πN and $\pi\pi N$ Couplings of the $\Delta(1232)$ and Its Chiral Partners

K. Nagata,[1,*] A. Hosaka,[2] and V. Dmitrašinović[3]

[1]Department of Physics, Chung-Yuan Christian University, Chung-Li 320, Taiwan
[2]Research Center for Nuclear Physics, Osaka University, Mihogaoka 10-1, Osaka 567-0047, Japan
[3]Vinča Institute of Nuclear Sciences, lab 010, P.O. Box 522, 11001 Beograd, Serbia

(Received 20 April 2008; published 29 August 2008)

We investigate the interactions and chiral properties of the four spin-$\frac{3}{2}$ baryons $N^-(D_{13})$, $N^+(P_{13})$, $\Delta^+(P_{33})$, and $\Delta^-(D_{33})$ together with the nucleon. We construct the $SU(2)_R \times SU(2)_L$ invariant interactions between the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons with the aid of a new, specially developed spin and isospin projection technique for these baryon fields, where the chiral invariant interactions contain one- and two-pion couplings. We obtain simple relations for the coupling constants of the one- and two-pion spin-$\frac{1}{2}$-$\frac{3}{2}$ transitions terms. The relation for the one-pion interactions reasonably agrees with the experiments, which suggests that these spin-$\frac{3}{2}$ baryons are chiral partners.

DOI: 10.1103/PhysRevLett.101.092001 PACS numbers: 14.20.Gk, 12.39.Fe, 13.30.Eg, 14.65.Bt

Chiral symmetry is a key to understanding strong interaction. When the spontaneous breakdown $SU(2)_R \times SU(2)_L \to SU(2)_V$ occurs, the broken symmetry plays a dynamical role in various processes accompanying the Nambu-Goldstone bosons, i.e., the pions. Hadrons are then classified according to the residual symmetry $SU(2)_V$. If chiral symmetry is restored at high temperature or density, hadrons should form degenerate multiplets of the full chiral group representations (I_R, I_L), where I_R $[I_L]$ is the isospin for the $SU(2)_R$ $[SU(2)_L]$. Even in the broken phase, we may expect that hadrons are expressed as one or a simple superposition of chiral multiplets [1]. Familiar examples are the chiral mesons $(\sigma, \vec{\pi})$ and the vector mesons $(\vec{\rho}, \vec{a}_1)$. However, the role of chiral symmetry in the classification of the baryons has been less explored. It is in this regard that we can shed some new light.

The linear realization of chiral symmetry offers two advantages. First, the properties of different hadrons in the same chiral multiplet are related by the larger symmetry $SU(2)_R \times SU(2)_L$ than $SU(2)_V$, which reduces the number of free parameters. Second, it is convenient for the study of property changes towards the chiral restoration as functions of the chiral condensate. Having these advantages, the purpose of this Letter is to investigate the properties of baryons in a manner that respects chiral symmetry.

It is particularly interesting that the masses of $\Delta^+_{P_{33}}(1232)$ and $N^-_{D_{13}}(1520)$ (the superscript indicates the parity) were reproduced in the *quenched* lattice QCD and QCD sum rule [2–7], which validates to some extent the empirical assumption that the baryons are dominated by their $3q$ Fock components. Recently, we clarified the relation between the baryon fields' chiral multiplets and their quark structures [8]. For instance, the interpolating fields used in Refs. [2–7] belong to a chiral multiplet $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ [8]. This is our starting assumption where a set of spin-$\frac{3}{2}$ baryons form the multiplet $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ as chiral partners. We extend this idea to include two other four-star resonances, the $N^+_{P_{13}}(1720)$ and $\Delta^-_{D_{33}}(1700)$, following Jido *et al.* [9], where the four spin-$\frac{3}{2}$ baryons form a certain set of chiral multiplets, the so-called quartet scheme. Reference [9] mostly investigated the interactions between hadrons within the same spin and chiral multiplet. The inclusion of other hadrons, in particular, the ground state nucleon, enables us to test such a framework in comparison with the experimental data not only for masses but also for other quantities such as resonance decays and scatterings.

