Geodesic Acoustic Mode Induced by Toroidal Rotation in Tokamaks

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The effect of toroidal rotation on the geodesic acoustic mode (GAM) in a tokamak is studied. It is shown that, in addition to a small frequency upshift of the ordinary GAM, another GAM, with much lower frequency, is induced by the rotation. The new GAM appears as a consequence of the nonuniform plasma density and pressure created by the centrifugal force on the magnetic surfaces. Both GAMs in a rotating plasma are shown to exist both as continuum modes with finite mode numbers m and n at the rational surfaces $q = m/n$ as well as in the form of axisymmetric modes with $m = n = 0$.

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The geodesic acoustic mode (GAM) is a well-established experimental phenomenon in tokamaks [1,2]. The mode is basically a magnetohydrodynamic (MHD) continuumlike mode localized on a flux surface and, in the original analysis by Winsor *et al.* [3], has the poloidal (m) and toroidal (n) mode numbers both equal to zero. The GAM with these mode numbers plays an important role in theories of turbulent transport in tokamaks [4]. The frequency of a GAM existing on a flux surface with safety factor q is given by [3] $\omega_{\rm GAM}^2 = \omega_s^2(2 + 1/q^2)$, where $\omega_s^2 = \Gamma p/(\rho R_0^2)$ is the sound frequency, Γ the adiabatic index, p the pressure, ρ the density and R_0 the (average) major radius of the flux surface. This expression is valid for a static, toroidal plasma with circular cross section, large aspect ratio and low pressure. The same frequency appears also in connection with Alfvén cascades (ACs) [5] and with so-called beta-induced Alfvén-acoustic eigenmodes (BAAEs) [6] in tokamaks with reversed or low magnetic shear. Although frequency chirping GAMs with $n = 0$ can be excited in tokamaks [7], the ACs and BAAEs are usually characterized by finite mode numbers [5,6], and analytical theories of ACs are often based on assumptions of large values of m and n [8]. The rather surprising property of the GAMs to exist both as axisymmetric modes as well as in the form of continuum modes with finite mode numbers is one of the issues that will be addressed in this Letter, including also effects of plasma rotation.

A phenomenon seemingly unrelated to the GAM is the stabilizing effect of toroidal plasma rotation observed in some tokamaks, especially in spherical tokamaks where rotation speeds approaching the sound speed can be achieved by neutral beam injection. Such an effect on the sawtooth instability has, for instance, been seen both in NSTX [9] and in MAST [10]. Furthermore, it appears that for toroidal flows of this order of magnitude the stabilization has to do with centrifugal effects that are present already within a MHD description of the plasma [9–11]. A stabilizing effect of this kind on the ideal $m = n = 1$ mode was analyzed in Refs. [12,13], and the same effect was later shown to be able to stabilize also Mercier modes [14] as well as the quasi-interchange mode [15]. In the present Letter it will be shown that there is a connection between the rotational stabilization described above and a GAM. More specifically, we show that a new GAM is induced by the plasma rotation, and that the stabilization analyzed in Refs. [12–15] has to do with the coupling of the instability to this rotation-induced GAM. Furthermore, the existence of the new GAM branch, and therefore this stabilizing mechanism, is shown to depend crucially on the tangential gradients of the plasma density and pressure on the flux surfaces created by the centrifugal force. For this reason it is not possible to capture this stabilizing effect in numerical codes that do not take the centrifugal effects on the plasma equilibrium into account [16].

Let us consider a tokamak plasma that rotates toroidally with frequency $\Omega(r)$, where we use the flux coordinates (r, θ, φ) defined in Refs. [13,14] in order to describe the plasma equilibrium as well as small perturbations of this equilibrium. If the equilibrium temperature is constant on the flux surfaces, $p/\rho = T(r)$, the centrifugal force creates a nonuniform plasma density on each flux surface given by $\rho(r, \theta) = \rho_0(r)e^{(R^2-R_0^2)\Omega^2/2T}$ where R is the major radius [12–15]. A similar relation is valid for the pressure. The following analysis will be based on the ordering $\mu_0 p/B_0^2 \sim \beta \sim \varepsilon^2$, where B_0 is the toroidal magnetic field and $\varepsilon = r/R_0 \ll 1$. Furthermore, we assume that the sonic Mach number, $\mathcal{M} = (\rho\Omega^2R_0^2/2p)^{1/2}$, is of order unity. The MHD spectrum of this plasma can be found by solving the Frieman-Rotenberg eigenvalue equation for the Lagrangian perturbation $\boldsymbol{\xi} \sim e^{-i\omega t}$ [17]:

