |\langle 1|\Phi/L|1\rangle|$. The tunneling energy of Eq. (1) is given by $\Delta = \langle e|\mathcal{H}|e\rangle - \langle g|\mathcal{H}|g\rangle$. Both $|I_p|$ and Δ are controlled by $\Phi_x^{\rm cjj}$. Maximum $\Delta \sim \omega_p$, where ω_p is the plasma energy of the rf SQUID, is obtained at $\Phi_x^{\rm cjj} = \Phi_0/2$. For $\Phi_x^{\rm cjj} \approx \Phi_0$, one expects $\Delta \to 0$, and the system becomes localized in $|0\rangle$ or $|1\rangle$. In this latter regime, $|I_p|$ generates an amount of flux that can be resolved by an inductively coupled dc SQUID (not shown) as described in Refs. [15,16].

The device from which the data presented herein were obtained was fabricated on an oxidized Si wafer with $Nb/Al/Al_2O_3/Nb$ trilayer junctions and multiple Nb layers separated by sputtered SiO_2. The qubit comprised a parallel plate transmission line with a CJJ and a short placed on opposing ends. Sections of the transmission line were broken out to form transformers for coupling sources of flux. The two Nb wiring layers used to form the qubit were 100 and 300 nm thick with an intervening 200 nm thick layer of SiO_2. Qubit wiring was 1.4 μm wide for the bulk of the structure, and the total length of the wiring was ~3.2 mm. The qubit body was located above a 200 nm thick SiO_2 layer atop the substrate and below a 450 nm thick layer of SiO_2 capped with a Nb ground plane. The device parameters were $L = 661 \pm 6$ pH, $C = 146 \pm 3$ fF, and $I_c = 1.95 \pm 0.05$ μA. L was measured using a breakout structure, and C was inferred from L and measurements of the MRT peak spacing [7]. From the MRT spacing, we also determined $\omega_p \sim 0.5$ K. The dc SQUID was observed to have a maximum switching current $I_{\rm sw}^{\rm DC} = 1.9 \pm 0.1$ μA, and the readout-qubit mutual inductance was $M_{{\rm ro}-q} = 16.7 \pm 0.2$ pH. The device was mounted in an Al box in a dilution refrigerator, and all on-chip cross couplings were calibrated *in situ* as described in Ref. [16]. While the qubit described herein is unconventional and couples differently to local flux noise sources than an open inductive loop, the method presented below is applicable to any CJJ rf SQUID qubit.

Our experimental procedure is a variant of the MRT technique first developed by Rouse, Han, and Lukens [7]. We exploit $\Phi_x^{\rm cjj}$ to modulate Δ using a bias line with bandwidth ~5 MHz. A depiction of the control sequence

