Isolating Segregation Mechanisms in a Split-Bottom Cell

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We study the segregation of mixtures of particles in a split-bottom cell to isolate three possible driving mechanisms for segregation of densely sheared granular mixtures: gravity, porosity, and velocity gradients. We find that gravity alone does not drive segregation associated with particle size without a sufficiently large porosity or porosity gradient. A velocity gradient, however, appears to be capable of driving segregation associated with both particle size and material density. In all cases, the final segregation state is approached exponentially.

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Mixtures of particles tend to *unmix* by particle property. One of the most dramatically destructive examples of this occurs in debris flow: Boulders, rocks, and sand tumble down a hillside, and the largest rocks migrate toward the top and then the front of the flow where they do the most damage [1]. Rotating drums and chute flows are two of the most common apparatuses used to study segregation in dense, gravity-driven granular flows [2,3]. In these cases, smaller or, alternatively, denser particles segregate away from the free surface, phenomena that have been modeled using mechanisms such as kinetic sieving and buoyancy, respectively [4–6].

The original formal development of kinetic sieving by Savage and Lun [4] uses a statistical mechanics approach to model the distribution of void sizes and subsequent size-dependent particle movement. Loosely speaking, small particles are more likely to move in the direction of gravity due to the statistically greater likelihood of voids of sufficient size for smaller particles. Gray and Thornton [5] expressed this kinetic sieving using a continuum formulation where the size-dependent resistance of particle movement is expressed in terms of a relative pressure that the differently sized particles support. Compared to the Savage and Lun model, void size distribution is not explicitly considered, but gravity is. Buoyancy effects were modeled by Khakhar, McCarthy, and Ottino [6] for segregation associated with particle density. Analogous to buoyancy in fluids, particles experience different vertical forcing according to their density relative to surrounding particles; resistance to transfer from one layer to the next is considered as equal for the two different (usually equal-sized) components.

Other segregation mechanisms have been identified in more energetic systems. For example, a gradient in *granular temperature*—the kinetic energy of velocity fluctuations—has been shown to drive segregation [7]; shear-driven segregation has been recently reported in particle suspensions [8], mitigated somewhat by curvature in the mean particle path [9]. All may, in fact, contribute to segregation in densely sheared systems, though with most experimental systems the dominant segregation mechanism is difficult to ascertain. In typical experimental systems designed to study segregation in dense granular flow (such as chutes and rotated drums), gravity, velocity gradients, and porosity gradients coexist in the direction of segregation [10,11].

To isolate some of these mechanisms, we use a split-bottom cylindrical cell, recently introduced by Fenistein *et al.* [12] [Fig. 1(a)]. In our system, the bottom of a cylindrical container of radius $R_o = 145$ mm is split at radius $r = R_s = 121$ mm, and the outer part is fixed to the vertical cylinder walls. We rotate the inner disk at an angular velocity $\Omega = \frac{1}{100}$ rev/sec for up to 10 revolutions and hold the outer surface fixed. The boundary friction is controlled using a layer of 2 mm glass particles glued to the base of the cell. We fill the cell with granular materials to a height $H \sim R_s/3$. As shown by Refs. [12–15], at these fill levels, a cylindrically symmetric—primarily vertical—shear band is produced in the bulk of the granular materials. Near the base, the shear band is narrow and centered at the split; with increasing distance from the bottom h, the band moves slightly inward, and its width increases [see Fig. 1(b)]. We found that, as shown by Fenistein *et al.* [12], the velocity profile may be fit using an error function

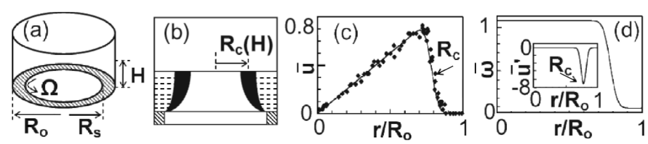

FIG. 1. (a) Sketch of a split-bottom cell; Ω is the rotation speed of the base. (b) Sketch of bulk shear band (black area). (c) Measured surface velocity as a function of radial position (symbols) for 1 and 3 mm glass particle mixtures filled to $H \approx R_s/3$ as described in text. The solid line is $\bar{u} = \bar{r} * \bar{\omega} = \bar{r} * \{1/2 - 1/2\,\mathrm{erf}[(r - R_c)/W]\}$, the shear band velocity described by Ref. [13], where $\bar{r} = r/R_o$; $\bar{\omega} = \omega/\Omega$, and ω is the local angular velocity. $R_c = 115.3$ mm and $W = 7.9$ mm for this mixture, in good agreement with $\approx$2 mm monosized systems in Ref. [12]; (d) shows the radial dependence of $\bar{\omega}$ and $\bar{u}' = \bar{r}\frac{d\bar{\omega}}{d\bar{r}}$.

