Spin-Charge Separation in the Quantum Spin Hall State

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The quantum spin Hall state is a topologically nontrivial insulator state protected by the time-reversal symmetry. We show that such a state always leads to spin-charge separation in the presence of a π flux. Our result is generally valid for any interacting system. We present a proposal to experimentally observe the phenomenon of spin-charge separation in the recently discovered quantum spin Hall system.

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Spin-charge separation is one of the deepest concepts in condensed matter physics. In the Su-Schrieffer-Heeger model of polyacetylene [1], a domain wall induces two midgap states, one for each spin orientation of the electron. If both states are unoccupied, or both states are occupied, the domain wall soliton has charge $\pm e$ but no spin. If only one of the states is occupied, the domain wall soliton has spin $S_z = \pm 1/2$ but no charge. In this remarkable way, the two fundamental degrees of freedom of an electron are split apart. Since then, the concept of spin-charge separation has become a cornerstone in condensed matter physics [2,3].

Recently, a new two-dimensional quantum state of matter has been theoretically proposed [4–6]. The quantum spin Hall (QSH) state is a topologically nontrivial state of matter protected by the time-reversal symmetry. This novel quantum state of matter has recently been theoretically predicted [6] and experimentally observed [7] in the HgTe quantum wells. The topological property of the quantum Hall (QH) state is described by an integer Chern number [8], defined over the single particle momentum space, and this integer is directly related to the experimentally observed quantum of Hall conductance. This construction can also be generalized to an interacting system, where the Chern number is defined over the space of twisted boundary conditions [9]. The topological property of the QSH state is currently described by a Z_2 topological number [10–13], which is also defined over the single particle momentum space. This Z_2 classification has provided an important insight on the topological nontriviality of the QSH state. However, unlike the situation in QH systems, there are several fundamental missing links in the QSH systems. Needed is a general classification of time-reversal invariant (TRI) topological insulators in two dimensions which is valid in the presence of arbitrary interactions. Such a general classification beyond the single particle band picture is especially called for since the concept of a topological Mott insulator has recently been introduced [14]. More importantly, we need to find experimentally measurable properties which directly demonstrate the topological nontriviality of the QSH state.

In this Letter, we solve both problems by providing a deep connection between the concept of spin-charge separation and the QSH effect. We consider the adiabatic insertion of a π gauge flux in the QSH state, which demonstrate the existence of the spin-charge separated holon, chargeon, and two spinon states which are exponentially localized near the flux. We then prove two general conclusions providing a Z_2 classification of TRI insulators in two dimensions. This new classification scheme is generally valid in the presence of many-body interactions, and leads to spin-charge separation as its direct physical consequence. Finally, we propose an experimental setting to observe the phenomenon of spin-charge separation in the recently discovered QSH system.

To make our discussions more concrete, we start by a typical model describing a QSH insulator—the effective tight-binding model of HgTe quantum well proposed in Ref. [6]. The model has a tight-binding Hamiltonian written as

$$H = \sum_{\mathbf{k}} \psi^\dagger(\mathbf{k}) \left[\epsilon(\mathbf{k}) \mathbb{I} + \begin{pmatrix} d_i(\mathbf{k})\sigma^i & \\ & d_i^*(-\mathbf{k})\sigma^i \end{pmatrix} \right] \psi(\mathbf{k}), \quad (1)$$

with σ^i, $i = 1, 2, 3$ the Pauli matrices and $\epsilon(\mathbf{k})$, $d_i(\mathbf{k})$ the matrix elements defined by $\epsilon(\mathbf{k}) = -2D(2 - \cos k_x - \cos k_y)$, $d_{1,2}(\mathbf{k}) = A \sin k_{x,y}$, and $d_3(\mathbf{k}) = M - 2B(2 - \cos k_x - \cos k_y)$. The upper and lower 2×2 blocks of the model are decoupled with each other, which describe spin-up and -down electrons, respectively. This model has a QSH phase which is simply defined as two copies of QH, with opposite Hall conductances of $+(-)e^2/h$ for the spin-up (-down) subsystem. In the following, we will first present an argument on the spin-charge separation for this special case, which is physically intuitive, but only valid when there is at least a $U_s(1)$ spin rotation symmetry. Afterwards, we will generalize the discussion to the most generic case.

