Direct Coupling Between Magnetism and Superconducting Current in the Josephson φ_0 Junction

A. Buzdin

Institut Universitaire de France and Université Bordeaux I; CNRS; CPMOH, F-33405 Talence, France
(Received 14 April 2008; published 5 September 2008)

We study the proximity effect between conventional superconductor and magnetic normal metal with a spin-orbit interaction of the Rashba type. Using the phenomenological Ginzburg-Landau theory and the quasiclassical Eilenberger approach it is demonstrated that the Josephson junction with such a metal as a weak link has a special nonsinusoidal current-phase relation. The ground state of this junction is characterized by the finite phase difference φ_0, which is proportional to the strength of the spin-orbit interaction and the exchange field in the normal metal. The proposed mechanism of the φ_0 junction formation gives a direct coupling between the superconducting current and the magnetic moment in the weak link. Therefore the φ_0 junctions open interesting perspectives for the superconducting spintronics.

DOI: 10.1103/PhysRevLett.101.107005 PACS numbers: 74.50.+r, 74.45.+c, 76.50.+g

Usually the current-phase relations in Josephson junctions near the critical temperature are sinusoidal $j(\varphi) = j_c \sin(\varphi)$, but with lowering temperature the contribution of the higher harmonic terms $\sim j_n \sin(n\varphi)$ can be observed. However, if the time reversal symmetry is preserved the current-phase relation is always antisymmetric $j(-\varphi) = -j(\varphi)$ [1]. Without this restriction a more general $j(\varphi) = j_0 \sin(\varphi + \varphi_0)$ dependence is also possible, and the generic expression for the current in the pioneering work of Josephson [2] incorporates this possibility. In fact, such current-phase relations have been predicted for Josephson coupling involving the unconventional superconductors [3–6]. The experimental verification of these predictions is still lacking.

In the present work we demonstrate that the Josephson superconductor/normal metal/superconductor junctions ($S/N/S$) provide the realization of such unusual current-phase relations $j(\varphi) = j_0 \sin(\varphi + \varphi_0)$ for the case of conventional superconductors when the normal layer is a noncentrosymmetric, i.e., with broken inversion symmetry (BIS) magnetic metal. Further on we will call this junction the "φ_0 junction." The phase shift φ_0 is proportional to the magnetic moment, and therefore these φ_0 junctions serve as examples of systems with direct coupling between magnetic moment (internal exchange field) and superconducting current. This opens an interesting field of application of φ_0 junctions in superconducting spintronics. Varying the N layer thickness we may easily control the phase shift φ_0. Note that the considered situation is different from the case of the Josephson junction with dominating second sinusoidal harmonic; see [6,7] and references cited therein. In such a case at the ground state an arbitrary phase drop across the junction may exist if the sign of the second harmonic is negative. However, in these systems it is impossible to have the direct coupling between magnetic exchange field and superconducting phase, and the properties of these junction are very different from that of the φ_0 junctions considered here.

Before addressing the problem of the proximity effect between conventional superconductor and BIS magnetic metal, it is useful to recall that recently BIS superconductors attracted a lot of attention. Namely, the heavy fermion superconductor $CePt_3Si$ provides a famous example of the superconductivity and antiferromagnetism coexistence in the noncentrosymmetric compound [8]. Now the number of superconductors without inversion symmetry approaches 12, and during the past years their properties were under intense studies from both theoretical and experimental points of view; see [9–14] and references cited therein. In the presence of the magnetic field the lack of inversion symmetry leads to the spatially modulated helical superconducting phase [10,13,15,16].

The Josephson junctions between conventional superconductor and BIS superconductors should reveal some special features [13,17]. We stress that the aim of this work is to study the very different situation: the Josephson junction between conventional superconductors with a weak link formed by a BIS magnet. As an example of the suitable candidates for such interlayer, we may cite MnSi and FeGe. The anomalous properties of studied junctions are related to the particularities of the superconducting proximity effect in the BIS metal.

