Admittance and Noise in an Electrically Driven Nanostructure: Interplay between Quantum Coherence and Statistics

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We investigate the interplay between the quantum coherence and statistics in electrically driven nanostructures. We obtain an expression for the admittance and the current noise for a driven nanocapacitor in terms of the Floquet scattering matrix and derive a nonequilibrium fluctuation-dissipation relation. As an interplay between the quantum phase coherence and the many-body correlation, the admittance has peak values whenever the noise power shows a step as a function of the nearby gate voltage. Our theory is demonstrated by calculating the admittance and noise of driven double-quantum dots.

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Quantum dynamics in a time-dependent potential is an important topic because of its unrevealed nonequilibrium phenomena as well as the increasing demand for manipulating coherent electronic states in quantum information science [1,2]. While the time-dependent potentials in diffusive conductors are known to have a destructive role in the quantum coherence of electrons, it is not always obvious which role is played on the quantum coherence by the time-periodic potential in ballistic nanostructures. Quantum coherence and quantum many-particle statistics of relevant particles are key concepts in understanding electron transport properties. For instance, the current noise, which is basically a two-particle property, contains the information on the quantum many-particle statistics [3]. Furthermore, the noise of a coherent conductor in the presence of a time-periodic external field is known to be sensitive to the phase of transmission amplitudes [4–6].

In this work, we study how the interplay between the quantum coherence and the statistics affects the transport properties in the presence of a time-periodic external potential. We particularly concentrate on the noise of the quantum displacement current through the driven capacitor, because in usual conductors, shot noise (in the classical granular nature of electrons) dominates other types of noises. By this means, we derive the analytic expression for the admittance and noise formula in terms of the Floquet scattering matrix for the driven nanocapacitor.

The linear response of the capacitor is described by the admittance $g(\omega) = I(\omega)/V(\omega)$, which relates the displacement current $I(\omega)$ to the applied voltage between capacitor $V(\omega)$. The expression which relates equilibrium admittance and the noise power $S(\omega)$ to the scattering matrix has been obtained by Büttiker and his co-workers [7,8]. Here we generalize the expression to the case of the nonequilibrium states generated by a time-periodic potential of frequency $\Omega \gg \omega$. It will be shown here that, as in the case of equilibrium capacitors, the admittance of the driven capacitor can also be understood in terms of the time delay of electrons near the Fermi level. Meanwhile, the nonequilibrium current noise power cannot be understood within a single-particle picture. The noise power shows step structure as a function of the nearby gate voltage, which is associated with the opening of temporal channels in the lead.

Let us begin by introducing a model for the system of a biased dynamic capacitor (see Fig. 1). A time-dependent potential with frequency Ω is applied in a nanostructure (dotted box) which is connected to a mesoscopic conductor. The chemical potential of the mesoscopic conductor is controlled by a nearby gate voltage V_g. The oscillating nanostructure and the mesoscopic conductor are in a loop enclosing a time-periodic magnetic flux $\Phi(t)$. The bias voltage $V(t)$ between the nanostructure and external lead is induced as an electromotive force along the loop: $V(t) = d\Phi/dt = V(\omega)e^{i\omega t} + \text{c.c.}$.

We assume the mesoscopic conductor is spatially one-dimensional and the electrons are a noninteracting spin-polarized gas for simple presentation. Generalization of our results to the case of multichannel conductors with spin is straightforward. The Hamiltonian for the conductor reads

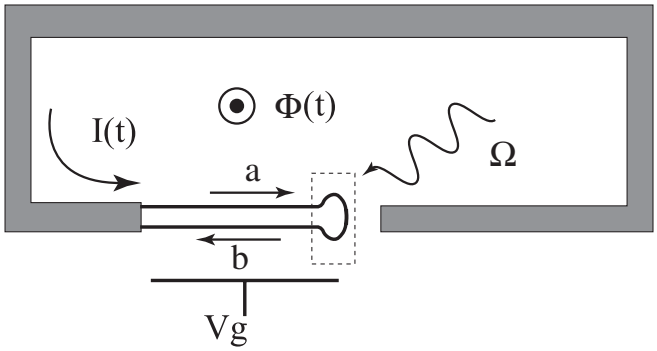

FIG. 1. A time-periodic driven nanostructure (dotted box) is connected to a mesoscopic conductor. For bias voltage, an external loop driven by slow-varying flux $\Phi(t) = \Phi_\omega \cos\omega t$ is connected.