Without loss of generality, we first consider a disk geometry with a gauge flux of $\phi_\uparrow = \phi_\downarrow = hc/2e$, or simply π in units of $\hbar = c = e = 1$, through a hole at the center; see Fig. 1. The gauge flux acts on both spin orientations, and the π flux preserves time-reversal symmetry. We consider adiabatic processes of $\phi_\uparrow(t)$ and $\phi_\downarrow(t)$, where $\phi_\uparrow(t) = \phi_\downarrow(t) = 0$ at $t = 0$, and $\phi_\uparrow(t) = \phi_\downarrow(t) = \pm\pi$ at $t = 1$. Since the flux of π is equivalent to the flux of

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$-\pi$, there are four different adiabatic processes all reaching the same final flux configuration, as illustrated in Fig. 1.

We consider a loop surrounding the flux, as shown schematically in Fig. 1(a). As the flux $\phi_\uparrow(t)$ is turned on adiabatically, Faraday's law of induction states that a tangential electric field $\mathbf{E}_\uparrow$ is induced along the loop. The quantized Hall conductance implies a radial current and results in a net charge flow $\Delta Q_\uparrow$ through the loop: $\Delta Q_\uparrow = -\int_0^1 dt \oint d\mathbf{n}\cdot\mathbf{j}_\uparrow = -\frac{e^2}{h}\int_0^1 dt \oint d\boldsymbol{l}\cdot\mathbf{E}_\uparrow = -\frac{e^2}{hc}\int_0^1 dt \frac{\partial\phi}{\partial t} = -\frac{e^2}{hc}\frac{hc}{2e} = -\frac{e}{2}$. An identical argument applied to the down spin component shows that $\Delta Q_\downarrow = -e/2$. Therefore, this adiabatic process creates the holon state with $\Delta Q = \Delta Q_\uparrow + \Delta Q_\downarrow = -e$ and $\Delta S_z = \Delta Q_\uparrow - \Delta Q_\downarrow = 0$. For later convenience, here and below we will call such a loop around the flux a "Gaussian loop."

Applying similar arguments to the process in Fig. 1(b) gives $\Delta Q_\uparrow = \Delta Q_\downarrow = e/2$, which leads to a chargeon state with $\Delta Q = e$ and $\Delta S_z = 0$. Processes (c) and (d) give $\Delta Q_\uparrow = -\Delta Q_\downarrow = e/2$ and $\Delta Q_\uparrow = -\Delta Q_\downarrow = -e/2$, respectively; we therefore obtain the spinon states with $\Delta Q = 0$ and $\Delta S_z = \pm 1/2$. Assuming that the ground state is unique at $t = 0$, we obtain four final states for the same system at $t = 1$, which are the holon, chargeon, and the two spinon states. Both the spin and the charge quantum numbers are sharply defined [15]. The insulating state has a bulk gap Δ and an associated coherence length $\xi = \hbar v_F/\Delta$. As long as the radius of the Gaussian loop r_G far exceeds the coherence length ξ, the spin and the charge quantum numbers are sharply defined within exponential accuracy. Recently, similar proposals of fractionalization have been studied in other systems [16–19].

