Methods for Detecting Acceleration Radiation in a Bose-Einstein Condensate

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We propose and study methods for detecting Unruh-like acceleration radiation effects in a Bose-Einstein condensate in a (1 + 1)-dimensional setup. The Bogoliubov vacuum of a Bose-Einstein condensate is used to simulate a scalar field theory, and accelerated atom dots or optical lattices serve as detectors of phonon radiation due to acceleration effects. In particular, we study the dispersive effects of the Bogoliubov spectrum on the ideal case of exact thermalization. Our results suggest that acceleration radiation effects can be observed using currently accessible experimental methods.

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One of the surprising fundamental consequences of relativistic quantum field theory is the dependence of the concept of particle number on the observer's state of motion. While inertial observers see the vacuum as empty, noninertial observers generally perceive this vacuum as populated with particles. Unruh [1] showed that a uniformly accelerated particle detector perceives the field in vacuum as a thermal state with temperature $k_B T_U = \hbar a/2\pi c$, where a is the proper acceleration. The Unruh effect is related to other particle creation effects in curved space-time, such as Hawking radiation and the Gibbons-Hawking cosmological thermalization effect [2]. Numerous experimental ideas for detecting the effect have been suggested. They include intense laser induced electron acceleration [3] and the passage of atoms through a cavity [4]. An expanding Bose-Einstein condensate (BEC) [5] or linear ion trap [6] have been proposed for detecting the Gibbon-Hawking effect [7].

In this Letter we propose to simulate and detect Unruh-like acceleration effects using atom dots (AD) [8] or optical lattices accelerated in a BEC. Since the relevant velocity for the Unruh temperature is the speed of sound, $c_s \approx 1$ mm/s, we find $T_U \approx 1\ \mathrm{nKs^2/m} \times a\ \mathrm{m/s^2}$. Currently feasible accelerations of optical lattices may reach $a \approx 5 \times 10^5\ \mathrm{m/s^2}$ so that the Unruh temperature can be significantly higher than the relevant energy scales, the AD minimal energy gap ($\omega_{\rm AD} \approx 100$ Hz ≈ 1 nK), and the BEC temperature.

Let us begin by recalling some features of the Unruh effect. A detector is modeled as a localized system with internal levels $|g\rangle$ and $|e\rangle = \sigma^+|g\rangle$ and energy gap ω_d (we use $\hbar = 1$), which moves along a trajectory $x_D(\tau)$ and $t(\tau)$, where τ is the detector's proper time. In the simplest case, a free scalar field ϕ, initially in its vacuum state, couples with the detector through

$$H_{\rm int} = g(e^{i\omega_d\tau}\sigma_+ + e^{-i\omega_d\tau}\sigma_-)\phi(x_D(\tau), t(\tau)). \quad (1)$$

By evaluating the transition amplitudes between the levels, it is then found that for inertial trajectories the detector remains unexcited, while for uniformly accelerated trajectories the detector becomes thermalized. This can be seen by evaluating the transition amplitudes to the lowest order in g. Inserting $x_D(\tau) = \frac{c^2}{a}\cosh\frac{a\tau}{c}$, $t(\tau) = \frac{c}{a} \times \sinh\frac{a\tau}{c}$, and the expression for a free field $\phi(x, t)$ in Eq. (1), one finds that a field mode ω has a time dependent coupling of the form $g_e(\tau, \omega) = \exp(i\frac{\omega c}{a}e^{-(a\tau/c)})$. This readily yields transition probabilities which satisfy $P_{\rm excitation}/P_{\text{de-excitation}} = e^{-\omega_d/k_B T_U}$.

It is important to note the following: (i) The appearance of the effective coupling $g_e(\tau, \omega)$ is sufficient in order to thermalize the detector. A similar coupling is also a landmark of the Hawking and cosmological thermalization effects. (ii) In the Unruh effect, property (i) is a direct consequence of the detector's *accelerated motion*. This can be easily seen [9] by noticing that the field mode ω is Doppler shifted in the detector's rest frame to $\omega'(\tau) = \omega_0 \frac{1-v/c}{\sqrt{1-(v/c)^2}} = \omega_0 e^{-a\tau/c}$. Therefore, the relevant collected phase factor becomes $\exp(i\int\omega(\tau)d\tau) = g_e(\omega, \tau)$. (iii) The Unruh effect is manifestly relativistic. Hence the interaction (1) is defined in the detector's rest frame, and the trajectory $x_{\rm DR}(t) = c\sqrt{t^2 + c^2/a^2}$ coincides with nonrelativistic acceleration only for short times.