$$\rho\omega^2\boldsymbol{\xi} + 2i\rho\omega\mathbf{v}\cdot\nabla\boldsymbol{\xi} - \rho\mathbf{v}\cdot\nabla[(\mathbf{v}\cdot\nabla)\boldsymbol{\xi}] + \nabla\cdot[\rho\boldsymbol{\xi}(\mathbf{v}\cdot\nabla)\mathbf{v}] + \mathbf{F}(\boldsymbol{\xi}) = 0. \quad (1)$$

Here, $\mathbf{v} = \Omega\mathbf{e}_\varphi$ is the equilibrium flow velocity, $\mathbf{F}(\boldsymbol{\xi}) = -\nabla\delta P + [(\mathbf{B}\cdot\nabla)\mathbf{Q} + (\mathbf{B}\cdot\nabla)\mathbf{Q}]/\mu_0$ the static force operator, $\delta P = -\boldsymbol{\xi}\cdot\nabla p - \Gamma p\nabla\cdot\boldsymbol{\xi} + \mathbf{B}\cdot\mathbf{Q}/\mu_0$ the perturbed total pressure and $\mathbf{Q} = \nabla\times(\boldsymbol{\xi}\times\mathbf{B})$ the perturbed magnetic field. A system of equations describing the coupling of a perturbation with $\xi^r = \xi_{m,n}(r)e^{i(m\theta-n\varphi)}$

to the sidebands $\xi^r = \xi_{m\pm1,n}(r)e^{i[(m\pm1)\theta - n\varphi]}$ of order ε was derived from Eq. (1) in Ref. [14]. The final equation for the main harmonic $\xi_{m,n}$ can be summarized as

$$(\mathcal{L}_{m,n} + \mathcal{T}_{m,n})\xi_{m,n} + \mathcal{G}\{\xi_{m,n}, \xi_{m+1,n}, \xi_{m-1,n}\} = 0, \quad (2)$$

where the operators $\mathcal{L}_{m,n}$ and $\mathcal{T}_{m,n}$ are given by

$$\mathcal{L}_{m,n} \equiv \frac{d}{dr}\left[r^3(m/q - n)^2\frac{d}{dr}\right] - r(m^2 - 1)(m/q - n)^2, \quad (3)$$

$$\mathcal{T}_{m,n} \equiv \frac{d}{dr}\left(r^3 A_1 \frac{d}{dr}\right) + r^2\frac{dA_2}{dr} - r(m^2 - 1)A_1, \quad (4)$$

and the coefficients $A_{1,2}$ in Eq. (4) have the form [14]

$$A_1 = -\frac{\omega_D^2 - \Omega^2\mathcal{M}^2}{\omega_A^2} - \frac{(2\omega_D^2 + \kappa_+\omega_D\Omega + \kappa_+^2\Omega^2/4)\Omega^2 + (\omega_D^2 + 2\kappa_+\omega_D\Omega)\omega_s^2}{\omega_A^2(\kappa_+^2\omega_s^2 - \omega_D^2)} - (\kappa_+ \to \kappa_-), \quad (5a)$$

$$A_2 = -\frac{\omega_D^2 + \Omega^2[(m^2 - 1)\mathcal{M}^2 + 2m^2 - n^2 - 4]}{\omega_A^2} + (m - 1)\frac{(\kappa_+\omega_D\Omega + \kappa_+^2\Omega^2/4)\Omega^2 + (\omega_D^2 + 2\kappa_+\omega_D\Omega + 2\kappa_+^2\Omega^2)\omega_s^2}{\omega_A^2(\kappa_+^2\omega_s^2 - \omega_D^2)} - (m + 1)(\kappa_+ \to \kappa_-). \quad (5b)$$

