Intra-Landau-Level Cyclotron Resonance in Bilayer Graphene

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Interaction driven integer quantum-Hall effects are anticipated in graphene bilayers because of the near degeneracy of the eight Landau levels which appear near the neutral system Fermi level. We predict that an intra-Landau-level cyclotron resonance signal will appear at some odd-integer filling factors, accompanied by collective modes which are nearly gapless and have approximate $k^{3/2}$ dispersion. We speculate on the possibility of unusual localization physics associated with these modes.

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Introduction.—Because the Zeeman spin splitting in most two-dimensional electron systems (2DESs) is much smaller than the Landau-level (LL) separation, the magnetic band spectrum usually consists of narrowly spaced doublets. When one of these doublets is half filled and disorder is weak, Coulomb interaction physics leads to ferromagnetism, i.e., to spontaneous spin polarization in the absence of a Zeeman field [1–3]. In some circumstances [4] other approximate Landau-level degeneracies occur, often associated with layer degrees of freedom. These can also lead to broken symmetries which induce quasiparticle gaps and hence interaction driven integer quantum-Hall effects. The case of bilayer 2DESs is particularly interesting because the *which-layer* degree of freedom doubles Landau-level degeneracies and leads to exciton condensation [5,6] at odd filling factors and to canted antiferromagnetic states [7] at even filling factors. In this Letter, we address the still richer case of graphene bilayer 2DESs in which chiral bands lead to an additional degeneracy doubling [8] at the Fermi energy of a neutral system. Bilayer graphene's Landau-level octet is already apparent in present experiments [9] through the $8(e^2/h)$ Hall conductivity jump between well formed plateaus at Landau-level filling factors $\nu = -4$ and $\nu = +4$. We anticipate that when external magnetic fields are strong enough or disorder is weak enough [10], interactions will drive quantum-Hall effects at the octet's seven intermediate integer filling factors. We predict that these quantum-Hall ferromagnets (QHFs) will exhibit unusual intra-Landau-level cyclotron modes at odd filling factors, and that the collective mode excitations at these filling factors are nearly gapless even when there is no continuous symmetry breaking. Because the conductivity has Drude weight centered near zero energy, we speculate that localization physics and quantum-Hall related transport phenomena will also be anomalous.

Graphene bilayer Landau levels.—When trigonal warping [11] and Zeeman coupling are neglected, the low energy properties of Bernal stacked unbalanced bilayer graphene are determined by electron-electron interactions and a band Hamiltonian [8] $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm ext}$, where

$$\mathcal{H}_0 = \frac{1}{2m}\begin{pmatrix} 0 & \pi^{\dagger 2} \\ \pi^2 & 0 \end{pmatrix} \tag{1}$$

and the influence of an external potential difference Δ_V between the layers is captured by

$$\mathcal{H}_{\rm ext} = \xi\Delta_V\left[\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{v^2}{\gamma_1^2}\begin{pmatrix} \pi^\dagger\pi & 0 \\ 0 & -\pi\pi^\dagger \end{pmatrix}\right]. \tag{2}$$

In Eqs. (1) and (2), $\vec{\pi} = \vec{p} + (e/c)\vec{A}$ is the 2D kinetic momentum, $\pi = \pi_x + i\pi_y$, the 2×2 matrices act on the pseudospin degree of freedom associated with the two low energy sites [8] (the top and bottom layer sites without a near neighbor in the opposite layer), v is the single-layer Dirac velocity, $\gamma_1 \sim 0.4$ eV is the interlayer hopping amplitude, and the effective mass $m = \gamma_1/2v^2 \approx 0.054 m_e$. $\mathcal{H}$ describes both K ($\xi = 1$) and K' ($\xi = -1$) valleys provided that we choose the pseudospin representation $(A, \tilde{B})$ for K and $(\tilde{B}, A)$ for K'.