The above points quantify, with increasing refinement, important aspects of the Unruh effect, which one wishes to simulate in a specific model. For example, (i) can be obtained by modifying the vacuum normal mode frequencies ω to $\omega(t) = \omega e^{-(at/c)}$, as proposed for an ion trap by changing the trap frequency [6], or by an expanding BEC [5]. In what follows we suggest a model that incorporates properties (i) and (ii), and finally shortly discuss possible realizations of (iii).

It is well known that small perturbations of the BEC Schrödinger field satisfy a relativisticlike Klein-Gordon equation with the speed of sound c_s playing the role of c [10]. Nevertheless, the transformation laws for a moving detector will remain nonrelativistic. We can therefore obtain the effective coupling constant (i) as a consequence of nonrelativistic Doppler shift by choosing a modified trajectory: $x_{\mathrm{Deff}}(t) = (c_s t + \frac{c_s^2}{a} e^{-at/c_s})$, which differs from the relativistic trajectory $x_{\mathrm{DR}}(t)$ above (when $c = c_s$), by $O(a^2 t^3/c_s)$ for short times and $O(c_s^2/a^2 t)$ for long times. The Doppler shift $\omega' = \omega_0(1 - v/c_s) = \omega_0 e^{-at/c_s}$ of the right moving phonons has the same time dependence as in the relativistic case, with $\tau \rightarrow t$. The left moving phonons will acquire a non-Unruh–like Doppler shift, but since the interaction with these phonons is off resonant their effect would be negligible. We hence expect that a detector that moves along x_{Deff} will be thermalized.

Consider then a setup with hyperfine levels a and d, where a forms a condensate described by the field Ψ. Level d will be used for an AD produced by a localized potential V_d [8,11] or by an optical lattice. It will be sufficient to consider only one level with a wave function $\psi_d(x)$ and the creation and destruction operators d and $d^\dagger$. Since V_d affects only atoms in the state d, in the absence of further coupling with the condensate, moving about V_d will not disturb the condensate state. We need, however, to make sure that nonadiabatic excitations of the AD are negligible. The adiabatic condition in this case can be derived by examining the extreme case for which the velocity changes abruptly from zero to c_s. In the frame of the harmonic oscillator the atom receives an energy of $mc_s^2/2$, which is 2 orders of magnitude smaller than $\hbar\omega_d$ [12]. Atomic levels then couple through elastic collisions $g_{ij} n_i n_j$, where g_{ij} is the interaction strength between state i and j of the atom. This interaction to lowest order redefines the detector's energy gap, producing self-interaction terms $g_{dd} d^\dagger d^\dagger dd$, where $d^\dagger$ is the operator that creates an atom in state d and displaces the field, but this displacement is negligible since the number of modes is large and the population of the detector would be chosen to be of the order of 1. A large g_{dd} is used [8] to simulate a two-level detector [Eq. (1)]. In the following we found it more convenient to assume small g_{dd}; hence the detector is a harmonic oscillator and we disregard elastic collisions from now on. Since the Unruh effect is unaffected by the nature of the detector, by using a harmonic oscillator we obtain a solvable model and still find the Unruh result.

We couple the AD and the BEC by laser induced Raman transitions described by interaction Hamiltonian

$$H_{\mathrm{int}} = \delta d^\dagger d + \Omega_a \int dx \psi_d(x)[d^\dagger \Psi(x) + \mathrm{H.c.}], \quad (2)$$

where Ω_a is the Rabi frequency, δ is the laser detuning, $\psi_d(x)$ the ground state wave function of the AD, and $\Psi(x)$ is the annihilation operator of an atom in state a at location x in the BEC. At first sight Eq. (2) lacks the number of nonconserving terms of Eq. (1), which are essential to the effect. However, our interest is in the resulting coupling with phonons. Using Bogoliubov theory we expand the field operator

$$\hat{\Psi}(x) = \phi(x) + \sum_k u_k(x) e^{-i\omega_k t} c_k + v_k(x) e^{+i\omega_k t} c_{-k}^\dagger, \quad (3)$$

where $\phi(x)$ is a c number, $u_k(x)$ and $v_k(x)$ are the phonon mode functions, and c_k their annihilation operators. This brings the BEC Hamiltonian to a free field form $H_{\mathrm{BEC}} = \sum_k \omega_k c_k^\dagger c_k$ and the spectrum $\omega_k = \sqrt{(c_s k)^2 + (\frac{k^2}{2m})^2}$ that is "relativistic," $\omega \approx k$, for $k < k_c = mc_s/\hbar$.