[Fig. 1(c)]. The shear rate is greatest near the middle of the shear band [R_c in Fig. 1(d)]. In contrast to free-surface gravity-driven flow, the velocity gradient is primarily perpendicular to gravity, though there is a small deviation from this particularly near the base. Near the free surface, presumably a solids fraction gradient is coincident to gravity.

We use three sets of experiments—i.e., initial conditions—to study the effects of three possible external segregation driving mechanisms: gravity, porosity, and velocity gradient (Fig. 2). Experiment set I is the simplest: We simply fill the cylinder to height H with a well-mixed binary mixture [Fig. 2(a)]. In experiment set II, we fill the container with the mixture to height h_1 and cover it by an overburden—a layer of thickness Δh of monosized particles [Fig. 2(b)]. In this way, we move any surficial solids fraction gradient away from the top of the mixed phase. To prevent exchange of different types of particles at the mixture-overburden interface, this top layer must consist of particles that *float* on the mixture. We have found that the choice does not influence the results, and typically we use the less dense or larger particles (depending on whether the components in the mixtures differ only in density or size, respectively). In experiment set III, we isolate the effects of the velocity gradient in the horizontal direction, by placing a thin layer of our mixtures on top of a supporting base of one type of particle [Fig. 2(c)]. To prevent exchange of different types of particles at the interface of the mixture and the supporting matrix, the particles in the base must consist of particles that would sink in the mixture; we use either the denser or smaller particles, depending on whether the components differ in density or size. Here we report results primarily using binary mixtures of particles chosen from four different sizes ($d = 0.5$, 1, 2, and 3 mm) and three materials (steel, glass, and plastic).

When mixtures of particles differing only in density are sheared in the container as in experiments of type I, a segregated ring of the lighter component appears at the center of the shear zone [Figs. 3(a) and 3(b)], visible within a single rotation of the base. As rotation continues, this segregation band widens until a maximum width is reached [Fig. 3(c)]. Figure 3(d) shows the evolution of the segregated band visible from the top using a portion of each image [see Fig. 3(b)] taken at one frame per second. There is no apparent depletion of the less dense particles (or higher fraction of denser particles) outside the band, indicating that the dominant segregating process is occurring in the vertical direction. To determine the evolution of segregation in the bulk for these experiments, we run several consecutive experiments of increasing duration. After each run of progressively longer duration, we remove the layers of particles incrementally and note the segregation state at each depth. Then we remix the particles and restart the experiment (from time $t = 0$). The results from the bulk excavations [sketched in Figs. 3(e)–3(g)] indicate that, within one rotation of the base, the segregated zone of less dense particles extends at least 1–2 layers beneath the free surface and that, further, there is a small segregated region of denser particles region near the base of the cell.

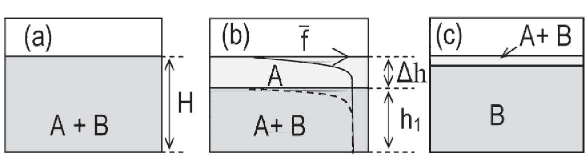

FIG. 2. (a) Sketch of the setup for experiment set I. (b) Sketch of the setup for experiment set II. The dashed line is the expected solids fraction $\bar{f}$ ($=1 -$ porosity [11]) for a mixture of components $A + B$ filled to height h_1; the solid line is the same for a system with an overburden of A, described in the text. (c) Sketch of the setup for experiment set III. The fill height $H \approx R_s/3$ is kept approximately the same for all experiments as described in the text, except to test sensitivity to fill height.