$$H_0 = \int d\epsilon \epsilon [a^\dagger(\epsilon)a(\epsilon) + b^\dagger(\epsilon)b(\epsilon)], \quad (1)$$

where $a(\epsilon)$ $[b(\epsilon)]$ is the annihilation operator for the incoming [outgoing] electron to the driven nanostructure; $\{a^\dagger(\epsilon), a(\epsilon')\} = \{b^\dagger(\epsilon), b(\epsilon')\} = \delta(\epsilon - \epsilon')$ and $\{a(\epsilon), a(\epsilon')\} = \{b(\epsilon), b(\epsilon')\} = \{a(\epsilon), b^{(\dagger)}(\epsilon')\} = 0$.

Time-periodic steady states are formed in the conductor and the driven nanostructure

$$i\hbar \frac{\partial}{\partial t}|\psi_\epsilon(t)\rangle = [H_0 + H_\Omega(t) + H_c]|\psi_\epsilon(t)\rangle, \quad (2)$$

where H_Ω is the time-periodic Hamiltonian for the electron in the driven nanostructure and H_c denotes the coupling between the lead and the nanostructure. $|\psi_\epsilon(t)\rangle$ is formed by the incoming electron state of energy ϵ and the linear combinations of its scattered states with energy $\epsilon + n\hbar\Omega$. We assume the external metallic lead is big enough so that it can be considered as a reservoir in thermal equilibrium. The occupation number for the incoming electron state of energy ϵ is given by Fermi-Dirac distribution function $f(\epsilon)$.

Let us consider the linear response of the electric current to the time-dependent magnetic flux. The perturbing time-dependent Hamiltonian is $H_\omega(t) = \Phi(t)\hat{I}$, where $\hat{I}$ is the current operator:

$$\hat{I} = \frac{e}{h}\int d\epsilon d\epsilon' [a^\dagger(\epsilon)a(\epsilon') - b^\dagger(\epsilon)b(\epsilon')]. \quad (3)$$

In fact, the above current formula is an approximation where the current value is taken as the spatial average value over the length of the conductor L. This approximation holds when the relevant frequency scale is much smaller than the Fermi velocity divided by the length of the conductor L, i.e., $\omega, \Omega \ll v_F/L$, so that the relevant wave number k satisfies $|k - k_F|L \ll 1$ and $k_F L \gg 1$ where the rapidly oscillating phase terms are washed out.

The adiabatic turning on of $\Phi(t) = \Phi_\omega \cos\omega t e^{0^+ t}$ gradually deforms $|\psi_\epsilon(t)\rangle$ to $|\Psi_\epsilon(t)\rangle = \int d\epsilon' C_{\epsilon\epsilon'}(t)|\psi_{\epsilon'}(t)\rangle$. The coefficients $C_{\epsilon\epsilon'}(t)$ are determined by solving the Schrödinger equation for the time-dependent Hamiltonian $H(t) = H_0 + H_\Omega(t) + H_c + H_\omega(t)$. By employing a perturbation expansion of $C_{\epsilon\epsilon'}$ in terms of Φ_ω, we get the first-order term $C^{(1)}_{\epsilon\epsilon'} = \frac{\Phi_\omega}{i\hbar} \times \int_{-\infty}^t dt' \cos\omega t' \langle\psi_{\epsilon'}(t')|\hat{I}|\psi_\epsilon(t')\rangle$.

Up to the first order of Φ_ω, the ω component of the displacement current $I^{(1)}_{\epsilon,\omega}$ is obtained through

$$\langle\Psi_\epsilon(t)|\hat{I}|\Psi_\epsilon(t)\rangle = \int d\epsilon' \langle\psi_\epsilon(t)|\hat{I}|\psi_{\epsilon'}(t)\rangle C^{(1)}_{\epsilon\epsilon'}(t) + \text{c.c.}$$
$$\approx I^{(1)}_{\epsilon,\omega} e^{i\omega t} + I^{(1)}_{\epsilon,-\omega} e^{-i\omega t}. \quad (4)$$