Inserting Eq. (3) into Eq. (2), and assuming that $\psi_d(x)$ extends over scales smaller than the phonon wavelength (the dominant coupling arises from long wavelengths), we obtain

$$H_{\mathrm{int}} = \delta d^\dagger d + \sqrt{n_a}\Omega_a(d + d^\dagger) + \sqrt{l_d}\Omega_a d^\dagger \times \sum_k (u_k(x_D) e^{-i\omega_k t} c_k + v_k(x_D) e^{i\omega_k t} c_{-k}^\dagger) + \mathrm{H.c.}, \quad (4)$$

where n_a and l_d are the effective number of condensate atoms at the AD and the size of the wave function ψ_d. For $k \ll k_c$, $u_k \approx v_k$, the last term coincides with Unruh's detector model Eq. (1). The second term $\sqrt{n_a}\Omega_a(d + d^\dagger)$ describes an additional interaction with the mean field, which can be eliminated using a two mode condensate with levels a and b that couple as in Eq. (2) via Raman transitions and with Rabi frequencies satisfying $\Omega_a = -\Omega_b$. A cancellation of the two terms is then obtained via the symmetry of the Hamiltonian. Alternatively, one can use a single mode condensate and remove the resulting displacement in the AD final state by applying the unitary $\exp[\frac{\sqrt{n_a}\Omega}{2\delta^2}(d - d^\dagger)]$. This approach requires a precise control of $\sqrt{n_a}$ [13].

Consider the effect of H_{int} on the AD when the condensate is in its ground state: $c_{k,\alpha}|\mathrm{BEC}\rangle = 0$. For uniform motion $x = vt$, the excitation amplitude is to first order:

$$i \int_{-T}^{T} dt \sum_k v_k(x_D(t)) e^{i[\omega_d + (1 - v/c_s)\omega_k]t} \xrightarrow[T \rightarrow \infty]{} \sum_k \delta(\omega_d + (1 - v/c_s)\omega_k).$$

Therefore as long as $v < c_s$, the detector remains unexcited. For the suggested noninertial trajectory x_D, and for positive momentum modes with $\omega < \omega_c$, $v_k(x) \sim u_k(x) \sim \exp(ikx)$, and the transition amplitudes reduces to $A_\pm(\omega) \simeq \Omega_a \propto \int_{-T}^{T} \exp(\pm i\omega_d t - i\omega_k c/a e^{-at/c}) dt$. This coincides with Unruh's expressions, with t replacing τ. Modes with $\omega > \omega_c$ (the cutoff frequency) will give rise to a deviation from Unruh's result.

The contribution to the transition probability arises mostly from the saddle point around the time $t \sim t_s(\omega) \equiv \frac{c_s}{a} \log\omega/\omega_d \pm \frac{1}{\sqrt{\omega_d a/c_s}}$ [14], and is given for modes with $\omega < \omega_c$ by $|A_\pm|^2 = \Omega_a^2(\pm 2\pi c_s/\omega_d a) \times [\exp(\pm 2\pi c\omega_d/a) - 1]^{-1}$, which is independent of ω. Since each mode effectively interacts with the AD at $t_s(\omega)$, it is possible to avoid the unwanted effect of modes with $\omega > \omega_c$ by limiting the duration of the experiment to $T < t_s(\omega_c)$. Then the total transition rate depends on the number of contributing modes N_c, the number of modes with $\omega < \omega_c$, and is given by $R_\pm = N_c|A_\pm|^2/T_c$, where $T_c = t_s(\omega_c)$. This term scales with a as the Bose-Einstein distribution.

It is important to note that the switching on and off of the laser for finite time also modifies the transition amplitude. In the simplest case of abrupt change in the coupling, the full transition probabilities can be approximated as $P_\pm(\omega) \approx |A_\pm(\omega) + \frac{1}{i\omega_d} e^{\mp i\omega_d t_s(\omega)}|^2$. Since different modes contribute at different times, the total contribution is effectively averaged and $P_\pm \propto |A_\pm|^2 + \frac{1}{\omega_d^2}$. The correction does not decrease with the energy gaps since $|A_\pm|^2$ scale as $(\omega_d a)^{-1}$. More generally we shall assume that the coupling starts and ends smoothly over a time scale γ by adding a regulator $e^{-t/\gamma}$, i.e., a slow decoupling function. It is important to note that the manipulating laser creates a stark shift of the order of Ω_a^2/δ; this stark shift is changed once the dot is present. Thus, a moving perturbing potential is created. This perturbing potential has to create phonons at a slower rate than the Unruh thermalization. This regime is achieved if the size of the dot is much smaller than the interparticle distance. Since the size of the dot is the size of the ground state which can be of the order of tens of nm, this requirement can be achieved for most experimental setups.