On the microscopical level the special character of the electron spectrum in BIS metal may be described by the Rashba-type spin-orbit coupling [18]: $\alpha(\vec{\sigma} \times \vec{p}) \cdot \vec{n}$, where $\vec{n}$ is the unit vector along the asymmetric potential gradient and parameter α describes its strength. To illustrate the unusual properties of the BIS Josephson junction we start with a simple Ginzburg-Landau (GL) approach. Describing the weak link by the GL theory we assume the temperature is above the critical temperature of the material of the weak link and the superconducting order parameter is induced only by the superconducting banks. As it has been noted in [10,13], the Rashba-type interaction in the presence of the field $\vec{h}$ acting on the electron spin leads to the following GL free energy density:

$$F = a|\psi|^2 + \gamma|\vec{\mathbf{D}}\psi|^2 + \frac{b}{2}|\psi|^4 - \varepsilon\vec{n}\cdot\{\vec{h}\times[\psi(\vec{\mathbf{D}}\psi)^* + \psi^*(\vec{\mathbf{D}}\psi)]\}, \quad (1)$$

where ψ is the superconducting order parameter, $D_i = -i\partial_i - 2eA_i$, and the coefficient a becomes zero at some temperature T_{c0}: $a \sim (T - T_{c0})$. The special character of the BIS superconductivity is described by the last term in (1) with the coefficient $\varepsilon \sim \alpha$. In principle the field $\vec{h}$ may be created by the applying external field, but we suppose that BIS metal is a ferromagnet and $\vec{h}$ is an internal exchange field (which is assumed to be small to avoid the necessity of adding higher order derivative terms in (1) [19]). Note that the origin of the spin-orbit contribution in (1) may be intrinsic resulting from the crystal symmetry or extrinsic. The latter case corresponds, for example, to the ferromagnetic layer with the in-plane magnetization and a varying thickness. To be more specific, below we consider the intrinsic BIS metal. Schematically the Josephson junction is presented in Fig. 1. Further on, to concentrate on the special properties of this junction we neglect the orbital effect. In Fig. 1 the magnetization is along the z axis and the demagnetization factor $N = 1$ and then the internal magnetic field in the junction $\vec{H}_i = -4\pi\vec{M}$. Therefore the magnetic induction $\vec{B} = \vec{H}_i + 4\pi\vec{M} = \vec{0}$ and then for this geometry the orbital effect is vanishing. Alternatively we may assume the magnetization lying in the x-y plane (in such a case the $\vec{n}$ vector must be along the z axis).

In the considered case of the weak link of the length $2L$, see Fig. 1, the order parameter depends only on the coordinate x and the corresponding GL equation is

$$a\psi - \gamma\frac{\partial^2\psi}{\partial x^2} + 2i\varepsilon h\frac{\partial\psi}{\partial x} = 0, \quad (2)$$

where we have neglected the nonlinear term assuming the superconducting banks being at a temperature slightly below their critical one. The solution of (2) is straightforward:

$$\psi = A\exp(q_1x) + B\exp(q_2x), \quad (3)$$

with $q_{1,2} = i\tilde{\varepsilon} \pm \sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2}$, where $\tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$ and the condition $\frac{a}{\gamma} > \tilde{\varepsilon}^2$ assures that the weak link is in the normal state (i.e., the temperature is above the intrinsic critical temperature of BIS metal $a > a_c = \gamma\tilde{\varepsilon}^2$). To illustrate the particularity of the proximity effect in the BIS system, let us consider the contact of superconductor with a metal occupying the $x > 0$ half-space. In such a case the order parameter distribution is described by the decaying exponent in (3) $\psi \sim \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}})$. The difference with the usual proximity effect is that the order parameter decay is accompanied by the superconducting phase rotation. Therefore, in the weak link the phase difference proportional to its length would be accumulated. Because of the 2π periodicity the actual phase difference is limited by the interval $(0, 2\pi)$.

To calculate the current we need to determine the coefficients A and B in (3) from the boundary conditions at the contact with the superconductors. For illustration we assume that there is no barrier at the interface and we may use the rigid boundary conditions [1] (i.e., the normal conductivity of the BIS metal is much smaller than that of the superconducting bank). Therefore the coefficients A and B are obtained from the continuity conditions for ψ at $x = \pm L$: $\psi(\pm L) = |\Delta|\exp(\mp i\frac{\varphi}{2})$, with $|\Delta|$ being the modulus of the order parameter in the banks and φ is the superconducting phase difference across the junction. Taking into account the new expression for the superconducting current coming from (1) (note that there is additional contribution to the current from the spin-orbit term) and performing the corresponding calculation, we readily find in the limit of the long junction $L\sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2} \gg 1$

$$j = 4e\gamma|\Delta|^2\sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2}\sin(\varphi + 2\tilde{\varepsilon}L)\exp\left(-2\sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2}L\right). \quad (4)$$