The Floquet theorem says that the eigenstates $|\psi_\epsilon(t)\rangle$ of a time-periodic Hamiltonian can be written in terms of time-independent basis $|\phi^{(l)}_\epsilon\rangle$ as $|\psi_\epsilon(t)\rangle = \exp(-i\frac{\epsilon}{\hbar}t) \times \sum_{l=-\infty}^{\infty} e^{-il\Omega t}|\phi^{(l)}_\epsilon\rangle$. Using Eqs. (3) and (4), one can relate $I^{(1)}_{\epsilon,\omega}$ to the Floquet eigenstates $|\phi^{(l)}_\epsilon\rangle$, which is useful for further calculations. The thermal-averaged displacement current is given by $I(\omega) = \int d\epsilon f(\epsilon) I^{(1)}_{\epsilon,\omega}$, because the incoming electrons are from the reservoir in equilibrium.

The admittance is given by the induced displacement current $I(\omega)$ divided by the applied voltage $V(\omega) = i\omega\Phi_\omega/2$. After some algebra we find

$$g(\omega) = \frac{1}{i\omega}\sum_m \int d\epsilon d\epsilon' \left|\sum_l \langle\phi^{(l)}_\epsilon|\hat{I}|\phi^{(l+m)}_{\epsilon'}\rangle\right|^2 \times \frac{f(\epsilon) - f(\epsilon')}{\epsilon - \epsilon' - \hbar\omega - m\hbar\Omega + i0^+}.$$

Here $|\omega| < |\Omega|$ is assumed.

In a quantum conductor with a time-periodic scatterer, the scattering relation between the incoming electron of energy ϵ and the outgoing electron of energy ϵ' is given by the Floquet scattering matrix [9] $S_F(\epsilon', \epsilon)$. The Floquet state in the quantum conductor can be written

$$|\phi^{(l)}_\epsilon\rangle = [a^\dagger(\epsilon)\delta_{l0} + s_l(\epsilon)b^\dagger(\epsilon + l\hbar\Omega)]|0\rangle, \quad (5)$$

where $s_l(\epsilon) = S_F(\epsilon + l\hbar\Omega, \epsilon)$. The unitarity of the scattering matrix gives $\sum_l |s_l(\epsilon)|^2 = 1$, and its time-reversal symmetry gives $s_{-l}(\epsilon + l\hbar\Omega) = s_l(\epsilon)$. The current matrix element in the Floquet basis simply reads

$$\langle\phi^{(l)}_\epsilon|\hat{I}|\phi^{(l+m)}_{\epsilon'}\rangle = \frac{e}{h}[\delta_{l,0}\delta_{m,0} - s^*_l(\epsilon)s_{l+m}(\epsilon')]. \quad (6)$$

The real part of the admittance $g'(\omega)$ is now written using $\frac{1}{x+i0^+} = P\frac{1}{x} - i\pi\delta(x)$:

$$g'(\omega) = \frac{e^2}{2h}\sum_m \int d\epsilon \left|\delta_{m0} - \sum_l s_{l+m}(\epsilon)s^*_l(\epsilon + \hbar\omega + m\hbar\Omega)\right|^2 \times \frac{f(\epsilon) - f(\epsilon + \hbar\omega + m\hbar\Omega)}{\hbar\omega}. \quad (7)$$

The above result is partly confirmed by the fact that if the high frequency Ω radiation were not there, then $s_l \propto \delta_{l,0}$ and Eq. (7) is equivalent to Eq. (2) in Ref. [7]. It is worth noting that the admittance is a quantity governed by the electron near the Fermi level. At low frequency and zero temperature, the admittance is approximated by

$$g'(\omega) \approx \frac{e^2}{2h}\omega^2[\tau_d^2(E_F) + \tau_p^2(E_F)], \quad (8)$$

where τ_d is the phase delay time [10] of the electron in the nanostructure, which is defined as $\tau_d(\epsilon) = -i\hbar\sum_l s^*_l(\epsilon) \times \frac{ds_l(\epsilon)}{d\epsilon}$, and τ_p (relatively smaller than τ_d for weaker electrical driving) is the nonequilibrium photoassisted phase delay time defined by $\tau_p^2(\epsilon) = \sum_{m\neq 0}|\hbar\sum_l s_{l+m}(\epsilon - m\hbar\Omega)\frac{ds^*_l(\epsilon)}{d\epsilon}|^2$.