Defining the usual raising and lowering Landau-level ladder operators $a^\dagger$, a with $a^\dagger = (l_B/\sqrt{2}\hbar)\pi$, where $l_B = (\hbar c/eB)^{1/2} = 25.6/(B[\mathrm{T}])^{1/2}$ nm is the magnetic length, the zero-energy eigenstates of $\mathcal{H}_0$ can be identified using the property that $a^2\phi_n = 0$ for 2D orbitals with Landau-level index $n = 0, 1$. In bilayer graphene the $n = 0$ and $n = 1$ orbital Landau levels are members of the same octet. This peculiarity is behind most of the physics explored in this Letter. Neutral bilayer graphene's Landau-level octet is the direct product of three $S = 1/2$ doublets: real spin and which-layer [12] pseudospins (as in a normal bilayer), and the Landau-level pseudospin $n = 0, 1$ degree of freedom which is responsible for new physics. Zeeman coupling produces real spin-splitting Δ_Z while Δ_V gives rise to layer splitting as in normal bilayers, but also to a small splitting of the Landau-level pseudospin which plays a central role in the physics: $\Delta_{\rm LL} = \Delta_V\hbar\omega/\gamma_1 \equiv \hbar\omega_{\rm LL}$, where $\hbar\omega = 2\hbar^2v^2/l_B^2\gamma_1 = 2.14(B[\mathrm{T}])$ meV.

Octet Hund's rules.—The octet HF Hamiltonian [13] contains single-particle pseudospin splitting fields and direct and exchange interaction contributions:

$$\langle n\tau\alpha|\mathcal{H}_{\rm HF}|n'\sigma\beta\rangle = E_H(\rho_\tau - \rho_\beta) - \sum_{n_1n_2}(X^{+}_{n_2n'nn_1} + \xi_\tau\xi_\sigma X^{-}_{n_2n'nn_1})\rho^{n_1n_2}_{\tau\sigma\alpha\beta} + \Big(\xi_\tau\Delta_{\rm LL}\delta_{n,1}\delta_{n',1} - \frac{\Delta_Z}{2}\xi_\alpha\delta_{nn'} - \frac{\Delta_V}{2}\xi_\tau\delta_{nn'}\Big)\delta_{\alpha\beta}\delta_{\tau\sigma}, \quad (3)$$

where $n = 0, 1$ are LL indices, $\tau, \sigma = t$ (b) are valley indices, $\alpha, \beta = \uparrow (\downarrow)$ are spin indices, and $\xi_{\tau(\alpha)} = 1(-1)$ for the t (b) layer and $\uparrow (\downarrow)$ spins, respectively. In Eq. (3) $\rho_\tau = \sum_{n\alpha}\rho^{nn}_{\tau\tau\alpha\alpha}$ is the total electron density in layer τ. The density-matrix $\rho^{n_1n_2}_{\tau\sigma\alpha\beta} = \langle c^\dagger_{n_2\sigma\beta}c_{n_1\tau\alpha}\rangle$ must be determined self-consistently by occupying the lowest energy eigenvectors of $\mathcal{H}_{\rm HF}$. The Hartree-field E_H captures the electrostatic contribution to the bilayer capacitance, $E_H = (e^2/\epsilon l_B)(d/2l_B)$, and the exchange fields capture fermion quantum statistics:

$$X^{\xi}_{n_2n'nn_1} = \int\frac{d^2\mathbf{p}}{(2\pi)^2}v_\xi(\mathbf{p})F_{n_2n'}(\mathbf{p})F_{nn_1}(-\mathbf{p}). \quad (4)$$

In Eq. (4) $v_\pm$ are the symmetric and antisymmetric combinations of same (s) and different (d) layer electron-electron interactions ($v_s = 2\pi e^2/\epsilon q$, $v_d = v_s e^{-qd}$), and the form factors [$F_{00}(\mathbf{q}) = e^{-(ql_B)^2/4}$, $F_{10}(\mathbf{q}) = (iq_x + q_y)l_B e^{-(ql_B)^2/4}/\sqrt{2} = [F_{01}(-\mathbf{q})]^*$, and $F_{11}(\mathbf{q}) = [1 - (ql_B)^2/2]e^{(-ql_B)^2/4}$] reflect the character of the two different quantum cyclotron orbits.