In the following we have assumed that the detector is accelerated for time T and that the experiment is repeated n times by moving the AD back and forth in the BEC. We have studied numerically a BEC with a finite number of N phonon modes and described the detector by a harmonic oscillator. The total state is then Gaussian and fully characterized by its covariance matrix, and the population and temperature of the detector are derived from the reduced covariance matrix of the AD [15].

We first considered the idealized case with $k_c \to \infty$. As is shown in Fig. 1 the effective temperature of the detector changes gradually until it reaches a final steady state after $n \sim 100$ repetitions. A large number of repetitions in needed since the time of each repetitions has to be short to avoid interaction with high frequency $\omega > \omega_c$, single particle modes. The temperature is slightly higher than the value of T_U since the finite decoupling time and the final coupling strength increase the average final energy of the steady state. With increasing γ we can approach the theoretical value of T_U. We have checked the final temperature for various values of the detector energy gaps, and the temperature remained unchanged in agreement with a thermal distribution, up to fluctuations of $\Delta T/T \sim 1\%$. The fluctuations are due to the finiteness of the number of modes and the interaction time.

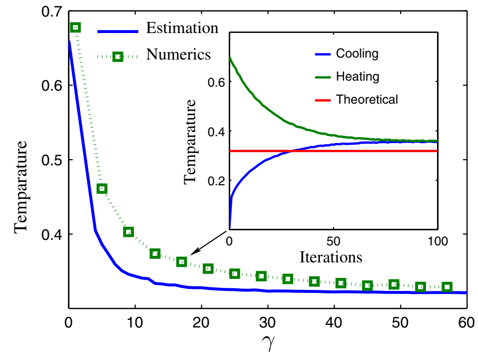

FIG. 1 (color online). Thermalization of an oscillator as a function of the smearing time γ. The field is modeled by 20 modes, $a = 2$, $\omega_d = 1$, and the coupling is $\Omega_a = 1/50$. The green squares are numerical results described in the text, and the solid blue line is derived by integration of the transition amplitudes $A_\pm(\omega)$. The inset shows the thermalization curve for $\gamma = 17$, where the red (medium gray) line shows the theoretical limiting temperature. The blue (light gray) and green (dark gray) graphs shows the cooling and the heating curves. The results are in units of ω_d. The thermal nature of the result is checked by calculating the trace norm distance between the final density matrix and the closet thermal state.

Next we extended the analysis to the full problem with a finite cutoff scale $\omega_c = c_s k_c$, which corresponds in a realistic BEC to more than 10 kHz and is 2 orders of magnitude larger than the minimal energy gap of the AD, which is limited by the fluctuations of the laser. There are two types of corrections. The first type is due to the changing dispersion relation; since the phase in the transition amplitude is now given by $e^{\pm i\omega_d t} e^{i(kx-\omega t)} = e^{\pm i\omega_d t} e^{i(ckt - cke^{-at}/a - \omega(k)t)} = e^{\pm i\omega_d t} e^{i((ck-\omega(k))t - cke^{-at}/a)}$, the detector's energy gap is corrected by $ck - \omega(k)$, which is always a negative quantity. For certain modes the effective detector gap can vanish, which implies a divergence in the resulting partial excitation probability. For higher modes the effective energy gap may then become negative, which causes a gradual population inversion since $P_\pm = \frac{2\pi c_s}{\pm\omega_d a} \times \frac{1}{e^{\pm 2\pi\omega_d c/a} - 1}$, the ratio for large frequency tends to unity. This cutoff effect would be felt once $\omega_d = ck - \omega(k)$, which is smaller than the field cutoff. The second type of correction comes from the modified momentum dependence of mode functions u_k and v_k. As k increases, v_k decreases to zero; hence for $T > t_c$ the temperature starts decreasing.