While the argument above is intuitive and generally valid in the presence of both interaction and disorder, it

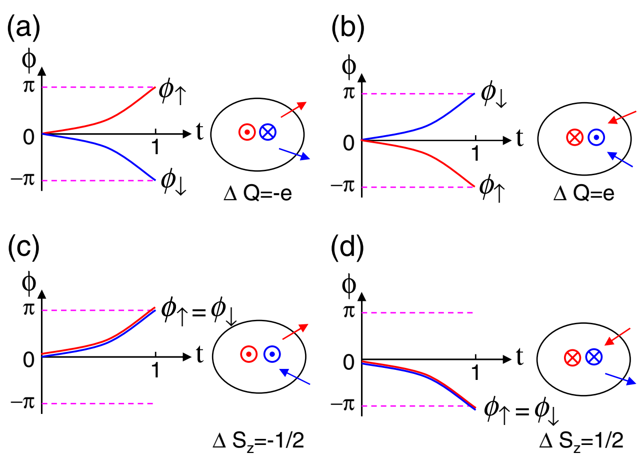

FIG. 1 (color online). Four different adiabatic processes from $\phi_\uparrow = \phi_\downarrow = 0$ to $\phi_\uparrow = \phi_\downarrow = \pm\pi$. The red (blue) curve stands for the flux $\phi_{\uparrow(\downarrow)}(t)$, respectively. The symbol "⊙" ("⊗") represents increasing (decreasing) fluxes, and the arrows show the current into and out of the Gaussian loop, induced by the changing flux. Charge is pumped in the processes with $\phi_\uparrow(t) = -\phi_\downarrow(t)$, while spin is pumped in those with $\phi_\uparrow(t) = \phi_\downarrow(t)$.

relies on the $U_s(1)$ spin rotation symmetry which is not generic in the presence of spin-orbit interactions. To generalize our results to a generic QSH insulator, we first need to generalize the concept of spin-charge separation to a system with no spin conservation. For a system with time-reversal symmetry, this is possible because the states with integer spin and those with half-integer spin can be distinguished by their time-reversal properties. Denoting the time-reversal operator as $\mathcal{T}$, and the electron number operator as N, we can define a quantum state $|\psi_c\rangle$ satisfying $(-1)^N|\psi_c\rangle = -|\psi_c\rangle$, $\mathcal{T}^2|\psi_c\rangle = |\psi_c\rangle$ as a generalized chargeon or holon state, and a state $|\psi_s\rangle$ satisfying $(-1)^N|\psi_s\rangle = |\psi_s\rangle$, $\mathcal{T}^2|\psi_s\rangle = -|\psi_s\rangle$ as a generalized *spinon state*. According to the Kramers theorem, each spinon state is degenerate with its Kramers partner $\mathcal{T}|\psi_s\rangle$ which is orthogonal to $|\psi_s\rangle$. It should also be noticed that the chargeon and holon states belong to the same topological class since they have the same time-reversal property and the same value of $(-1)^N$.

We now consider a TRI insulator without any additional spin rotational symmetry. An example is given by introducing bulk inversion asymmetry terms in the model (1), as discussed in Refs. [20,21]. A generic tight-binding model can always be written in the form of $H = \sum_{\langle ij\rangle}\psi_i^\dagger t_{ij}\psi_j + H_{\rm int}$, where ψ_i is the annihilation operator of electron on the site i which is in general an N component spinor. Correspondingly, the matrix element t_{ij} is a $N\times N$ matrix. $H_{\rm int}$ is a generic local interaction. For such a generic model, the flux threading processes can be generalized by replacing the hopping matrix elements t_{ij} with $t_{ij}e^{i\theta(t)\Gamma}$ [22,23] on all links along a string extending from the flux tube to infinity. Here Γ is a generic $N\times N$ matrix in the spin space. For example, the model (1) corresponds to $N = 4$, and the processes (a) and (c) in Fig. 1 correspond to $\Gamma = \sigma_z \otimes \mathbb{I}_{2\times 2}$ and $\Gamma = \mathbb{I}_{4\times 4}$, respectively. All the following discussions are valid even if Γ is not conserved. For any choice of Γ which is time-reversal odd and satisfies $e^{i\pi\Gamma} = -1$, the following conclusions are true for a generic system as long as the ground state is nondegenerate and the Γ flux threading does not close the bulk energy gap. (1) The charge pumped towards the isolated flux tube during the adiabatic evolution $\theta(t) = 0 \to \pi$ is an integer N_Γ (in units of the electron charge e). (2) A topological index, defined by $(-1)^{N_\Gamma}$, is independent of the choice of Γ.