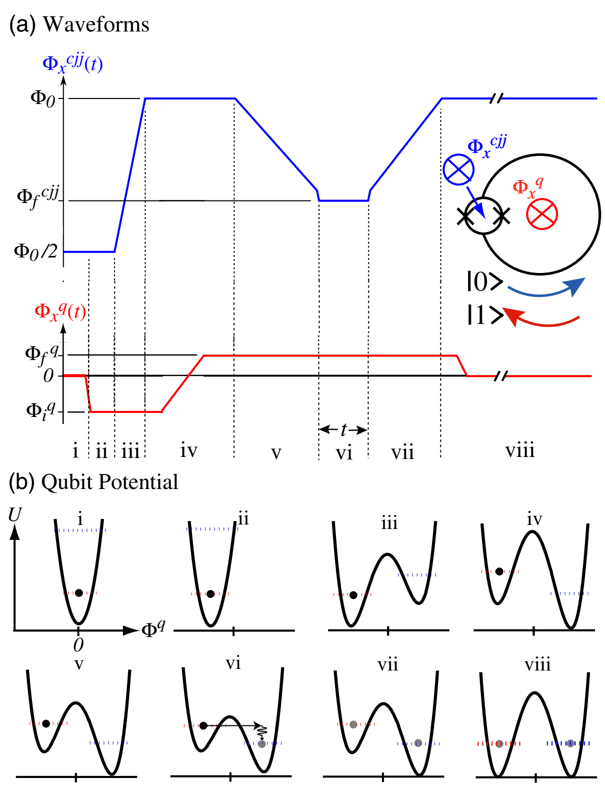

FIG. 1 (color online). (a) MRT flux control bias sequence as a function of time. The inset depicts a CJJ rf SQUID subjected to the external biases $\Phi_x^{\rm cjj}$ and Φ_x^q. The sense of the macroscopic persistent current states $|0\rangle$ (counterclockwise) and $|1\rangle$ (clockwise) is noted. (b) Evolution of the qubit potential $U(\Phi)$.

and the evolution of the qubit potential $U(\Phi)$ are shown in Fig. 1. The state of the qubit is initialized by setting $\Phi_x^{\rm cjj} \approx \Phi_0/2$ (i) and then tilting $U(\Phi)$ via Φ_x^q to an initial value of Φ_i^q (ii). Slowly raising $\Phi_x^{\rm cjj}$ to Φ_0 in the presence of the tilt traps the system in its ground state (iii). With tunneling suppressed, Φ_x^q is reset to a target value Φ_f^q (iv). Thereafter $\Phi_x^{\rm cjj}$ is lowered (v) to $\Phi_f^{\rm cjj}$ for a prescribed amount of time t during which the system can tunnel from the initial state to the lowest lying state in the opposite well (vi). Raising $\Phi_x^{\rm cjj}$ (vii) to Φ_0 then localizes the qubit state in $|0\rangle$ or $|1\rangle$ (viii) which can then be distinguished by a single shot readout. For a given Φ_f^q and t the probability of the system being found in $|0\rangle$ can be calculated from balancing $\Gamma_{01}(\Phi_f^q)$ and $\Gamma_{10}(\Phi_f^q)$: $dP_0/dt = -\Gamma_{01}P_0(t) + \Gamma_{10}P_1(t)$, where $P_0(t) + P_1(t) = 1$. In the limit $t \to 0$ the system starts in a definite state, and this expression reduces to $dP_0/dt = -\Gamma_{01}$ $[P_0(0) = 1]$ or Γ_{10} $[P_1(0) = 1]$.

The results shown herein were generated using a value of $\Phi_f^{\rm cjj}$ for which we measured $|I_p| = 0.56 \pm 0.02$ μA. This particular $\Phi_f^{\rm cjj}$ was chosen as $1/\Gamma$ varies by nearly 4 orders of magnitude (10 μs $\to$ 100 ms) as a function of Φ_f^q in the vicinity of $\Phi_f^q = 0$, which then takes full advantage of the dynamic range of our apparatus. Using the calibrated device parameters and Eq. (4), it was determined that the above-mentioned value of $|I_p|$ corresponds to $\Phi_f^{\rm cjj} = 0.606 \pm 0.002\Phi_0$. The tunneling energy was estimated to be $\Delta_0 = 0.10^{+0.28}_{-0.07}$ mK. The asymmetric error bars on Δ_0 are a consequence of its exponential sensitivity to errors in L, C, and I_c.

We have measured $P_0(\Phi_f^q, t)$ over the range $\Phi_f^q \in [-3, 3]$mΦ_0 for both initial polarizations $[P_0(t = 0) = 1$ and $P_1(t = 0) = 1]$ as a function of $T_{\rm th}$. The point $\Phi_f^q = 0$ was calibrated at each $T_{\rm th}$ by determining the flux bias at which $P_0(t \to \infty) = 0.5$. Fitting the thermal distribution $P_0(\Phi_f^q, t \to \infty) = \frac{1}{2}[1 - \tanh(2|I_p|\Phi_f^q/2T_{\rm th})]$ yielded $T_{\rm th}$. Example MRT data are shown in Fig. 2(a) for $P_0(0) = 1$ at three values of Φ_f^q and $T_{\rm th} = 28$ mK. A summary of initial decay rates versus Φ_f^q is shown in Fig. 2(b). Note that Γ_{01} and Γ_{10} consist of broad peaks displaced from $\Phi_f^q = 0$ in the direction opposing that in which the system was initialized. These peaks can be attributed to MRT between the two lowest lying states of the bistable rf SQUID. Beyond