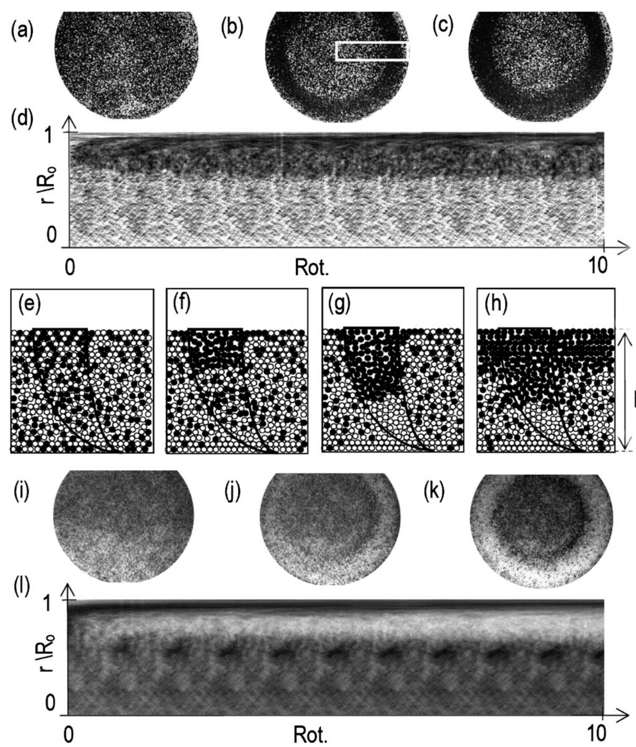

FIG. 3. Measurements of segregation in particle mixtures differing only in density for experiments described in the text. (a)–(c) Pictures of the surface for experiments of type I after (a) 0, (b) 1, and (c) 10 rotations for a mixture of 2 mm steel (white) and glass (black) particles. (d) A compilation of this information using average pixel values in the region indicated in (b) from each image taken throughout the experiment (at a rate of 1 Hz). (e)–(g) Sketches of bulk segregation from similar experiments at (e) 0, (f) 5, and (g) 10 rotations. (h) Sketch of segregation of the final segregation state for these mixtures from experiments of type II. (i)–(k) Pictures of the surface segregation for an experiment of type III using a mixture of 3 mm beads, glass (dark), and plastic (white) after (i) 0, (j) 1, and (k) 10 rotations. (l) A compilation of the experiment similar to that shown in (d).

With further rotation, the two segregated zones grow toward each other until they meet in the middle. Based on comparisons with 3D measurements [12–14], the segregation is primarily limited to the bulk shear zone.

In experiments of type II, segregation of these mixtures progresses in the very same way that it does without the overburden. Segregation zones of lighter particles form at the top of the sheared region, and a segregation zone of denser particles forms at the bottom; they both grow and meet in the middle [final state illustrated in Fig. 3(h)]. Experiments of types I and II show that mixtures of particles differing only in density behave as one might expect from an emulsion of immiscible fluids of different densities. That is, particles of different density than the surrounding sheared fluidlike mixture rise or fall according to their relative densities, and a solids fraction (gradient) has little or no effect. Experiments of type III illustrate the effect of the isolated shear gradient on these mixtures: The particles that are less dense drift horizontally relative to the mixture to regions of lower shear rate [Figs. 3(i)–3(k)]. A quick initial migration of a few lighter particles to the outer edge of the shear band is followed by a slower vacation of most of these particles to the inner edge of the shear band. Figure 3(l) shows the time dependence of the top segregated band similar to that shown in Fig. 3(d) for experiment set I.

When the three types of experiments are performed using mixtures of particles differing only in size, there are a number of similarities but a few key differences. In experiments of type I, a band of large beads quickly forms and widens, similar to the band of lighter beads in mixtures differing only in density [Fig. 4(a)]. Progressive bulk excavations as described above show that this band grows downward, but the vertical growth is limited to a maximum depth $\approx 10\bar{d}$ [sketched in Figs. 4(b) and 4(c)] [16]. A segregation zone of small particles forms at the base as for the denser particles in the mixtures described above, but the zone of small particles does not grow. In experiments of type II, segregation in the mixtures of particles differing only in size is either reduced or eliminated by the overburden. That is, for small enough values of Δh, a segregated region grows from the top of the mixture, but the final depth is reduced, approximately by Δh, and eliminated for $\Delta h \geq 10\bar{d}$ [Fig. 4(d)]. These results appear independent of our choice of h_1 and indicate that either a solids fraction gradient or a velocity gradient must be present to assist gravity-driven size segregation. In experiments of type III, mixtures of particles differing only in size behave similarly to those differing only in density: A quick initial outward migration of the larger particles is followed after some delay by a slower vacation of the rest of these particles inward [Fig. 4(e)]. These two stages of segregation may be due to two separate phenomena. The initial quick movement of the larger or less dense particles outward appears similar to the curvature-induced migration of larger spheres in a bidisperse suspension reported by Krishnan, Beimfohr, and Leighton [9]. The slower more complete evacuation of the shear band is more likely due to dynamics associated with the velocity gradient in the shear band such as a temperature gradient in experiments reported by Zuriguel *et al.* [17].

We investigate the time dependence of the segregation in experiments I and III via the width of the surface band Δr in two mixtures of different sized beads. In both cases, we