To prove the conclusion (1), we denote the Hamiltonian with a Γ flux as $H_\Gamma(\theta(t))$. Because of the condition $e^{i\pi\Gamma} = -1$, we know that $H_\Gamma(\pi)$ is the same as the Hamiltonian with a *charge* π-flux tube. Consequently, two distinct adiabatic evolutions can be defined between $H_\Gamma(0)$ and $H_\Gamma(\pi)$, that is, the process l_Γ through spin Γ flux threading, and the process l_c through charge flux threading. The combination of them $l = l_c^{-1}l_\Gamma$ leads to a closed path in the parameter space [24]. Given the condition that the system with no flux has a unique ground state, the charge pumped during such a process must be an integer, denoted as N_Γ. Moreover, the charge pumped during the path l_c^{-1} has to be zero, since the Hall conductivity of the system vanishes due to time-reversal symmetry. Consequently, an integer number of charge $N_\Gamma e$ is pumped towards the flux tube during the first half of the adiabatic process, l_Γ.

To prove the conclusion (2), we consider two different operators Γ_1, Γ_2 satisfying $e^{i\pi\Gamma_{1,2}} = -1$, from which two different adiabatic paths l_1, l_2 connecting $\theta = 0$ and $\theta = \pi$ Hamiltonians are defined. Consequently, a closed path can be formed as $l = l_2^{-1} l_1$. If the number of charge pumped during l_1 and l_2 is N_{Γ_1} and N_{Γ_2}, respectively, the net charge pumped during l is given by $N_{\text{tot}} = N_{\Gamma_1} - N_{\Gamma_2}$. Since the spin Γ flux preserves time-reversal symmetry, if the initial state $|G\rangle$ is a Kramers singlet, so is the final state $|F\rangle$. This is only possible if the charge N_{tot} pumped during the closed path $l = l_2^{-1} l_1$ is an even integer, leading to the conclusion that $(-1)^{N_{\Gamma 1}} = (-1)^{N_{\Gamma 2}}$ for any two choices Γ_1 and Γ_2.

Based on these two general conclusions, we obtain a way to determine whether a π flux in a generic TRI insulator is spin-charge separated or not. By taking any choice of Γ satisfying $\mathcal{T}^{-1}\Gamma\mathcal{T} = -\Gamma$, $e^{i\pi\Gamma} = -1$ and doing the Γ flux threading, one can measure the number of charge N_Γ pumped towards the flux tube. If N_Γ is an odd integer, then from conclusion (2) we know that for any other choice Γ' the number of charge pumped is also odd. Consequently, the final state $|\phi_c\rangle$ of this flux threading process carries odd number of charge around the flux tube, but at the same time is a Kramers singlet. In other words, $|\phi_c\rangle$ is a chargeon or holon state as defined earlier. Once the flux is fixed to be π, no local perturbation can change the spin-charge separation nature of the system. In general a local operator $\hat{O}_+$ can be defined, which acts only around the flux tube and carries charge $-N_\Gamma e$. When N_Γ is odd, $\hat{O}_+$ has to form a doublet representation of time-reversal transformation $\mathcal{T}$ together with its partner $\hat{O}_- = \mathcal{T}^{-1}\hat{O}_+\mathcal{T}$. Thus the quantum states $|\psi_{s\pm}\rangle = \hat{O}_\pm|\psi_c\rangle$ carry vanishing electric charge but form a doublet representation of $\mathcal{T}$, which are thus a pair of spinon states.