The notation $(\kappa_+ \to \kappa_-)$ above stands for a term similar to the previous one, but with κ_+ replaced by κ_-, where $\kappa_\pm = (m \pm 1)/q - n$. Furthermore, $\omega_A^2 = B_0^2/(\mu_0\rho_0 R_0^2)$ is the Alfvén frequency and $\omega_D = \omega + n\Omega$ the Doppler-shifted mode frequency. The various frequencies in Eq. (5) are ordered as $\omega_D \sim \omega \sim \Omega \sim \omega_s \sim \varepsilon\omega_A$, and the denominators $\kappa_\pm^2\omega_s^2 - \omega_D^2$ come from the toroidal sidebands $\xi_{m\pm1,n}^\varphi$ that appear to first order in ε, and manifest the coupling of the Alfvén and slow continua by the geodesic curvature of the magnetic field [18,19]. In the present analysis, which is mainly focused on the continuous spectrum, the operator $\mathcal{G}$ in Eq. (2) is of minor importance. The reason for this is that since the $\mathcal{T}_{m,n}$ term and the $\mathcal{G}$ term in Eq. (2) both are of order ε^2, these terms become comparable to the $\mathcal{L}_{m,n}$ term and therefore of interest only if $m/q - n$ is of order ε (or smaller). With this ordering of $m/q - n$ it turns out that the $\mathcal{G}$ term does not contribute to the continuous spectrum [14]. Indeed, it can be shown that in low-shear plasmas where $m/q - n \sim \varepsilon$ in a finite region, the equations for the sidebands $\xi_{m\pm1,n}$ can be solved exactly, and by using these solutions in Eq. (2), the $\mathcal{G}$ term simplifies to the "Mercier term" $r^2(m^2 - n^2)(\beta_0 + \mathcal{M}^2\beta_0)'\xi_{m,n}$, where $\beta_0(r) = 2\mu_0 p_0(r)/B_0^2$, plus a term that comes from a homogeneous solution of $\xi_{m+1,n}$ [14,20,21]. We point out that, apart from the expansion in ε and the ordering $\beta \sim \varepsilon^2$, the only assumption used in the derivation of Eq. (2) is that the plasma cross section is circular. Thus, no assumption of large mode numbers has been made, and it will be seen that the equation even predicts the frequencies of the two GAMs existing for $m = n = 0$ correctly, in spite of the fact that the derivation in Ref. [14] is invalid for $m = 0$.

Equation (2) plays a similar role for compressible, low-frequency MHD phenomena in toroidal plasmas with circular cross section, large aspect ratio and $\beta \sim \varepsilon^2$ as the Hain-Lüst equation plays for such phenomena in cylindrical plasmas [22]. The way the denominator of dA_2/dr generates the continuum $\omega_D^2 = \omega_s^2/q^2$ also somewhat resembles the way the apparent singularities of the Hain-Lüst equation appear [22]. In the present case, however, the continuum $\omega_D^2 = \omega_s^2/q^2$ is real, as noted previously both for static [6] and rotating [23,24] plasmas. The continua of most interest here, however, are those associated with the equation $(m/q - n)^2 + A_1 = 0$, and we first consider this equation in the case of a nonrotating plasma. For $m = n = 0$ and $\Omega = 0$ it is seen that the equation has *two* roots, $\omega^2 = \omega_s^2(2 + 1/q^2) = \omega_{\mathrm{GAM}}^2$ and $\omega^2 = 0$. However, we get the same two roots also with finite values of m and n and $q = m/n$. In Fig. 1 these two continua are shown by the solid lines as functions of q for the mode numbers $m = 6$ and $n = 3$ in a plasma with $\omega_s^2/\omega_A^2 = \Gamma\beta/2 = 0.01$. Whereas the low-frequency branch has a minimum at $\omega^2 = 0$ and approaches $\omega^2 = \omega_s^2/q^2$ when $m/q - n$ becomes finite, the high-frequency branch has a minimum at $\omega^2 = \omega_{\mathrm{GAM}}^2$ and approaches the shear Alfvén root $\omega^2 = \omega_A^2(m/q - n)^2$ at finite $m/q - n$. This is consistent with the way the GAM frequency appears in the initial, low-frequency range of the ACs [5,8]. Furthermore, since the shear Alfvén wave vanishes in the limit $q \to m/n$, it is meaningful to classify such modes as GAMs with finite mode numbers [6–8]. We point out that the curves in Fig. 1 are based on the expression $\kappa_\pm \approx \pm 1/q$, and if the full form of $\kappa_\pm$ is used instead, the acoustic branch is some-