Now we turn to the (nonsymmetrized) current noise $S(\omega)$ defined by

$$S(\omega)\delta(\omega + \omega') = \frac{1}{\pi}\int dt dt' e^{i\omega t} e^{i\omega' t'} \langle\delta\hat{I}(t)\delta\hat{I}(t')\rangle, \quad (9)$$

where $\delta I(t) = I(\hat{t}) - \langle \hat{I}(t) \rangle$ and $\hat{I}(t) = e^{iH_0 t/\hbar} \hat{I} e^{-iH_0 t/\hbar}$. Again ω and ω' are assumed low enough that terms of higher harmonics involving $\delta(\omega + \omega' + n\Omega)$ $(|n| > 1)$ vanish. Here the average $\langle \ldots \rangle$ means the spatial average after both quantum mechanical and statistical average over many-particle states $\frac{1}{2^{(N-1)/2}} \prod_{\epsilon_j} [a^\dagger(\epsilon_j) + \sum_l s_l(\epsilon_j) b^\dagger(\epsilon_j + l\hbar\Omega)]|0\rangle$ with thermodynamic weighting factor $e^{-\beta \sum_j (\epsilon_j - E_F)}$. The current correlation $\langle \hat{I}(\omega) \hat{I}(\omega') \rangle$ is written in terms of the incoming and outgoing particle operators (a, $a^\dagger$, b, and $b^\dagger$). The calculation can be easily done by projecting the total many-particle states into incoming particle states. In the projected basis, $b(\epsilon)$ is replaced with $\sum_l s_l(\epsilon) a(\epsilon + l\hbar\Omega)$.

The correlation between the outgoing particles comes from the exchange correlation among incoming particles. After some algebra, the nonsymmetrized noise power of the driven conductor is given by

$$S(\omega) = \frac{e^2}{h} \sum_m \int d\epsilon \times \left| \delta_{m,0} - \sum_l s^*_{l+m}(\epsilon) s_l(\epsilon + \hbar\omega + m\hbar\Omega) \right|^2 \times f(\epsilon)[1 - f(\epsilon + \hbar\omega + m\hbar\Omega)]. \quad (10)$$

By keeping only the $m = 0$ term, we recover the result for the case of time-independent potential [7].

Equations (7) and (10) are the main results of this work. One can notice that current noise power $S(\omega)$ is related to the admittance $g'(\omega)$ via

$$S(\omega) - S(-\omega) = 2\hbar\omega g'(\omega). \quad (11)$$

Lesovik and Loosen [11] have shown that the above fluctuation-dissipation relation is valid in a nonequilibrium case where the particle current flows at a small finite bias.

The noise power in Eq. (10) can be divided into two different parts $S(\omega) = S_0(\omega) + S_P(\omega)$. They are the equilibrium noise $S_0(\omega)$ $(m = 0)$ and the nonequilibrium noise S_P $(m \neq 0)$. At low frequency and low temperatures, the nonequilibrium noise $S_P(\omega)$ is more important than the equilibrium noise $S_0(\omega)$. The equilibrium noise is proportional to ω^3, and the nonequilibrium noise is proportional to ω^2. So there is always a frequency regime where the nonequilibrium noise $S_P(\omega)$ dominates the equilibrium noise $S_0(\omega)$ at low frequencies.

To demonstrate our theory, we consider electrically driven double-quantum dots (DQDs) connected to a single (spatial) channel lead. We employ the Floquet scattering theory based on the tight-binding approximation to obtain the Floquet scattering matrix element $s_l(\epsilon)$ [12].

In the tight-binding model, the localized states in the dots and the leads are created by $d^\dagger_{1(2)}$ and $c^\dagger_j$ $(j = -1, -2, -3, \ldots)$, respectively. The Hamiltonian for the lead and the dot-lead coupling are given by $H_0 = -\frac{V_0}{2} \sum_{j<-1} (c^\dagger_{j+1} c_j + c^\dagger_j c_{j+1})$ and $H_c = -\gamma \sum_{i=1}^2 (d^\dagger_i c_{-1} + c^\dagger_{-1} d_i)$, respectively. $V_0/2$ is the hopping parameter for the leads which controls the kinetic energy. γ is the tunnel coupling between the dots and the lead. The Hamiltonian for the driven double dots in the base of localized state is