In order to observe the Unruh effect, we can reduce the effects of the above "ultrahigh" frequency corrections by selecting a sufficiently short time scale. Figure 2 displays the resulting final temperature for a numerical computation which includes all Bogoliubov theory corrections. The expected thermalization effect can be observed but due to the shorter interaction time requires a slightly higher number of repetitions, $n \sim 300$. In order to decrease the number of repetitions the initial state may be chosen close to the final temperature. Note that this choice does not affect the limiting temperature, as this is unique. In case a higher

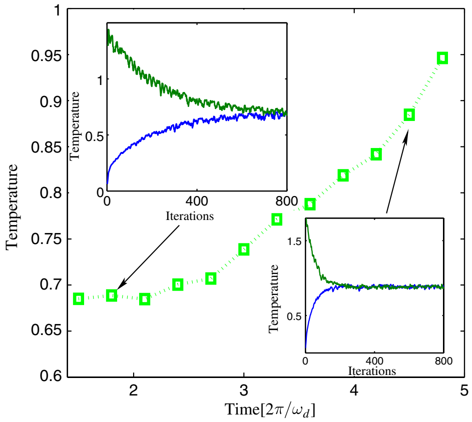

FIG. 2 (color online). The final temperature using the full Bogoliubov theory, as a function of the interaction time t_0 (with $\gamma = 0$). The ratio of the energy gap and the cutoff energy was taken as 1:500; hence deviations due to the effective energy gap are expected for $\omega \geq 120\omega_d$. This corresponds to acceleration times $t_0 \approx 1/a \log 120 \approx 2.4$. Indeed, the theoretically predicted temperature, $T \approx 0.67$, is obtained in this simulation for $t < t_0$.

initial temperature is given, the Unruh effect would result in cooling, as shown in the insets of Figs. 1 and 2. To avoid finite temperature corrections, we need to have the phonon number in the relevant interacting modes to be smaller than 1. For a BEC temperature of 50 nk this requires to start the interaction by tailoring the path at 1 kHz or an energy gap of this scale. It is important to stress that this proposal could only work in case state dependent potentials are realized; if the potential is only semistate dependent then the movement of the AD will excite the BEC resulting in non-Unruh excitations in the detector.

The measurement of the temperature could be realized by measuring the number of atoms in the d state using state selective resonance fluorescence detection [16] via quantum jump detection [17], which is highly efficient. It is important to stress that the measurement can be done at the end of the experiment after the coupling is switched off and the detector is fixed and if preferable experimentally is located far from the BEC.

The orders of magnitude are the following. The energy gap (detuning) can be between a minimal value of 100 Hz, which is limited by experimental precision, to a value of 1 kHz, which is limited by the field cutoff which is of the order of tens of kHz. The Rabi frequency Ω_a should be no more than 0.1δ and thus scales between 10 and 100 Hz. Assuming Unruh temperature of the order of the energy gap a maximum acceleration of 1 to 10 m/s^2 is required. The time of each repetition is then between 0.6 and 6 ms. This means that the rates for large BEC and strong coupling should be of the order 400 Hz, and these would decrease for smaller BEC and weaker couplings. The number of repetitions varies from a few hundreds to few repetitions depending on the size of the BEC and the required precision (the coupling strength).

The proposed scheme would only give the required results in a $1-d$ setting or in an elongated BEC for which the radial frequency is small enough not to interact with the detector. For higher dimensions the effective path we chose would not produce the correct relativistic behavior and the final temperature would be different to the Unruh temperature. Nevertheless, such an experiment would still have the potential to demonstrate the inequivalence of physical vacua for noninertial observers. We remark that a full relativisticlike realization may be achieved by choosing $\delta = 0$ in Eq. (2). This yields the Hamiltonian $H_r \approx \Omega(t) \times [\phi(d + d^\dagger) + (d - d^\dagger)\sum_k u_k(b_k - b_k^\dagger)] \equiv \Omega H'$, for $k < k_c$. We can then set the overall factor $\Omega(t)$ by shaping the laser intensity: $\Omega(t) \propto \frac{d\tau}{dt} = \frac{1}{\sqrt{t^2 + c_s^2/a^2}}$. Since $\int H_r dt = \int H' d\tau$, this effectively recovers the Unruh effect for a uniformlike accelerating trajectory.

In conclusion, we found that a moving AD or an atomic lattice in a condensate can be used to detect acceleration radiation effects that are analogous to the Unruh effect. Our results indicate that the measurability of such effects is within reach of current methods. We hope that the analogy that we are drawing may also be useful to interpret what happens when one moves a particle in a condensate with some acceleration.

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