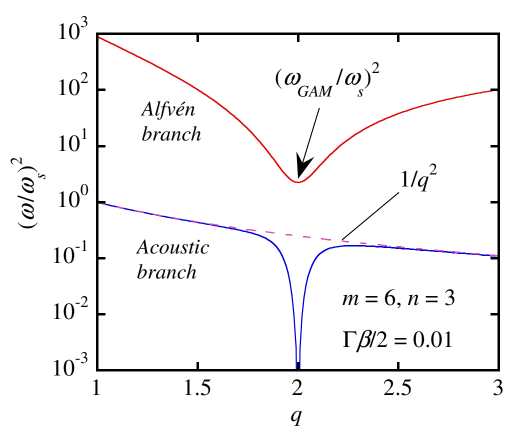

FIG. 1 (color online). The two continua associated with the equation $(m/q - n)^2 + A_1 = 0$ in a static plasma, calculated for $\Gamma\beta/2 = 0.01$, $m = 6/n = 3$, and $\kappa_\pm \approx \pm 1/q$.

what modified away from $m/q - n = 0$. We also mention that equations for the continua similar to the equations discussed here recently have been derived by Gorelenkov *et al.* [6] for static plasmas and by van der Holst *et al.* [24] for rotating plasmas. On the basis of Eqs. (2)–(5), however, it is possible to study also the properties of global eigenmodes analytically, and some aspects of this were recently discussed in Ref. [21] for a low-shear tokamak with parabolic pressure profile, using a simplified version of A_1 and neglecting the dA_2/dr term. For a more thorough analysis, the complete form of Eqs. (2)–(5) provides a suitable framework for studying both the continua as well as global eigenmodes, for instance the flow induced Alfvén eigenmode found in Refs. [23,24]. This is a large area for future research, and we limit the discussion here to the continua only, and examine, in particular, the validity of the "continuum frequencies" predicted for $m = n = 0$.

Including now also finite rotation in the equation $(m/q - n)^2 + A_1 = 0$, the two solutions $\omega^2 = \omega^2_{\rm GAM}$ and $\omega^2 = 0$ valid for a static plasma are modified to

$$\omega_D^2 = \omega_s^2\Bigg[1 + \frac{1}{2q^2} + \frac{\mathcal{M}^2(\mathcal{M}^2+4)}{\Gamma} \pm \sqrt{\left(1 + \frac{1}{2q^2} + \frac{\mathcal{M}^2(\mathcal{M}^2+4)}{\Gamma}\right)^2 - \frac{2\mathcal{M}^4}{q^2\Gamma}\left(1 - \frac{1}{\Gamma}\right)}\Bigg]. \tag{6}$$

These roots are also obtained both for $m/n = q$ and for $m = n = 0$. Furthermore, both of the frequencies above, as well as $\omega_D^2 = \omega_s^2/q^2$, are included in Eqs. (81)–(83) in Ref. [24]. It will be shown rigorously later that both of the roots in Eq. (6), but not $\omega_D^2 = \omega_s^2/q^2$, indeed are the eigenfrequencies of two axisymmetric GAMs with $m = n = 0$. For slow rotation, $\mathcal{M} \ll 1$, the large and the small root in Eq. (6) are approximated by

$$\omega_D^2 = \omega_{\rm GAM1}^2 = \omega_s^2\left(2 + \frac{1}{q^2} + \frac{8\mathcal{M}^2}{\Gamma} + \ldots\right), \tag{7a}$$

$$\omega_D^2 = \omega_{\rm GAM2}^2 = \omega_{\rm BV}^2 = \frac{\mathcal{M}^2\Omega^2}{1+2q^2}\left(1 - \frac{1}{\Gamma}\right) + \ldots, \tag{7b}$$

respectively. The solid and dotted curves in Fig. 2 illustrate the two roots in Eqs. (6) and (7), respectively, for $q = 1, 2, 3$ and $\Gamma = 5/3$.