Since the topological index $(-1)^{N_\Gamma}$ does not depend on the choice of Γ, it can be considered as an intrinsic property of the TRI insulator. In this way, we see that $(-1)^{N_\Gamma}$ can be considered as a generic *definition* of the Z_2 topological invariant for $(2+1)$-dimensional TRI insulators, which is applicable to interacting and disorder system, and has its direct physical meaning of spin-charge separation in a flux tube.

To see more explicitly the realization of spin-charge separation, we study the flux threading processes numerically for the model (1) accompanied by the bulk inversion asymmetry terms as written in Ref. [21]. We also use the realistic parameters obtained from $\mathbf{k}\cdot\mathbf{p}$ calculations of HgTe quantum wells. Both the spin Γ and the charge flux on a plaquette induce midgap states inside the gap. In Fig. 2, we show the local density of states $\rho_i(E) = \sum_n |\langle i|n\rangle|^2 \delta(E_n - E)$ in the core site i of both the charge and the spin Γ-flux tubes. As shown in Figs. 2(a) and 2(b), when a spin Γ flux is threaded from 0 to 2π, the time-reversal symmetry is preserved and a Kramers pair of midgap states moves from the valence band to the conduction band, or vice versa, depending on the choice of Γ matrix. Consequently, $\pm 2e$ units of charge are pumped during each period. From the discussions above, we know that the state at $\phi = \pi$ is a chargeon (holon) for the case of Fig. 2(a) [2(b)], respectively. In a similar way, the spinon states can be obtained by threading a charge π flux, as shown in Fig. 2(c).

We now discuss the experimental realization of spin-charge separation. Consider a hybrid structure between a type-II superconductor (SC) and the HgTe quantum well with perpendicular magnetic field, as shown in Fig. 3(a). Consider a flux tube with flux of $hc/2e$, or π, with a radius of the penetration depth λ. In this case, the time-reversal symmetry is broken due to finite λ, leading to a splitting of the two midgap states, as shown in Fig. 3(b). However, at a Gaussian loop with radius $r_G \gg \lambda$, there is no observable difference between such a realistic π flux and an ideal π flux threading into a plaquette, and the spinon and holon or chargeon states are still well-defined. Denoting E_1 and E_2 the energy of the two midgap states, and E_v and E_c the energy of valence band top and conduction band bottom, at zero temperature the ground state of the system is given by (i) the holon state, when the chemical potential $E_v < \mu < E_1$; (ii) the spinon state with a preferred spin in the magnetic field, when $E_1 < \mu < E_2$; (iii) the chargeon state,