In this Letter, we construct an effective Lagrangian for four types of four-star resonances, $\Delta(1232)$, $N(1520)$, $N(1720)$, and $\Delta(1700)$ together with the ground state nucleon. We investigate the structures of the one- and two-pion couplings. We derive a relation among the one-pion coupling constants of the four baryon resonances, which agree well with the experimental data.

We begin with the nucleon's chiral multiplet. There are two possibilities, $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$, for the nucleon as a three-quark field. As usual, we assume the nucleon to be dominated by the fundamental representation. In addition, we have shown [8] that the nucleon belongs only to the fundamental representation, irrespective of the choice of the nucleon operators as long as it is a local three-quark field. There are also two possible chiral representations for the $\Delta(1232)$: $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ and $(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$. We choose the former one as is common in the literature (see Ref. [9]).

Now we define two types of diquarks: (i) a Lorentz vector isoscalar diquark V^μ $[I(J)^P = 0(1)^-]$ and (ii) an axial-vector isovector diquark $A^{\mu i}$ $[1(1)^+]$,

$$V^\mu = \tilde{q}\gamma^\mu q, \qquad A^{\mu i} = \tilde{q}\gamma^\mu\gamma_5\tau^i q, \tag{1}$$

where $\tilde{q} = q^T C(i\tau_2)\gamma_5$ is a transposed quark field. These diquarks form the chiral multiplet $(\frac{1}{2}, \frac{1}{2})$ similar to the σ and $\vec{\pi}$, which is a key ingredient to constructing chiral invariant interactions. We will come back to this point later. There is one possible operator with $I(J) = \frac{3}{2}(\frac{3}{2})$,

$$\Delta_A^{\mu i} = A_\nu^j \Gamma_{3/2}^{\mu\nu} P_{3/2}^{ij} q, \tag{2a}$$

and two with $I(J) = \frac{1}{2}(\frac{3}{2})$,

$$N_V^\mu = V_\nu \Gamma_{3/2}^{\mu\nu} \gamma_5 q, \tag{2b}$$

$$N_A^\mu = A_\nu^i \Gamma_{3/2}^{\mu\nu} \tau^i q. \tag{2c}$$

Here the isospin projection operators P_I^{ij} ($I = \frac{1}{2}, \frac{3}{2}$) satisfy the completeness relation $\delta^{ij} = P_{1/2}^{ij} + P_{3/2}^{ij}$. Similarly the *local* spin projection operators $\Gamma_J^{\mu\nu}$ ($J = \frac{1}{2}, \frac{3}{2}$) satisfy $g^{\mu\nu} = \Gamma_{1/2}^{\mu\nu} + \Gamma_{3/2}^{\mu\nu}$. The explicit form of the projection operators is given in Ref. [8]. Note that we need to use the *nonlocal* projection operators in order to obtain the spin-$\frac{3}{2}$ baryons containing only the physical degrees of freedom. However, the type of spin projection operators, local or nonlocal, makes no difference in the chiral transformation of the spin-$\frac{3}{2}$ baryons [8]. Our strategy is then firstly to use the local projection operators in the construction of the Lagrangian, and later to eliminate the spin-$\frac{1}{2}$ components in the calculations of the physical quantities, the one-pion decays in the present context.

Taking into account the Pauli principle, as implemented by the Fierz transformation here, and proper normalization, we define the baryon fields as

$$\Delta_1^{\mu i} = \frac{\Delta_A^{\mu i}}{2}, \tag{3a}$$

$$N_1^\mu = \frac{\sqrt{3}}{2}\frac{N_V^\mu}{2} + \frac{1}{2}\frac{N_A^\mu}{2\sqrt{3}}, \tag{3b}$$

where we factor out certain coefficients so as to show explicitly the normalized baryon fields $\Delta_A^{\mu i}/2$, $N_V^\mu/2$, and $N_A^\mu/2\sqrt{3}$. Note that the mixing between N_V^μ and N_A^μ results from the chiral transformations of V^μ and $A^{\mu i}$, and the coefficients of N_V^μ and N_A^μ are determined by the Fierz transformation [8]. The chiral transformation properties are given by