The current-phase relation

$$j(\varphi) = j_c\sin(\varphi + \varphi_0) \quad (5)$$

implies that the junction energy

$$E_J \sim -j_c\cos(\varphi + \varphi_0), \quad (6)$$

and the minimum energy corresponds to the nonzero phase difference $\varphi = -\varphi_0$. Naturally the Josephson junction energy may be also directly obtained from the functional (1). Note that (6) describes the transition from 0 to π junction when φ_0 vary from 0 to $-\pi$. However, in contrast to the 0-π transition in superconductor/ferromagnet/superconductor ($S/F/S$) junctions [19], the critical current does not vanish at the transition but remains constant. The presence of the ground state phase difference φ_0 is a

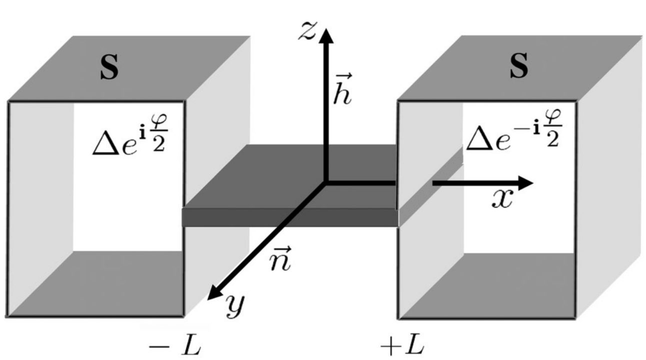

FIG. 1. Geometry of Josephson junction with BIS metal as a weak link. The exchange field is directed along the z axis and the **n** vector is along the y axis. The total length of the weak link is $2L$.

consequence of BIS and the $h = h_z$ component of the spin field in the weak link: $\varphi_0 = \frac{2\varepsilon hL}{\gamma}$. Note that we assumed the continuity of the order parameter at the bank. In general, the interface barrier provokes a jump of the order parameter and in our case it produces some additional phase rotation which may even exceed φ_0 for the large values of the interface barrier.

The approach on the basis of the GL functional though very insightful cannot adequately describe the systems with strong internal exchange field $h > T_c$, which is usually the case in the magnetic metals. Therefore we also present the theory of the φ_0 junction on the basis of the quasiclassical Eilenberger equations [20].

Provided the spin-orbit interaction is smaller than the characteristic electron energy scale E_F we may consider the Rashba term as an external potential in the standard scheme of derivation of the Eilenberger equations (this implies $\alpha \ll v$, where v is a Fermi velocity). This approach has been successfully applied for the description of the superconducting state in $CePt_3Si$ [21]. In principle, to have a complete description of the BIS junction we need to solve the Eilenberger equations also in the superconducting banks and take into account the suppression of the superconducting order parameter near the interfaces. The full treatment of this problem requires the extended numerical calculations [1]. Below we would like to concentrate on the peculiar properties of the BIS junctions. That is why we provide the results for some cases which can be treated analytically. The resulting coupled equations for anomalous Green function f_{ij} $(\mathbf{v}, \mathbf{r})$ (matrix in spin space) are rather cumbersome for the 3D case but strongly simplified in the 2D or 1D case—they are decoupled. Namely, for the geometry in Fig. 1 and supposing the weak link to be quasi-2D (in x-y plane), the Eilenberger equations in clean limit in the region $-L < x < L$ read

$$\begin{aligned}\left(\omega + \frac{v_x}{2}\frac{\partial}{\partial x}\right)f_{12} + \left(ih + \frac{\alpha}{2}\frac{\partial}{\partial x}\right)f_{12} = 0,\\ \left(\omega + \frac{v_x}{2}\frac{\partial}{\partial x}\right)f_{21} - \left(ih + \frac{\alpha}{2}\frac{\partial}{\partial x}\right)f_{21} = 0.\end{aligned} \tag{7}$$

For the junction Fig. 1 the superconducting order parameter in the banks may be considered constant as a transverse dimension of the BIS metal is small. Near T_c the Eilenberger equation in the bank reads $(\omega + \frac{v_x^s}{2}\frac{\partial}{\partial x})f_{12} = \Delta_{R,L}$, where v^s is a Fermi velocity in superconductor and the equation for f_{21} is obtained by the substitute $\Delta \to -\Delta$. From this equation it follows that for $\omega > 0$ and $v_x^s > 0$ the function f_{12} is constant in the left bank $f_{12} = (|\Delta|/\omega) \times \exp(i\frac{\varphi}{2})$, while for $v_x^s < 0$ it is constant in the right bank $f_{12} = (|\Delta|/\omega)\exp(i\frac{\varphi}{2})$ [22,23].