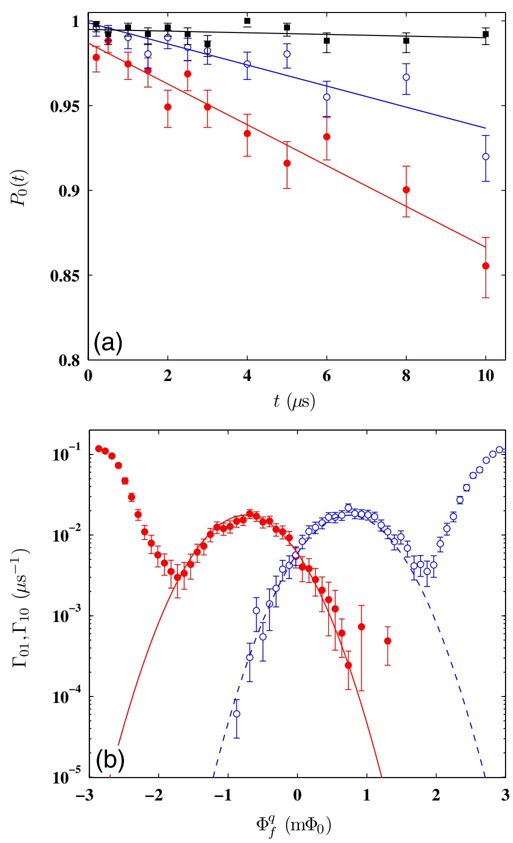

FIG. 2 (color online). (a) MRT from the initial state $|0\rangle$ versus t at $T_{\rm th} = 28$ mK. Results are shown for $\Phi_f^q = -0.488$ mΦ_0 (solid squares), -0.014 mΦ_0 (hollow circles), and 0.554 mΦ_0 (solid circles). Slopes of the linear fits yield $-\Gamma_{01}(\Phi_f^q)$. (b) Γ_{01} (hollow symbols) and Γ_{10} (solid symbols) versus Φ_f^q at $T_{\rm th} = 28$ mK. The peaks nearest $\Phi_f^q = 0$ have been fit to Eq. (2).

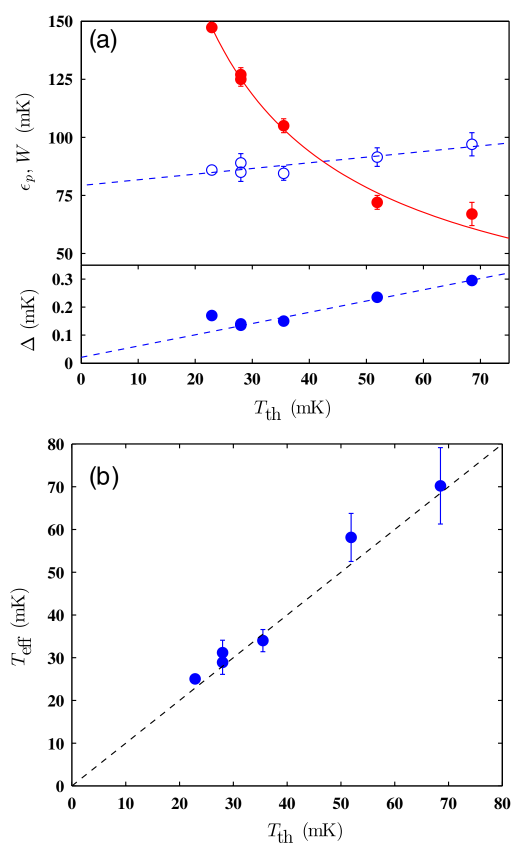

FIG. 3 (color online). (a) ϵ_p (solid symbols, upper plot), W (hollow symbols, upper plot), and Δ (solid symbols, lower plot) versus $T_{\rm th}$. W and Δ have been fit to lines (dashed) and ϵ_p to $C/T_{\rm th}^\alpha$ (solid). (b) $T_{\rm eff} = W^2/2\epsilon_p$ versus $T_{\rm th}$. The dashed line indicates $T_{\rm eff} = T_{\rm th}$.