$$\delta_5^{\vec{a}} N_1^\mu = \eta\left(\frac{5}{3} i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5 N_1^\mu + \frac{4}{\sqrt{3}} i\gamma_5 \boldsymbol{a}\cdot\boldsymbol{\Delta}_1^\mu\right), \tag{4a}$$

$$\delta_5^{\vec{a}} \Delta_1^{\mu i} = \eta\left(\frac{4}{\sqrt{3}} i\gamma_5 a^j P_{3/2}^{ij} N_1^\mu - \frac{2}{3} i\tau^i\gamma_5 \boldsymbol{a}\cdot\boldsymbol{\Delta}_1^\mu + i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5\Delta_1^{\mu i}\right), \tag{4b}$$

where $\eta = +1$. Equations (4) show that N_1^μ and $\Delta_1^{\mu i}$ are chiral partners forming the multiplet $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$.

Even after establishing these chiral transformations, it is a nontrivial task to build chirally invariant interactions for these fields, so we shall develop a new method to project out the good spin and isospin parts from chiral invariant operators containing reducible products of three-quark fields. This projection technique is performed in two steps. First, we adopt interactions containing the quark, diquark, and meson fields, where baryons are described as direct products of a quark and a diquark. Chiral invariance of such interactions is shown by using the equivalence of the chiral transformations of $(V^\mu, A^{\mu i})$ and (σ, π^i). Second, because such composite operators are reducible under the chiral, spin, and isospin transformations, we perform the decomposition into irreducible spin and isospin parts containing only chiral-, spin-, and isospin-projected baryons.

As an illustration, let us consider the vector and axial-vector diquarks (V^μ, $A^{\mu i}$). As explained above, they belong to the chiral multiplet $(\frac{1}{2}, \frac{1}{2})$ similar to $(\sigma, \vec{\pi})$. The combination $V_\mu^2 + A_\mu^2$ is therefore a chiral scalar, which immediately leads to the chiral invariant term $\bar{q}(V_\mu^2 + A_\mu^2)U_5 q$, where $U_5 = \sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}$. The direct products of a quark and a diquark $V^\mu q$ and $A^{\mu i} q$ contain several kinds of baryons with $I(J) = \frac{1}{2}(\frac{1}{2})$, $\frac{1}{2}(\frac{3}{2})$, $\frac{3}{2}(\frac{3}{2})$ [10]. The decomposition into irreducible parts is carried out by using the completeness relations for both the spin and isospin projection operators. The resulting interaction Lagrangian is given by

$$\mathcal{L}_{\pi BB}^1 = g_1\left(\bar{\Delta}_{1\mu}^i U_5 \Delta_1^{\mu i} - \frac{3}{4}\bar{N}_{1\mu} U_5 N_1^\mu + \frac{1}{12}\bar{N}_{1\mu}\tau^i U_5 \tau^i N_1^\mu + \frac{\sqrt{3}}{6}\bar{N}_{1\mu}\tau^i U_5 \Delta_1^{\mu i}\right). \tag{5}$$

Here and throughout the present Letter, we omit the Hermite conjugate terms. Note that the relative weights for N_1^μ and $\Delta_1^{\mu i}$ are fixed in terms of Eqs. (4) without any dependence on free parameters [11]. In general, it is possible to insert a chiral invariant operator such as $(\sigma^2 + \pi^2)^n$ in Eq. (5). Here, however, we concentrate on the minimal interactions to reduce the number of free parameters.