The solution of the Hartree-Fock equations for balanced bilayers ($\Delta_V = 0$) is summarized in Fig. 1 using a Zeeman field strength corresponding to $B = 20$ T. The large gaps [$\sim(\pi/8)^{1/2}$ in $e^2/\epsilon l_B$ units] separating occupied and empty states at the odd-integer filling factors of primary interest justify our weak-coupling theory. The octet filling, proceeding in integer increments starting from filling factor $\nu = -4$, follows Hund's rule behavior: first maximize spin polarization; then maximize layer polarization to the greatest extent possible; then maximize Landau-level polarization to the extent allowed by the first two rules. For balanced bilayers the layer symmetric states (S) are filled before the layer antisymmetric states (AS). The first four states to be filled are (S, $n = 0, \uparrow$), (S, $n = 1, \uparrow$), (AS, $n = 0, \uparrow$), and (AS, $n = 1, \uparrow$) in this order. This sequence is then repeated for the next four states with down ($\downarrow$) spin. The Hund rules imply that the Landau-level pseudospin is polarized at all odd-integer filling factors between $\nu = -4$ and $\nu = 4$. The physics of this new type of pseudospin polarization is the main focus of this Letter. An important distinction between layer and Landau-level polarization is that the former is associated with spontaneous interlayer phase coherence whenever a Landau level occupies both layers simultaneously, whereas the latter polarization is driven by the Landau-level dependence of the microscopic Hamiltonian.

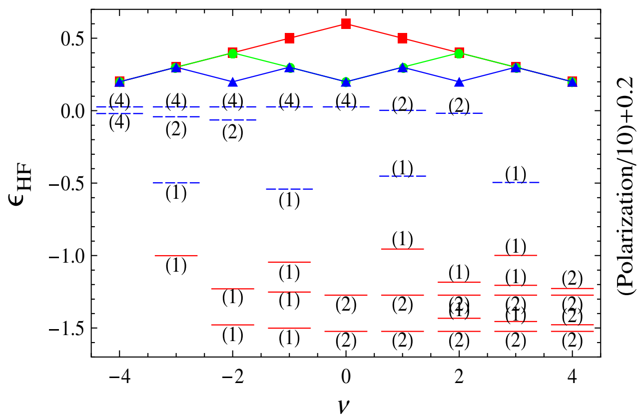

FIG. 1 (color online). Filling factor dependence of the integer filling factor HF theory occupied state (spectrum of the bilayer graphene octet at $\Delta_V = 0$). Energies of occupied (solid red lines) and unoccupied (dashed blue lines) are in units of $(\pi/2)^{1/2}e^2/\epsilon l_B$. The Zeeman field Δ_Z value in these units is 0.023 at a magnetic field of 20 T. Octet space fractional pseudospin polarizations offset for clarity: spin (red squares), valley (green circles), and LL pseudospin (blue triangles).

Octet quantum-Hall ferromagnets have an interesting and intricate dependence on the external potential Δ_V. Because the two layers are close together, a small value of Δ_V is sufficient to change the character of the layer polarization from the XY spontaneous-coherence form to an Ising polarization form in which one layer is occupied before the other. We find that for Δ_V larger than a critical value Δ_V^*, the layer filling proceeds by filling the top layer first. [For $\nu = -3$, $\Delta_V^* = 0.103(0.40)$ meV at $B = 20(50)$ T.] As we explain later, this filling sequence has qualitative consequences for the odd-integer filling factor LL pseudospin polarized states.