The low-frequency root in Eq. (7b) is written in the same form as in Refs. [13–15], with the subscript BV denoting the relationship with the Brunt-Väisälä frequency of a stably stratified fluid in a gravitational field [22]. In a local frame following the plasma rotation, the nonuniform density $\rho(r,\theta) = \rho_0(r)e^{(R^2-R_0^2)\Omega^2/2T}$ on the flux surfaces represents a stably stratified fluid in the "effective gravity" $\mathbf{g} = \Omega^2\mathbf{R}$, and the associated BV frequency [22] can be shown to be of order $\omega_{\rm BV}^2 \sim \mathcal{M}^2\Omega^2(1 - 1/\Gamma)$ [13–15]. This indicates, but does not prove, that the root in Eq. (7b) has to do with the nonuniform density and pressure on the magnetic surfaces only. Such a relationship can be proved, however, by inserting a tag σ in the equation $\rho(r,\theta) = \rho_0(r)e^{\sigma(R^2-R_0^2)\Omega^2/2T}$ (and in the corresponding pressure equation) and in this way trace the effects of these expressions for ρ and p in the derivation of Eq. (2). Doing this, it turns out that the main consequence of using $\sigma \neq 1$ is a modified coefficient A_1, and therefore also modified continua. One finds that the modified continuum frequencies are again given by Eq. (6), but with the last term inside the square root now given by $-2\sigma\mathcal{M}^4(1 - 1/\Gamma)/(q^2\Gamma)$ and with the factor $\mathcal{M}^2 + 4$ replaced by $\sigma\mathcal{M}^2 + \sigma + 3$. The equation replacing Eq. (7b) then becomes $\omega_D^2 = \omega_{\rm GAM2}^2 = \omega_{\rm BV}^2 = \sigma\mathcal{M}^2\Omega^2(1 - 1/\Gamma)/(1 + 2q^2)$, showing that the low-frequency root in Eq. (6) and (7b) indeed vanishes if a uniform plasma density and pressure on the flux surfaces is assumed ($\sigma = 0$). The effect of using $\sigma = 0$ for the high-frequency root in Eq. (7a) is much smaller. The importance of the density and pressure distribution on the flux surfaces for the value of ω_D^2 in Eq. (7b) can be understood also from the stability condition for the continua in Eq. (56) in Ref. [24]. This stability condition involves only the tangential gradients of ρ and p on the flux surfaces and therefore predicts marginal stability if ρ and p are functions of r only, in agreement with the σ-analysis above. Since the frequency in Eq. (7b) is the key quantity responsible for the stabilization in Refs. [12–15], it follows that the centrifugal effects on the equilibrium are of crucial importance for the MHD stability of plasmas rotating at sonic or near-sonic speeds.

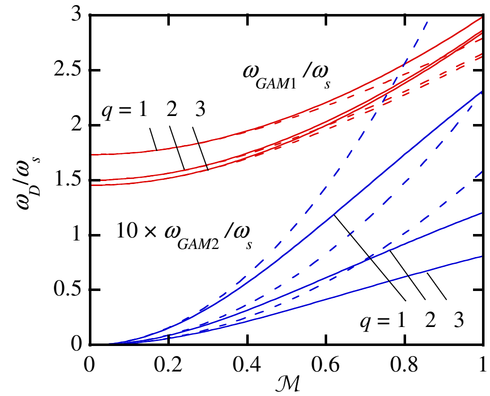

FIG. 2 (color online). The two GAM frequencies in Eq. (6) as functions of the sonic Mach number (solid lines), and the approximations in Eqs. (7a) and (7b) (dotted lines).

It should be noted that the analysis in Refs. [14,24] assumes finite mode numbers, and that axisymmetric modes never are discussed in those papers. To complete the present study, we therefore outline a theory of $m = n = 0$ GAMs in rotating plasmas below from which both the frequencies as well as the structure of the axisymmetric modes are obtained. Anticipating that the plasma perturbation $\boldsymbol{\xi}$ for such modes is tangential to the flux surfaces [3], and that ξ^θ is dominating, we use the following ansatz for $\boldsymbol{\xi}$: $\xi^\theta = \xi_0^{\theta(0)} + \varepsilon(\xi_1^{\theta(1)}e^{i\theta} + \xi_{-1}^{\theta(1)}e^{-i\theta}) + \ldots$, and $\xi^\varphi = \varepsilon(\xi_1^{\varphi(1)}e^{i\theta} + \xi_{-1}^{\varphi(1)}e^{-i\theta}) + \ldots$. The subscripts denote the poloidal mode number and the superscripts the order with respect to ε. It turns out that the form of $\mathbf{Q}$ needed to match these expressions for ξ^θ and ξ^φ is given by $Q^\varphi = \varepsilon^4(Q_1^{\varphi(4)}e^{i\theta} + Q_{-1}^{\varphi(4)}e^{-i\theta}) + \ldots$ to leading order in ε. From the toroidal component of the equation $\mathbf{Q} - \nabla \times (\boldsymbol{\xi} \times \mathbf{B}) = 0$ of order ε^2 one then finds that $\xi_{\pm 1}^{\theta(1)} = \xi_{\pm 1}^{\varphi(1)}/q$. Thereafter we obtain $B_0 Q_{\pm 1}^{\varphi(4)} = \pm i(\Gamma p_0/qR_0)\xi_{\pm 1}^{\varphi(1)} \pm ir(\Gamma p_0/R_0^2 + \rho_0\Omega^2/2)\xi_0^{\theta(0)}$ from the θ-component of Eq. (1) of order ε^3. The toroidal component of Eq. (1) then gives the toroidal sidebands