Now, following Ref. [9], we introduce a new set of spin-$\frac{3}{2}$ baryons (N_2^μ, $\Delta_2^{\mu i}$) that have the $SU(2)_A$ transformation properties opposite in sign to those of $(N_1^\mu, \Delta_1^{\mu i})$; $(N_2^\mu, \Delta_2^{\mu i})$ transform as in Eqs. (4) but with $\eta = -1$. Owing to this minus sign, the $N_{2r}(N_{2l})$ fields have the same chiral transformations as $N_{1l}(N_{2r})$, respectively, where $N_{1r,l} = \frac{1}{2}(1 \pm \gamma_5)N_1$, and similarly for N_2, Δ_1, and Δ_2. The interchange of the chiral transformation properties of the left- and right-handed parts of N_1 and N_2 is the main feature of the mirror models [9,13–15]. Keeping this property in mind, the diagonal interactions for the mirror baryons takes a form similar to Eq. (5):

$$\mathcal{L}_{\pi BB}^2 = g_2\left(\bar{\Delta}_{2\mu}^i U_5^\dagger \Delta_2^{\mu i} - \frac{3}{4}\bar{N}_{2\mu} U_5^\dagger N_2^\mu + \frac{1}{12}\bar{N}_{2\mu}\tau^i U_5^\dagger \tau^i N_2^\mu + \frac{\sqrt{3}}{6}\bar{N}_{2\mu}\tau^i U_5^\dagger \Delta_2^{\mu i}\right). \tag{6}$$

Owing to the interchange of the left- and right-handed properties of $(N_1^\mu, \Delta_1^{\mu i})$ and $(N_2^\mu, \Delta_2^{\mu i})$, the following mass term is allowed:

$$\mathcal{L}_{BB} = -m_0(\bar{\Delta}^i_{1\mu}\Delta^{\mu i}_2 + \bar{N}_{1\mu}N^\mu_2). \tag{7}$$

Combining Eqs. (5)–(7), the quartet scheme in Ref. [9] is reproduced. These interactions can be obtained also by using the projection method if one knows suitable baryon operators for the mirror fields. For instance, using nonlocal baryon fields

$$\Delta'^{\mu i}_A = \not{\partial} A^j_\nu \Gamma^{\mu\nu}_{3/2} P^{ij}_{3/2} q, \tag{8a}$$

$$N'^{\mu}_V = \not{\partial} V_\nu \Gamma^{\mu\nu}_{3/2} \gamma_5 q, \tag{8b}$$

$$N'^{\mu}_A = \not{\partial} A^i_\nu \Gamma^{\mu\nu}_{3/2} \tau^i q, \tag{8c}$$

the mirror fields $(N^\mu_2, \Delta^{\mu i}_2)$ are obtained by the same equations as Eqs. (3) with substitution of the primed fields for the original fields. Because of the derivatives in Eqs. (8), $(N^\mu_2, \Delta^{\mu i}_2)$ transform as in Eqs. (4) with $\eta = -1$. Now, the chiral invariant terms Eqs. (6) and (7) are obtained by the insertion of the derivatives into Eq. (5).

Next, we consider the interactions between the nucleon and the spin-$\frac{3}{2}$ baryons, which is new in this work. As in the above discussion, $(V^\mu, \vec{A}^\mu)$ and $(\sigma, \vec{\pi})$ form a chiral scalar $\sigma V_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{A}_\mu$. Hence we find two chirally invariant interactions: (i) $\bar{N}U_5[(\partial^\mu\sigma)V_\mu + i(\partial^\mu\boldsymbol{\pi})\cdot\boldsymbol{A}_\mu]q$ and (ii) $\bar{N}(\partial^\mu U_5)(\sigma V_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{A}_\mu)q$. Using the irreducible decomposition, we obtain

$$\mathcal{L}^1_{\pi NB} = \frac{g_3}{\Lambda^2}\bigg[\bar{N}U_5(i\partial_\mu\pi^i)\Delta^{\mu i}_1 + \frac{\sqrt{3}}{2}\bar{N}U_5(\gamma_5\partial_\mu\sigma + \frac{i}{3}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau})N^\mu_1\bigg], \tag{9}$$