Using the corresponding continuity conditions at $x = \pm L$ for the functions f_{ij} at the boundary with superconductors we may readily calculate them [22]. Note that the triplet components of f_{ij} vanish: $f_{11} = f_{22} = 0$. Knowing the Green functions readily permits us to calculate the supercurrent density flowing trough the junction. At temperature close to the critical temperature T_c of the banks at the lowest $|\Delta|^2$ approximation

$$\begin{aligned} j = -ieN(0)\pi T_c \sum_{\omega} \langle v_x [f_{12}(\mathbf{v}, x) f_{12}^{+}(\mathbf{v}, x) \\ + f_{21}(\mathbf{v}, x) f_{21}^{+}(\mathbf{v}, x)] \rangle, \end{aligned} \tag{8}$$

where $N(0)$ is the density of state at the Fermi level. In the limit of the long junction $L > v/h$, the main contribution in (8) comes from the directions $|v_x| \lesssim v$, and the formula for the current takes a very simple form:

$$j(\varphi) = j_0 \sin\left(\varphi + \frac{4\alpha hL}{v^2}\right) \frac{\cos(\frac{4|h|L}{v} + \frac{\pi}{4})}{\sqrt{\frac{4|h|L}{v}}}. \tag{9}$$

Here $j_0 = eN(0)\frac{v\Delta^2}{T_c}(\frac{\pi}{2})^{3/2}$ and in the absence of the spin-orbit interaction $\alpha = 0$ this expression coincides with that for $j(\varphi)$ for the 2D $S/F/S$ junction [24]. Comparing (9) with $j(\varphi)$ from GL theory (4), we see that the phase shift $\varphi_0 = \frac{4\alpha hL}{v^2}$ in both cases is proportional to the strength of the spin-orbit interaction and the product hL. On the other hand, the critical current in (9) oscillates with L changing its sign. This is a typical behavior inherent to the $S/F/S$ junctions with the strong exchange field $h \gg T_c$ [19]. Such oscillations are absent in our GL approach (4) as it is adequate for $h \lesssim T_c$, otherwise the gradient term in [1] changes its sign and it is needed to introduce the higher derivatives terms. Such a modified GL functional indeed qualitatively describes the oscillatory behavior of the superconducting order parameter at S/F proximity effect [19].

In the 1D model of the weak link (single channel approximation) the very similar to (9) current-phase dependence is obtained

$$j(\varphi) = eN(0)\frac{\pi v \Delta^2}{2T_c} \sin\left(\varphi + \frac{4\alpha hL}{v^2}\right)\cos\left(\frac{4|h|L}{v}\right). \tag{10}$$

We considered a weak link in the framework of Eilenberger equations in the clean limit (ballistic regime). In the diffusive regime the very convenient approach is provided by the Usadel equations [25] for the Green functions integrated over Fermi surface $F_{ij}(\mathbf{r}) = \langle f_{ij}(\mathbf{v}, \mathbf{r})\rangle$. The calculation of the current on the basis of the Usadel approach gives us also the expression which may be presented in the form (5). Therefore the formation of the φ_0 junction by the BIS magnets is a very general phenomenon which may be observed in both clean or dirty limits.

To summarize, in all approaches we obtain the Josephson junction with unusual current-phase relations $j(\varphi) = j_c \sin(\varphi + \varphi_0)$, where the phase shift φ_0 is determined by the z component of the internal magnetic field. Though our model is applied for the weak spin-orbit interaction $\alpha \ll v$, we may expect that qualitatively the phase-shift effect would be the same for the systems with strong spin-orbit interaction $\alpha \sim v$. In this case the characteristic length of the phase shift is the same as for the 0-π transition in $S/F/S$ junctions, i.e., several nanometer. Therefore we may believe that the formation of the φ_0 Josephson junction is inherent to all weak links or barriers with magnetic BIS metals.