Landau-level pseudospin dipoles.—We now focus on the LL pseudospin fluctuations of a state with odd-integer filling factor, freezing spin, and layer degrees of freedom. The collective excitation spectrum of graphene bilayer octets as a function of ν and Δ_V will be described in full detail elsewhere [14]. Fluctuating LL spinors are linear combinations of $n = 0$ orbitals (even with respect to their cyclotron orbit center) and $n = 1$ orbitals (odd with respect to orbit center), and therefore carry an electric dipole proportional to the in-plane component of their pseudospin. Because dipole-dipole interactions are long range, they play a dominant role in the QHF long-wavelength effective action [3]. We find that

$$S[\vec{m}] = \int dt\Big[\int d^2q\,\vec{\mathcal{A}}\cdot\partial_t\vec{m} - E[\vec{m}]\Big], \quad (5)$$

where the first term is the Berry-phase contribution [3,15] and for small fluctuations away from $m_z = 1$ (full $n = 0$ polarization)

$$E[\vec{m}] = \frac{e^2}{\varepsilon l_B}\int d^2q\left[\frac{1}{2|q|}(\vec{q}\cdot\vec{m})^2 + \frac{\tilde{\Delta}_{\rm LL}}{2}(m_x^2 + m_y^2)\right], \quad (6)$$

where $\tilde{\Delta}_{\rm LL} = \Delta_{\rm LL}/(e^2/\epsilon l_B)$. The mass terms in Eq. (6) are due to the single-particle splitting between $n = 0$ and $n = 1$ levels, and the interaction term is due to electric-dipole interactions. The absence of interaction contributions to the mass terms is a surprise, since the interaction is Landau-level pseudospin dependent. We address this point below. Because of the in-plane electric dipoles associated with LL pseudospinors, the long-wavelength pseudo–spin-wave collective mode dispersion is not analytic: $\hbar\omega \to (\Delta_{\rm LL}^2 + \Delta_{\rm LL}e^2q/\epsilon)^{1/2}$, and for $\Delta_{\rm LL} \to 0$ is proportional to $q^{3/2}$ when exchange interactions are included in the energy functional. The in-plane dipoles are also responsible for the intra-Landau-level cyclotron resonance discussed below.

To explain the absence of interaction contributions to the mass terms and to address shorter-wavelength fluctuations, it is necessary to derive the action microscopically. It is convenient to temporarily restrict fluctuations to one space direction by considering Landau-gauge states in which the LL pseudospins at different guiding centers X fluctuate independently:

$$|\psi[z]\rangle = \prod_X (z_{0X}c_{0X}^\dagger + z_{1X}c_{1X}^\dagger)|0\rangle, \quad (7)$$

where the spinor components z_{nX} satisfy the normalization constraint $|z_{0X}|^2 + |z_{1X}|^2 = 1$. The corresponding imaginary-time action is

$$\mathcal{S}[\bar{z}, z] = \mathcal{S}_B + \mathcal{E} = \int_0^\beta d\tau \sum_{Xn} \bar{z}_{nX}\partial_\tau z_{nX} + \sum_{XX'}\left(\frac{1}{2}\sum_{n_i}[H(X-X') - F(X-X')]\bar{z}_{n_1X}z_{n_3X}\bar{z}_{n_2X'}z_{n_4X'} + \xi\Delta_{LL}\bar{z}_{1X}z_{1X'}\right), \quad (8)$$

where $\mathcal{S}_B$ is the Berry phase term and $\mathcal{E} = \langle\psi[z]|(\mathcal{H} + \mathcal{H}_{\rm int})|\psi[z]\rangle$ is the energy functional. In Eq. (8) the direct (H) and exchange (F) energy contributions depend on the LL pseudospin labels,

$$H_{n_3,n_4}^{n_1,n_2}(X) = \frac{1}{L_y}\int\frac{dq}{2\pi}v_qF_{n_1n_4}(q)F_{n_2n_3}(-q)e^{-iq_xX},$$
$$F_{n_3,n_4}^{n_1,n_2}(X) = \frac{1}{L^2}\sum_{\mathbf{q}}v_{\mathbf{q}}\delta_{q_y,X}F_{n_1n_3}(\mathbf{q})F_{n_2n_4}(-\mathbf{q}). \quad (9)$$