$$\mathcal{L}^2_{\pi NB} = \frac{g_4}{\Lambda^2}\bigg[\bar{N}(\partial_\mu U_5)(i\pi^i)\Delta^{\mu i}_1 + \frac{\sqrt{3}}{2}\bar{N}(\partial_\mu U_5)(\gamma_5\sigma + \frac{i}{3}\boldsymbol{\pi}\cdot\boldsymbol{\tau})N^\mu_1\bigg], \tag{10}$$

where the dimensional parameter Λ is introduced to keep g_3 and g_4 dimensionless. We ignore higher-order terms containing only the nucleons $\bar{N}U_5\not{\partial}U^\dagger_5 N$ and $\bar{N}(\partial^\mu U_5)U^\dagger_5\gamma_\mu N$. We find one chiral invariant operator for the mirror baryons $\bar{N}\not{\partial}[(\partial^\mu\sigma)V_\mu + i(\partial^\mu\boldsymbol{\pi})\cdot\boldsymbol{A}_\mu]q$. We obtain the one-meson interaction

$$\mathcal{L}^3_{\pi NB} = \frac{g_5}{\Lambda}\bigg(\bar{N}(i\partial^\mu\pi^i)\Delta^i_{2\mu} - \frac{\sqrt{3}}{2}\bar{N}\partial^\mu(\gamma_5\sigma - \frac{1}{3}i\boldsymbol{\tau}\cdot\boldsymbol{\pi})N_{2\mu}\bigg), \tag{11}$$

where one of the derivatives is absorbed into the mirror fields. Again, we ignore the nucleon term $\bar{N}\not{\partial}U^\dagger_5 N_m$, where N_m is another nucleon field having the mirror properties. Note that the interactions Eqs. (9) and (10) involve two mesons, while Eq. (11) contains only the single meson coupling.

Having constructed the Lagrangian with the nucleon and spin-$\frac{3}{2}$ baryons, we follow Ref. [9] to determine the parameters $g_{1,2}$ and m_0. After the spontaneous breakdown $SU(2)_R \times SU(2)_L \to SU(2)_V$, Eqs. (5) and (6) describe the diagonal mass terms proportional to the chiral condensate $\langle\sigma\rangle = f_\pi$, while Eq. (7) describes the off-diagonal mass terms between N^μ_1 and N^μ_2 and between $\Delta^{\mu i}_1$ and $\Delta^{\mu i}_2$. Note that the mixings between $N^\mu_1(\Delta^{\mu i}_1)$ and $N^\mu_2(\Delta^{\mu i}_2)$ occur only after the mass diagonalization when the so-called mirror mass m_0 is finite [9,14,15]. The masses of the four baryons are obtained by the diagonalization of the mass matrices. The results are shown in Table I [16]. With the corresponding parity (re)definition, the mass eigenstates are obtained as follows: for the Δs, $\Delta^{\mu i}_+ = (\Delta^{\mu i}_1 + \Delta^{\mu i}_2)/\sqrt{2}$, $\Delta^{\mu i}_- = \gamma_5(-\Delta^{\mu i}_1 + \Delta^{\mu i}_2)/\sqrt{2}$, and for the N^*s, $N^\mu_- = \gamma_5(-N^\mu_1 + N^\mu_2)/\sqrt{2}$, $N^\mu_+ = (N^\mu_1 + N^\mu_2)/\sqrt{2}$, where the subscripts $\pm$ denote the parity [17].

After the spontaneous breaking, the one-pion interactions in Eqs. (9)–(11) are reduced to

$$\begin{aligned}\mathcal{L}_{\pi NB} = &\frac{g_{\pi N\Delta^+}}{\Lambda}\bar{N}(i\partial_\mu\pi^i)\Delta^{\mu i}_+ + \frac{g_{\pi N\Delta^-}}{\Lambda}\bar{N}(i\gamma_5\partial_\mu\pi^i)\Delta^{\mu i}_- \\ &+ \frac{g_{\pi NN^{*-}}}{\Lambda}\bar{N}(i\gamma_5\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau})N^\mu_- \\ &+ \frac{g_{\pi NN^{*+}}}{\Lambda}\bar{N}(i\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau})N^\mu_+,\end{aligned} \tag{12a}$$

where the coupling constants are given by

$$g_{\pi N\Delta^\pm} = \frac{1}{\sqrt{2}\Lambda}(g_5\Lambda \pm g_3 f_\pi), \tag{12b}$$

$$g_{\pi NN^{*\pm}} = \frac{\sqrt{6}}{12\Lambda}[g_5\Lambda \pm (g_3 + 3g_4)f_\pi]. \tag{12c}$$