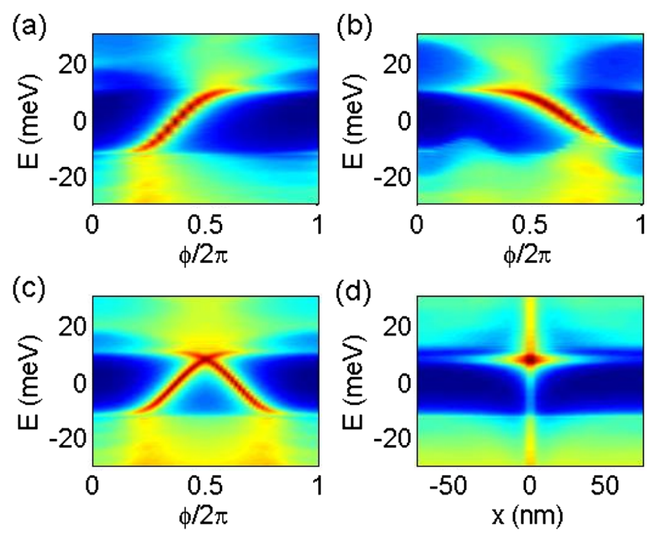

FIG. 2 (color online). Local density of states in the core of a flux tube. The dark (blue) color shows the bulk energy gap and the red line shows the midgap states. (a),(b) The evolution of the midgap state upon spin Γ-flux threadings, with $\Gamma = \sigma_z \otimes \mathbb{I}$ and $\Gamma = -(\sigma_z + \sigma_x) \otimes \mathbb{I}/\sqrt{2}$, respectively, in the representation used by Ref. [6]. (c) The case of charge flux threading. The two midgap states cross at $\phi = \pi$. The spatial distribution of the midgap state at $\phi = \pi$ is shown in (d), in which the zero of x axis corresponds to the position of the flux tube. Here and below, all the calculations are done for the HgTe model of $d = 70$ Å quantum well, with a lattice constant of $a = 30$ Å.

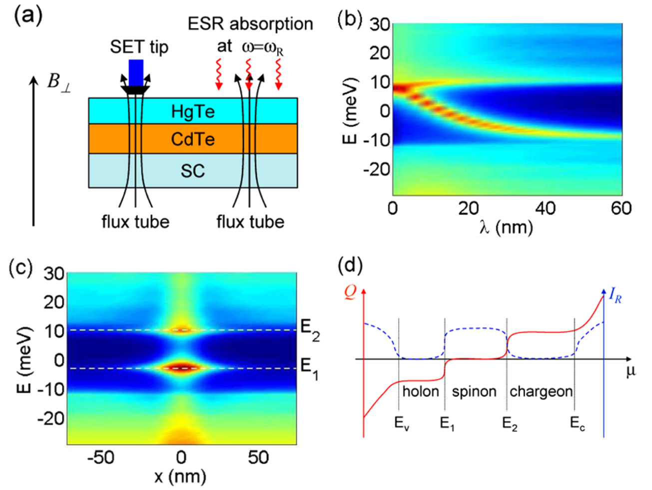

FIG. 3 (color online). (a) The superconductor-quantum well hybrid structure, with the π flux tubes generated by a perpendicular magnetic field. (b) The splitting of two midgap states upon increasing λ. (c) The spatial distribution of the two midgap states for $\lambda = 18$ nm. (d) Illustration of the measured charge Q (red solid line) and intensity of ESR signal $I_R = I(\omega_R)$ at resonance frequency $\omega_R = (E_2 - E_1)/\hbar$ (blue dashed line) as functions of the chemical potential.

when $E_2 < \mu < E_c$. Consequently, the spin charge separation can be observed if we can measure the charge and spin induced by a flux tube independently. The local charge distribution can be probed by scanning single-electron transistor (SET) [25], while the spin can be observed by electron spin resonance (ESR). Only when the system is in the spinon state, a transition between the two midgap states can occur, leading to a sharp resonance peak in the ESR signal at frequency $\omega_R = (E_2 - E_1)/\hbar$. The qualitative behavior of local charge and ESR spectrum is summarized in Fig. 3(d), as a function of the chemical potential, which can be tuned by the back gate voltage. Moreover, it can be seen from Fig. 3(b) that the energy splitting $E_2 - E_1$ is proportional to λ for small λ. Thus one expects to see $\omega_R \propto \lambda$ for small λ, which can demonstrate the existence of a Kramers pair in the ideal case with time-reversal symmetry.

In conclusion, we have given a Z_2 classification of TRI insulators which is generally valid in the presence of interactions and disorder. We showed that this topological property can be measured experimentally by the spin charge separation in the presence of a π flux. We provided an experimental proposal to observe the spin charge separation in an SC-QSH hybrid structure. Fractionalization is usually accompanied by fractional statistics. By studying the topological effective theory of the SC-QSH hybrid system, it can be shown that the spinon, holon, and chargeon are all bosons, but each spinon has nontrivial *mutual statistical* angle π with chargeon and holon [26]. Such a relation between spin charge separation and TRI topological insulators can also be generalized to higher dimensions. In a 3D "strong" topological insulator [11,27,28], a closed string with π flux is a spin charge separated extended object.

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