The $S/F/S$ Josephson junction may have zero or π phase difference in the ground state depending on the length of the weak link. In contrast, in the φ_0 junction the ground state is always different from zero and π states (except the occasional events $\varphi_0 = \pi n$). In the superconducting ring with the π junction the spontaneous current appears [26] if the parameter $k = \frac{c\Phi_0}{2\pi L j_c} < 1$, here L is the inductance of the system. For the φ_0 junction the system energy

$$E(\varphi) = \frac{j_c}{2e}\left(-\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2}\right). \tag{11}$$

Therefore the minimum energy is achieved for the phase difference satisfying the equation

$$\sin(\varphi + \varphi_0) + k\varphi = 0, \tag{12}$$

which always has a nonzero solution and then the φ_0 junction will always generate the spontaneous current with the flux $\Phi = -\Phi_0(\frac{\varphi_0}{2\pi})(1 - k)$ in the $k \ll 1$ limit. The SQUID with one normal and another φ_0 junction would reveal the shift of the diffraction pattern by φ_0. Note also that the φ_0 Josephson junctions may serve as a natural phase shifter in the superconducting electronics circuits.

The very important property of the discussed φ_0 junction is that it provides a direct mechanism of the coupling between supercurrent and magnetic moment—indeed, the phase shift φ_0 is proportional to the z component of the spin field. This means that the precessing magnetization will be directly coupled with the current which opens new interesting perspectives to study the coupled magnetic and current dynamics in Josephson junctions. Applying the voltage to the φ_0 junction we obtain the Josephson generation, and the magnetic moment of the weak link will experience the effective field varying with Josephson frequency. If this frequency is close to the ferromagnetic resonance frequency, it may be an efficient way to generate the spin precessing. Inversely, the spin precessing in the weak link would generate superconducting current in the circuit with the φ_0 junction.

Finally, we note that even in the centrosymmetric compounds the inversion symmetry is broken near the surface. This means that locally the Rashba-type interaction will be present there and then the Josephson junction made by two superconducting electrodes attached to the surface of ferromagnetic metal would be a φ_0 junction.

The author is grateful to M. Houzet, E. Goldobin, J. Cayssol, and F. Konschelle for useful discussions and comments. This work was supported by French ANR project "ELEC-EPR."

[1] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[2] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[3] V. B. Geshkenbein and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 306 (1986) [JETP Lett. **43**, 395 (1986)].
[4] S. Yip, Phys. Rev. B **52**, 3087 (1995).
[5] M. Sigrist, Prog. Theor. Phys. **99**, 899 (1998).
[6] S. Kashiwaya, and Y. Tanaka, Rep. Prog. Phys. **63**, 1641 (2000).
[7] E. Goldobin *et al.*, Phys. Rev. B **76**, 224523 (2007).
[8] E. Bauer *et al.*, Phys. Rev. Lett. **92**, 027003 (2004).
[9] S. Fujimoto, J. Phys. Soc. Jpn. **76**, 051008 (2007).
[10] K. V. Samokhin, Phys. Rev. B **70**, 104521 (2004).
[11] K. V. Samokhin, E. S. Zijlstra, and S. K. Bose, Phys. Rev. B **69**, 094514 (2004).
[12] K. V. Samokhin, Phys. Rev. Lett. **94**, 027004 (2005).
[13] R. P. Kaur, D. F. Agterberg, and M. Sigrist, Phys. Rev. Lett. **94**, 137002 (2005).
[14] P. A. Frigeri, D. F. Agterberg, A. Koga, and M. Sigrist, Phys. Rev. Lett. **92**, 097001 (2004).
[15] V. M. Edelstein, Phys. Rev. Lett. **75**, 2004 (1995).
[16] O. Dimitrova, and M. V. Figelman, Phys. Rev. B **76**, 014522 (2007).
[17] N. Hayashi, Ch. Iniotakis, M. Machida, and M. Sigrist, Physica (Amsterdam) **468C**, 844 (2008).
[18] E. I. Rashba, Fiz. Tverd. Tela (Leningrad) **2**, 1224 (1960) [Sov. Phys. Solid State **2**, 1109 (1960)]; Yu. A. Bychkov and E. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)].
[19] A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
[20] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[21] N. Hayashi, K. Wakabayashi, P. A. Frigeri, and M. Sigrist, Phys. Rev. B **73**, 024504 (2006).
[22] A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 147 (1982) [JETP Lett. **35**, 178 (1982)].
[23] I. Baladié and A. Buzdin, Phys. Rev. B **64**, 224514 (2001).
[24] F. Konschelle, J. Cayssol, and A. Buzdin (to be published).
[25] L. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[26] L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 314 (1977) [JETP Lett. **25**, 290 (1977)].