The three coupling constants $g_{3,4,5}$ are determined from the one-pion decay widths of the resonances as shown in Table I. We obtain quantitatively reasonable results for all the four coupling constants in Eqs. (12). Eliminating $g_{3,4,5}$ from Eqs. (12), we obtain a new relation:

$$(g_{\pi N\Delta^+} + g_{\pi N\Delta^-}) = 2\sqrt{3}(g_{\pi NN^{*-}} + g_{\pi NN^{*+}}), \tag{13}$$

TABLE I. Masses (second column) and coupling constants (third column). For masses, we follow Jido *et al.* [9]. The experimental values are taken from the Particle Data Group tables [12]. The experiments determine only the absolute values of the coupling constants; the positive values are our assumption.

States	Masses [MeV]	$g_{\pi NB}/\Lambda$ [MeV^{-1}]	$\Gamma_{B\to\pi N}$ [MeV]
	Theor (Exp)	Theor (Exp)	
$\Delta^{\mu i}_+(P_{33})$	1320 (1232)	18 (16)	118
$\Delta^{\mu i}_-(D_{33})$	1770 (1700)	10 (9.5)	45
$N^\mu_-(D_{13})$	1430 (1520)	6.1 (8.6)	69
$N^\mu_+(P_{13})$	1660 (1720)	2.2 (2.4)	30
		$m_0 = 1550$	$g_1 = g_2 = 2.4$
	$g_3 f_\pi/\Lambda^2 = 5.6$	$g_4 f_\pi/\Lambda^2 = -5.0$	$g_5/\Lambda = 20$

which is satisfied by the experimental data with a numerical error of about 20%. Considering the simplicity of the present description, this is an encouraging result suggesting that the spin-$\frac{3}{2}$ baryons are good candidates for the chiral partners.

An interesting feature of the present model is its two-pion contact terms, which are an inevitable consequence of the chiral invariance. They involve only the g_3 and g_4, while g_5, which is a leading contribution to the one-pion couplings, does not contribute to the two-pion couplings. The two-pion decay of $\Delta(1232)$ is therefore suppressed compared with the one-pion decay. On top of this, the derivative coupling causes an additional suppression of the two-pion decay rate, due to the small final state pion momentum. Hence we can expect strong suppression of the two-pion decay of $\Delta(1232)$. Explicitly, the two-pion contact interactions are given by

$$\mathcal{L}_{2\pi NB} = \frac{1}{\sqrt{2}\Lambda^2}\bar{N}\alpha^i_\mu \Delta^{\mu i}_+ - \frac{1}{\sqrt{2}\Lambda^2}\bar{N}\alpha^i_\mu \gamma_5 \Delta^{\mu i}_- - \frac{\sqrt{6}}{12\Lambda^2}\bar{N}\beta_\mu \gamma_5 N^\mu_- + \frac{\sqrt{6}}{12\Lambda^2}\bar{N}\beta_\mu N^\mu_+, \quad (14a)$$

with

$$\alpha^i_\mu = g_3(i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})(i\partial_\mu \pi^i) + g_4(i\gamma_5\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau})(i\pi^i), \quad (14b)$$

$$\beta_\mu = g_3(i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})(i\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau}) + g_4(i\gamma_5\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau})(i\boldsymbol{\pi}\cdot\boldsymbol{\tau}). \quad (14c)$$

Hence we obtain a relation for the 2-π contact terms:

$$|g_{2\pi N\Delta^+}| = |g_{2\pi N\Delta^-}| = 2\sqrt{3}|g_{2\pi NN^{*+}}| = 2\sqrt{3}|g_{2\pi NN^{*-}}|. \quad (15)$$