$$H_\Omega(t) = \frac{1}{2} \begin{pmatrix} \Delta - eV_\Omega \cos\Omega t & -\Delta_0 \\ -\Delta_0 & -\Delta + eV \cos\Omega t \end{pmatrix}, \quad (12)$$

where Δ and Δ_0 are the asymmetry energy and tunnel splitting energy, respectively, of the double-quantum dot. The scattering matrix elements $s_l(\epsilon)$ are obtained by a phase-matching method using the incoming state $a^\dagger(\epsilon)|0\rangle = [1/\sqrt{v(\epsilon)}] \sum_j e^{ik_0 j} c^\dagger_j |0\rangle$ and its outgoing states $b^\dagger(\epsilon + l\hbar\Omega)|0\rangle = [1/\sqrt{v(\epsilon + l\hbar\Omega)}] \sum_j e^{-ik_l j} c^\dagger_j |0\rangle$, where $\epsilon + l\hbar\Omega = -V_0 \cos k_l$ and $v(\epsilon)$ is the group velocity.

We show the admittance of driven double-quantum dots in Fig. 2(a). It shows peak structure as a function of Fermi energy. The admittance has peaks when the Fermi energy matches with the resonant energy levels of the DQD as well as photon sidebands $E_F \approx E_r \pm \hbar\Omega$, where E_r is the Floquet eigenvalue of $H_\Omega(t)$. For weak driving, E_r is the energy eigenvalue for DQD $E_r \approx \pm \frac{1}{2} \sqrt{\Delta^2 + \Delta_0^2}$. Why does it show peaks? The nonzero admittance of a capacitor

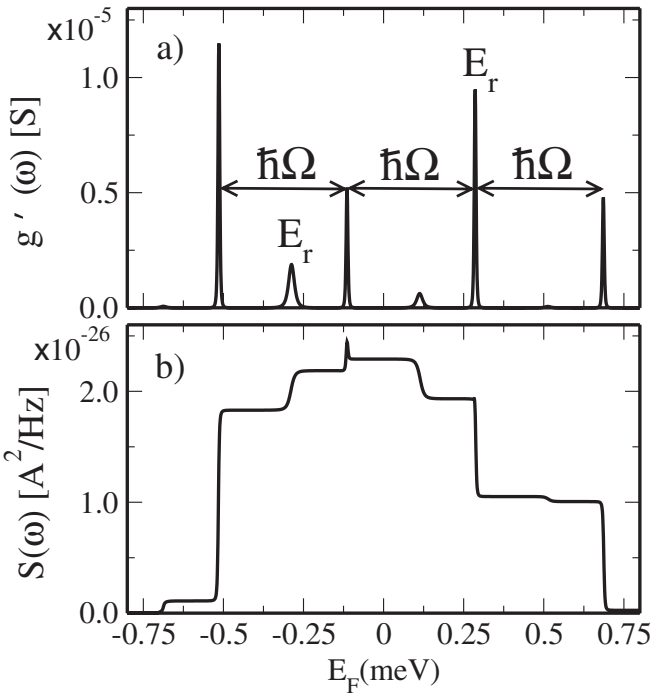

FIG. 2. (a) Admittance of the driven double-quantum dot in a capacitor as a function of the Fermi energy at zero temperature. (b) Nonsymmetrized current noise power $S(\omega)$ for the same system as a function of the Fermi energy of the external lead. $E_F = 0$ is the case when the Fermi level matches the center of the two eigenenergies of the DQD. The parameters in use are $\gamma = 0.16$ meV, $\Delta_0 = 0.3$ meV, $\Delta = 0.4$ meV, $\Omega = 60.7$ GHz, and $eV_\Omega = 0.3$ meV, and $\omega = 4.55$ GHz and $V_0 = 5.1$ meV, respectively.

is due to the time delay of the electrons at the capacitor. Since only states near the Fermi level are excited by an oscillating magnetic flux of low frequency ω, the admittance is naturally given by the Fermi level quantity. So the peak values of the dwell time at a certain Fermi energy give rise to the peak structure of the admittance as is clear in Eq. (8). Meanwhile, the role of the driving electric field of high frequency Ω is to help the incoming electron at the Fermi level jump into the resonant levels in double dots via photon absorption or emission. Whenever the Fermi level matches with the resonant energy plus an integer multiple of $\hbar\Omega$, the electron can dwell in the dots and the admittance has peaks. This process is depicted in Fig. 3(a).