This action can be identified as the Schwinger boson [15] coherent state path integral representation of a model with pseudospins at each guiding center. We can introduce a bosonic creation operator $a_{nX}^\dagger$ corresponding to $\bar{z}_{nX}$ and let $\mathcal{E}[\bar{z}, z] \to \mathcal{H}[a^\dagger, a]$.

To analyze fluctuations around the HF mean field state, we use the linear spin wave approximation

$$a_{0X} \to 1 - \tfrac{1}{2}a_X^\dagger a_X, \qquad a_{1,X} \to a_X. \quad (10)$$

Taking the continuum limit $(1/L_y)\sum_X = \int dX/(2\pi l_B)$, the action describing harmonic fluctuations can be written in Fourier space as $S = S_0 + \delta S$, where

$$\delta S = \frac{e^2}{\varepsilon l_B}\int_0^\beta d\tau\sum_{\mathbf{q}}\left[\left(\frac{1}{2}\sqrt{\frac{\pi}{2}} + \xi_q\right)a_q^\dagger a_q + \frac{\lambda_q}{2}(a_qa_{-q} + a_q^\dagger a_{-q}^\dagger)\right], \quad (11)$$

with

$$\xi_q = \frac{|ql_B|}{2}e^{-(ql_B)^2/2} - \int dp\left(1 - \frac{p^2}{2}\right)J_0(ql_Bp)e^{-p^2/2} + \xi\tilde{\Delta}_{\rm LL},$$
$$\lambda_q = \frac{|ql_B|}{2}e^{-(ql_B)^2/2} - \int dp\frac{p^2}{2}J_2(ql_Bp)e^{-p^2/2}, \quad (12)$$

In Eq. (11) we have restored [16] two-dimensional wave vectors to recognize the system's spatial anisotropy. The first and second terms in the above expressions capture the direct (H) and exchange (F) contributions, respectively, and J_0 and J_2 are the zeroth and second order Bessel functions. The quadratic action in Eq. (11) has the familiar Bogoliubov form, and the energy dispersion of the collective mode is given by

$$\omega(q) = \frac{e^2}{\varepsilon l_B}\left[\left(\frac{1}{2}\sqrt{\frac{\pi}{2}} + \xi_q\right)^2 - |\lambda_q|^2\right]^{1/2}. \quad (13)$$

As shown in Fig. 2, this collective mode has a roton minimum at $ql_B \approx 2.3$ and approaches the Hartree-Fock theory band splitting for $q \to \infty$ as expected [1]. The surprising absence of interaction contributions to the gap at $q = 0$ can be understood by examining the dependence

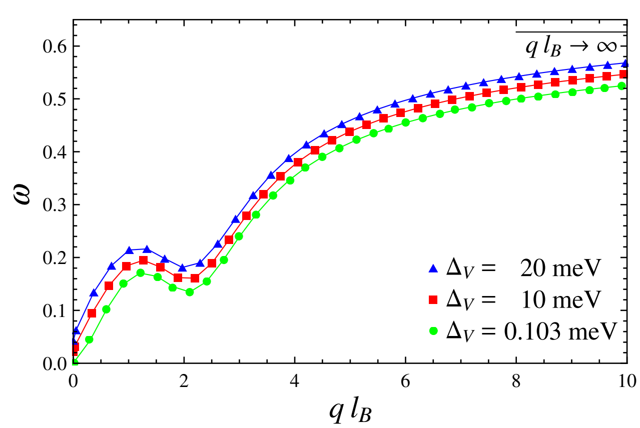

FIG. 2 (color online). Collective mode ω_q of the Landau-level pseudospin polarized state in units of interaction strength $e^2/\epsilon l_B = 11.2(B[\mathrm{T}])^{1/2}$ meV as a function of ql_B at different values of the external potential difference Δ_V at a magnetic field of 20 T. The solid black line indicates the $ql_B \to \infty$ asymptote for $\Delta_B = 0$.