$$\xi_{\pm 1}^{\varphi(1)} = -\frac{r}{qR_0}\frac{\omega_s^2 - \Omega^2/2 \mp q\omega\Omega}{(\omega_s^2/q^2 - \omega^2)}\xi_0^{\theta(0)}, \tag{8}$$

whereas the poloidal component, with $\xi_{\pm 1}^{\theta(1)}$, $\xi_{\pm 1}^{\varphi(1)}$, and $Q_{\pm 1}^{\varphi(4)}$ above inserted, becomes

$$\mathbf{e}_\theta \cdot [FR] = -(r^2/R_0^2)A_1\xi_0^{\theta(0)} + \varepsilon^4[(\ldots)e^{2i\theta} + (\ldots)e^{-2i\theta}], \tag{9}$$

with $[FR]$ denoting the left-hand side of Eq. (1) and A_1 the coefficient in Eq. (5a) with $m = n = 0$. From the θ-independent part of Eq. (9) it follows that $\xi_0^{\theta(0)}(r) = \text{const} \times \delta(r - r_0)$, where r_0 is a radius where $A_1 = 0$. This proves that both of the roots in Eq. (6) represent the eigenfrequencies of two axisymmetric $m = n = 0$ GAMs.

The presence of the $e^{\pm 2i\theta}$ Fourier components of order ε^4 in Eq. (9) has interesting consequences. It turns out that in order to be able to eliminate these components, radial components of $\boldsymbol{\xi}$ (of order ε^2) and of $\mathbf{Q}$ (of order ε^3), both with poloidal mode numbers $m = \pm 2$, and existing outside $r = r_0$, are required (also for $\Omega = 0$). This shows that the character of the $m = n = 0$ GAMs as modes localized on a flux surface is valid only to leading order in ε. These results on the radial and poloidal structure of the axisymmetric GAMs, and more details concerning the perturbation analysis above will be published elsewhere.

In summary, theory for geodesic acoustic modes (GAMs) in toroidally rotating tokamak plasmas has been developed in this Letter. From the normal mode equation for low-frequency MHD phenomena in rotating, toroidal plasmas with large aspect ratio derived in Ref. [14], and summarized in Eq. (2), it is shown that a new GAM, of very low frequency, is induced by the rotation. Both the ordinary GAM as well as the new GAM exist in the form of continuum modes with finite mode numbers at the rational surfaces, and both of them display increasing frequencies with increasing plasma rotation, as shown in Fig. 2. Similar results were predicted in a previous study of Alfvén continua in rotating, toroidal plasmas by van der Holst *et al.* [24]. As a major result of the present Letter, it is shown that both of these modes in addition exist in the form of "genuine", axisymmetric GAMs with mode numbers $m = n = 0$, thereby extending the result by Winsor *et al.* [3] to rotating plasmas. It is, furthermore, shown that the new, low-frequency GAM exists as a consequence of the nonuniform plasma density and pressure created by the centrifugal force on the magnetic surfaces, and that the oscillation frequency of the mode therefore is of similar origin as the Brunt-Väisälä frequency of a stably stratified fluid in a gravitational field [22]. The stabilizing effect of toroidal rotation on the MHD instabilities analyzed in Refs. [12–15] is caused by the coupling of the instabilities to this low-frequency, finite (m, n) GAM. Because of its existence also as an axisymmetric mode, however, the new GAM may in addition be of importance for the turbulent transport in rotating plasmas [4].

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