$|\Phi_f^q| \gtrsim 2$ mΦ_0, Γ_{01} and Γ_{10} increase due to MRT between the initial state and the first excited state in the opposing well.

The data in Fig. 2(b) have been fit to Eq. (2) using W, ϵ_p, and Δ as free parameters. We repeated the measurements for five values of $T_{\rm th}$ and summarized the best fit ϵ_p, W, and Δ in Fig. 3(a). The fact that the lowest order MRT peaks fit well to Gaussian line shapes is strong evidence that $S(\omega)$ is peaked at low ω and that it possesses a high ω cutoff below the width W. Note that W is only weakly dependent upon $T_{\rm th}$. Fitting W to a line yielded $W(T \rightarrow 0) \approx 80$ mK, which corresponds to a total integrated flux noise $\delta\Phi \sim 5 \times 10^{-4}\Phi_0$ [17]. The observed weak T dependence corroborates with Rabi decay time measurements from phase qubits as $T \rightarrow 0$ [18]. In contrast, ϵ_p behaves as $1/T_{\rm th}^{0.80\pm0.05}$. The fitted values of Δ are comparable to Δ_0 as estimated from qubit parameters; however, the uncertainty in Δ_0 makes it difficult to draw quantitative conclusions. Nonetheless, the results show a dependence of Δ on T. A possible explanation is a renormalization of Δ by noise [10,13].

Figure 3(b) shows $T_{\rm eff}$ [see Eq. (3)] versus $T_{\rm th}$. Agreement between $T_{\rm eff}$ and $T_{\rm th}$ is critical as it demonstrates that our analysis is self-consistent. It is emphasized that finite ϵ_p is required by thermodynamics per Eq. (3). Had ϵ_p been due to systematic errors, such as an initialization-dependent magnetization in our apparatus, then Eq. (3) would not be satisfied. We have experimentally verified that ϵ_p is independent of $|\Phi_i^q|$ and that Eq. (3) is satisfied for data from several CJJ rf SQUIDs of different design in three different cryogenic apparatuses. Thus the experimental results agree with the theory of Ref. [9] in which the dominant source of flux noise is a quantum mechanical environment in thermal equilibrium.

Interestingly, if the environment is magnetic with susceptibility $\chi(\omega, T)$, then one can write $S_s(\omega, T) = {\rm Im}\chi(\omega, T)\coth(\frac{\omega}{2T})$. Furthermore, a Kramers-Kronig relation requires that ${\rm Re}\chi(\omega = 0, T) = \frac{1}{\pi}\int d\omega\, {\rm Im}\chi(\omega, T)/\omega$. Using Eq. (2), it can then be shown that $W^2 \approx T{\rm Re}\chi(\omega = 0, T)$. The observed weak T dependence of W implies that the dc susceptibility ${\rm Re}\chi(\omega = 0, T) \propto 1/T$, which is consistent with the behavior of a paramagnetic bath.

Conclusions.—Measurements of MRT between the lowest energy states of a bistable rf SQUID have been used to characterize flux noise. Observation of shifted Gaussian-shaped tunneling rate peaks indicates that the dominant noise source is a quantum mechanical environment in thermal equilibrium whose spectral density $S(\omega)$ contains both symmetric and antisymmetric components and is peaked at low ω with a high ω cutoff below $W \lesssim 100$ mK. These properties of $S(\omega)$ provide guidance for developing theories of flux noise.

We thank J. Hilton, G. Rose, C. J. S. Truncik, A. Tcaciuc, and F. Cioata. Samples were fabricated by the Microelectronics Laboratory of the Jet Propulsion Laboratory, operated by the California Institute of Technology under a contract with NASA. S. H. was supported in part by NSF Grant No. DMR-0325551.

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