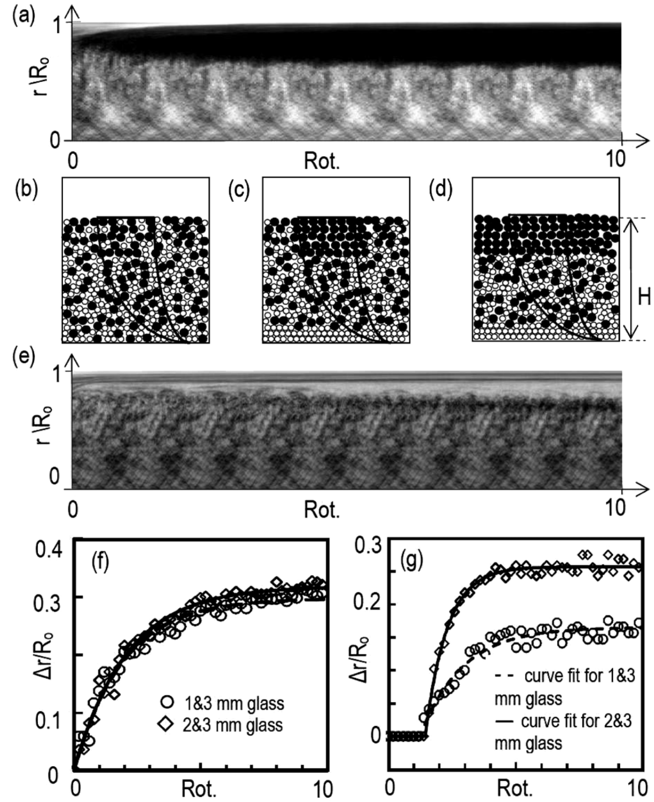

FIG. 4. Measurements of segregation in particle mixtures differing only in size for experiments described in the text. (a) Compilation of segregation in a mixture of 1 (white) and 3 mm (black) glass particles in experiments of type I obtained as in Fig. 3(d). (b)–(c) Sketches of bulk segregation for this experiment after (b) 0 and (c) 10 rotations. (d) Sketch of segregation (none) for experiments of type II when the overburden of larger particles $\Delta h \geq 10\bar{d}$, the final depth of the top segregation band from experiment set I. (e) The segregation evolution for a mixture of 1 (white) and 3 mm (black) glass particles in experiment set III obtained as described in the caption for Fig. 3(d). (f)–(g) show the width of surface band versus time for experiment set I and III, respectively, using mixtures of different sized particles: 1 and 3 mm glass and 2 and 3 mm glass, respectively. Here we consider the width Δr nonzero only for a region of the band purged of one type of particle. The lines are exponential fits $\frac{\Delta r(t)}{R_0} = A_0(1 - e^{-(t-t_d)/t_0})$ to the experimental data for $t > t_d$. For experiment I, $(A_0, t_0, t_d)_{1\&3} = (0.32, 207s, 0s)$ and $(A_0, t_0, t_d)_{2\&3} = (0.30, 192s, 0s)$. For experiment III, $(A_0, t_0, t_d)_{1\&3} = (0.17, 163s, 126s)$ and $(A_0, t_0, t_d)_{2\&3} = (0.26, 85s, 146s)$.

determine the width of the band according to maximum width, where there is only one type of particle. For both types of experiments, once the band starts to grow, the width of the band is well fit with an exponential $\frac{\Delta r(t)}{R_0} \approx A_0(1 - e^{-t/t_0})$. However, the initial growth of the band for experiments of type III is delayed by some time t_d nearly equal to the typical growth time for experiment type I. This delay may explain the lack of evidence for horizontal segregation in experiments of type I.

In summary, we emphasize new results obtained using a split-bottom cell to isolate various segregation driving mechanisms. We have found that isolating the effect of gravity from possible effects of shear rate and porosity (gradients) essentially eliminates the effect of gravity on size segregation. Further, even in the regions where a small porosity gradient exists in the direction of gravity, size segregation effects are significantly reduced relative to density segregation effects. Consider the experiments in rotating drums reported in Ref. [18]. Here, for a mixture of particles of a density ratio $R_\rho \approx 3.1$, a crossover between density-dominated segregation and size-dominated segregation occurred for particles of a diameter ratio $R_d = \frac{d_{\text{dense}}}{d_{\text{less dense}}} \approx 2$. In our experiment set I, no such reversal was observed; density segregation dominated for all size ratios, for $R_d \approx 1$–6 for $R_\rho \approx 3.1$. The results from experiment set III further indicate the importance of mechanisms related to shear rate. These driving mechanisms may be related to granular temperature gradients induced by shear rate gradients as described in more energetic systems such as Ref. [7].

These results indicate that, while kinetic sieving and buoyancy models using gravity alone as a segregation driving force work well as a first-order representation, a more complete model for segregation in densely flowing granular mixtures must include other kinematic details. Porosity gradients and velocity gradients appear to be significant factors for segregation in dense granular flows. At this time, it is not obvious which of these are most significant. For example, the apparent porosity dependence of segregation associated with particle size might be driven solely by the local porosity; it might also be facilitated by a sufficient porosity gradient. Similarly, the shear-driven segregation might be driven by gradients of the velocity fluctuations as in Ref. [17]. On the other hand, the presence of a shear rate might drive segregation in some other way, for example, through inertial effects and/or global circulation patterns [19]. To address these issues, additional computational studies are underway where both size and density may be more continuously varied, detailed bulk segregation and other kinematic details are more accessible, and the system can be studied with or without curvature effects.

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