In contrast to the $\Delta(1232)$ case, it is expected that the two-pion contact terms lead to larger contributions for the other baryons, because of the larger final state pion momenta. Especially, the two-pion coupling constants of $N^{*+}(\Delta^-)$ have the same magnitude as compared with that of $N^{*-}(\Delta^+)$, while the one-pion coupling constants are suppressed by the negative sign in Eqs. (12). This qualitatively explains the observed enhancement of the branching ration of the two-pion decays of $N(1720)$ and $\Delta(1700)$. Because of the absence of other resonances, such as the ρ meson and $N(1440)$, from the present analysis, we do not consider this point here in detail.

In summary, we have investigated the properties of four spin-$\frac{3}{2}$ baryon resonances together with the ground state nucleon. We have constructed the chiral invariant Lagrangian with the aid of the spin and isospin projection formalism for the baryon fields comprised of three-quark fields. Of course, we can prove the chiral invariance of the derived interactions directly from the chiral transformation laws, but the results can be understood from the group-theoretical point of view. Within the $J = \frac{3}{2}$ sector, the projection formalism reproduces the quartet scheme proposed by Jido *et al.* [9]. In addition, we derived the minimal chiral invariant off-diagonal one- and two-meson couplings between the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons. We found that the one-pion couplings describing the spin-$\frac{1}{2}$-$\frac{3}{2}$ transitions are constrained by chiral symmetry via Eq. (13), couplings which quantitatively agree with the experiment. Considering the simplicity of our assumptions on the effective Lagrangian, it is an encouraging result suggesting that these baryons are chiral partners. We also obtain chiral two-pion couplings, whose strengths are entirely determined by the one-pion coupling constants. This enables us to predict two-pion decays of the resonances that can be tested in experiments.

We thank Professor D. Jido for fruitful discussions. K. N. and V. D. thank Professor H. Toki for the hospitality during the stay at RCNP. K. N. is supported by the National Science Council (NSC) of Republic of China under Grant No. NSC96-2119-M-002-001.

*nagata@rcnp.osaka-u.ac.jp

[1] S. Weinberg, Phys. Rev. Lett. **65**, 1177 (1990).
[2] D. B. Leinweber, T. Draper, and R. M. Woloshyn, Phys. Rev. D **46**, 3067 (1992).
[3] J. M. Zanotti *et al.* (CSSM Lattice), Phys. Rev. D **68**, 054506 (2003).
[4] D. B. Leinweber, W. Melnitchouk, D. G. Richards, A. G. Williams, and J. M. Zanotti, Lect. Notes Phys. **663**, 71 (2005).
[5] K. Sasaki and S. Sasaki, Phys. Rev. D **72**, 034502 (2005).
[6] L. Zhou and F. X. Lee, Phys. Rev. D **74**, 034507 (2006).
[7] F. X. Lee, Nucl. Phys. **A791**, 352 (2007).
[8] K. Nagata, A. Hosaka, and V. Dmitrasinovic, arXiv:0705.1896.
[9] D. Jido, T. Hatsuda, and T. Kunihiro, Phys. Rev. Lett. **84**, 3252 (2000).
[10] The $I(J) = \frac{3}{2}(\frac{1}{2})$ state is forbidden by the Pauli principle [8] when it is described as a local three-quark state.
[11] Equation (5) predicts a mass relation $m(\Delta^{\mu i}_1)$:$m(N^\mu_1) =$ 2:1. There are no candidates for this sort of N^* and Δ in the observed spectrum [12].
[12] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[13] B. W. Lee, *Chiral Dynamics* (Gordon and Breach, New York, 1972).
[14] C. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989).
[15] D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. **106**, 873 (2001).
[16] In Fig. 1 of Jido *et al.* [9], the states labeled as 1770 and 1660 ought to be turned upside down.
[17] The slight difference from Ref. [9] is caused by our definition that all basis $\Delta_{1,2}$ and $N_{1,2}$ have positive parity.