of the uniform state interaction energy on global rotations in LL pseudospin space:

$$\frac{2\mathcal{E}[z]}{N_\phi} = -\frac{e^2}{\varepsilon l_B}\sqrt{\frac{\pi}{2}}\left[|z_0|^4 + \frac{3}{4}|z_1|^4 + 2|z_0|^2|z_1|^2\right]. \quad (14)$$

The factor in square brackets above is $1 - |z_1|^4/4$, independent of z_1 to quadratic order. Notice that because $\Delta_{\rm LL} < 0$ for $\nu = -1, 3$, the absence of interaction contributions to the gap implies that the fully spin-polarized state is unstable. The ground state at these filling factors is instead [14] an XY state with spontaneous phase order.

Intra-Landau-level cyclotron resonance.—Finally we show that the octet QHF will exhibit unusual intra-LL cyclotron modes at odd filling factors, focusing on the fully polarized $\nu = -3$, 1 cases. The dynamical conductivity $\sigma_\pm = \sigma_{xx} \pm i\sigma_{xy}$ can be evaluated using linear response theory. The projection of the current operator, $j_i = d\mathcal{H}/d\pi_i$, onto the octet space can be expressed in terms of LL pseudospins:

$$j_i = \frac{\xi\Delta_B}{m\gamma_1}\left(\frac{\hbar}{\sqrt{2}l_B}m_i + \frac{e}{c}\mathcal{A}_i^{\rm ext}(t)\right), \quad (15)$$

where the ac electric field $E_i = (1/c)d\mathcal{A}_i^{\rm ext}/dt$. The ac conductivity ($\xi = 1$) is most simply evaluated by solving the LL pseudospin equation of motion with the $j\cdot\mathcal{A}^{\rm ext}$ coupling included in the energy functional. We find that

$$\sigma_\pm(\omega) = \frac{N_\phi e\Delta_B}{m\gamma_1}\frac{1}{i(\omega \pm \omega_{\rm LL})}. \quad (16)$$

In the absence of interactions the conductivity has intra-octet peaks at the LL band-splitting frequency $\omega_{\rm LL}$, in addition to inter-Landau-level peaks which do not appear in the projected theory. Trigonal warping is expected [8] to have little influence on electronic properties over the broad field range over which $\hbar l_B^{-1} > v_3 m$. By performing explicit numerical calculations on the four-band model of bilayer graphene for typical bilayer quantum-Hall parameters, a magnetic field strength of 10 T and $\Delta_V \approx 10$ meV, we have verified that both the position and the oscillator strength of the intra-Landau-level resonance are shifted by less than 2%.

These low-frequency absorption peaks should be visible in microwave absorption experiments. The appearance of tunable low-frequency peaks in $\sigma(\omega)$ is a surprise that might be quite interesting from the point of view of the quantum-Hall localization physics, even in systems for which disorder dominates interactions. In normal quantum-Hall systems, peaks in $\sigma_\pm$ appear near the characteristic inter-Landau-level energy ω_c and the strong localization physics which leads to flat broad quantum-Hall plateaus occurs only in systems with $\omega_c\tau > 1$. We conjecture that one requirement for odd-integer filling factor plateaus within the graphene bilayer octet is that $\omega_{\rm LL}\tau > 1$. Since $\omega_{\rm LL}$ is proportional to Δ_V, the strength of the quantum-Hall effect can be tuned by a gate voltage which does not influence either the system's disorder or its total carrier density.

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