In Fig. 2(b), we show the noise power as a function of the Fermi level. The contribution from equilibrium Nyquist noise is due to electron states near Fermi level. At low frequency and zero temperature, $S_0 \approx \frac{e^2}{4\pi}\omega^3\tau_d^2(E_F)$ [7] and $S_P(\omega) \approx \frac{e^2\hbar}{2\pi}\omega^2\sum_{m\neq 0}\int d\epsilon|\sum_l s^*_{l+m}(\epsilon)\frac{d}{d\epsilon}s_l(\epsilon + m\hbar\Omega)|^2 \times f(\epsilon)[1 - f(\epsilon + m\hbar\Omega)]$. The sharp peaks in Fig. 2(b) are attributed to the nonzero dwell time at the Fermi levels. In contrast, we find that the nonequilibrium part of noise $S_P(\omega)$ shows stepwise behavior. Why does it show steps? The electrons below the Fermi level contribute to the nonequilibrium noise through photon absorption or emission. Note that, in contrast to the case of the admittance, there is no driving probe field of frequency ω. Therefore, the noise power is not necessarily a quantity for the Fermi level.

The incoming electron states of energy $E < E_F$ contribute to the noise when $E - E_r$ is the integer multiple of $\hbar\Omega$ [Fig. 3(b)]. Since we consider the current noise power $S(\omega)$ at low frequency $\omega < \Omega$, the outgoing electron states of the energy other than $E + \hbar\omega$ are not involved in the low frequency noise. Therefore, the number of pairs of incoming electrons of energy E and outgoing states of energy $E + \hbar\omega$ determine the strength of the current noise [Fig. 3(b)]. As the Fermi level increases, the number of the pairs increases, which gives rise to the step structure. The step arises whenever the Fermi level matches the resonant energy plus an integer multiple of $\hbar\Omega$.

While there have been experimental works on the electrical noise under ac excitation for diffusive conductors [13] and tunnel junctions [14,15], so far there has been no experimental realization of the driven nanostructure tunnel coupled to a single lead. To study the quantum aspect of the admittance discussed in this work, the experimental system by Gabelli *et al.* [2] seems most relevant to the present theoretical work where the dc conductance is zero. For experimental observation of the resonant admittance peaks predicted in this work, the quantum dot in use in Ref. [2] should be electrically driven and small enough to ensure that the dot's quantized energy spacing is larger than the temperature energy scale.

In conclusion, we investigate the low frequency admittance and current noise of a nanostructure which is driven by a high frequency field. A fluctuation-dissipation relation for the driven system is obtained. The phase delay time defined through the Floquet scattering matrix is essential to understand the admittance. The current noise power shows steps as a function of the Fermi energy when the admittance shows peaks. The fermionic nature of electrons or the exchange correlation of the incoming electrons is important to the step structure of the noise power.

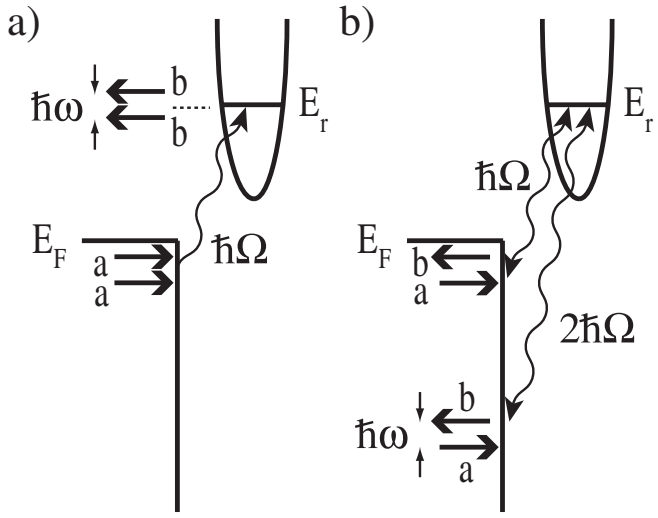

FIG. 3. Schematic figures explaining the processes involved in (a) the admittance and (b) the current noise. See the text.

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