# Anomalous Attenuation of Transverse Sound in $^3$He

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We present the first measurements of the attenuation of transverse sound in superfluid $^3$He-$B$. We use fixed path length interferometry combined with the magnetoacoustic Faraday effect to vary the effective path length by a factor of 2, resulting in absolute values of the attenuation. We find that attenuation is significantly larger than expected from the theoretical dispersion relation, in contrast with the phase velocity of transverse sound. We suggest that the anomalous attenuation can be explained by surface Andreev bound states.

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Over 50 years ago Landau published his seminal works on Fermi liquid theory [1,2]. In the second of these works, on the collective dynamics of Fermi liquids, he predicted that there would be collisionless sound modes outside the hydrodynamic limit, called zero sound. The crossover from hydrodynamic sound to longitudinal zero sound was discovered in the normal state of $^3$He in 1966 by Abel, Anderson, and Wheatley [3]. Along with longitudinal zero sound, Landau predicted that for certain values of the Fermi liquid interaction parameters [4] there should be a collisionless collective mode called transverse zero sound. The constraint on the Fermi liquid interaction parameters is essentially that the transverse sound velocity $c_t$ be greater than the Fermi velocity $v_F$, otherwise the transverse wave can decay into incoherent quasiparticles in a process called Landau damping [5]. This condition is likely satisfied over the entire range of liquid $^3$He [6]. However, attempts to observe transverse sound (TS) in the normal state of $^3$He [7] have proven unsuccessful [8], due in part to high attenuation.

Predictions for the fate of TS in the superfluid state of $^3$He were pessimistic since the number of unpaired quasiparticles decreases as the energy gap opens up [9,10]. In 1993, Moores and Sauls (MS) [11] showed that these ideas were incomplete and instead TS would be enhanced in the $B$ phase of superfluid $^3$He due to the off-resonant coupling of transverse currents to an order parameter collective mode, called the imaginary squashing mode (ISQ). They showed that the dispersion relation for TS, in the long wavelength limit, was given by

$$\frac{\omega^2}{q^2 v_F^2} = \Lambda_0 + \Lambda_{2^-} \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2^-}^2 - \frac{2}{5} q^2 v_F^2}, \qquad (1)$$

where $q$ is the complex wave vector, $q = k + i\alpha$, $k$ is the real wave vector, $\alpha$ is the attenuation, and the phase velocity is $c_t = \omega/k$. The ISQ-mode frequency closely follows the temperature and pressure dependence of the energy gap, $\Delta(T, P)$, $\Omega_{2^-}(T, P) = a_{2^-}(T, P)\Delta(T, P)$, where $a_{2^-} \approx \sqrt{12/5}$ [11–13] and the ISQ-mode width is given by $\Gamma$ [11,14]. It is customary to label this mode $2^-$, according to its total angular momentum quantum number and its parity under particle-hole conversion. The first term on the right-hand side of Eq. (1) is the quasiparticle background, the contribution to the dispersion in the absence of coupling to the ISQ mode, and the second term gives the off-resonant coupling strength to the ISQ mode. This off-resonant coupling produces a dramatic increase in the phase velocity of TS near the mode [15], lifting it well above the Fermi velocity and thereby reducing Landau damping. But this is only allowed above the ISQ-mode energy and below the pair-breaking energy, shown as the blue (or gray) shaded region in Fig. 1.

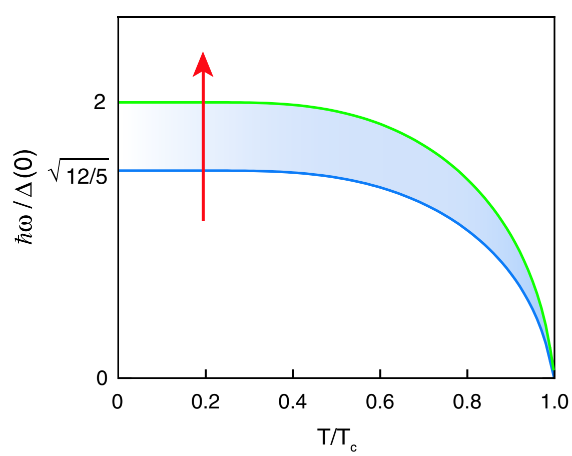


FIG. 1 (color online). The energy of pair-breaking (green or gray curve) and the ISQ mode (blue or dark gray curve) as a function of temperature normalized to $T_c$. TS propagates only in the shaded blue region. The low temperature pressure sweep technique follows a path represented by the red (or black) arrow.

 

The predictions of MS [11] prompted a new exploration for TS in $^3$He, which yielded more fruitful results than in the normal state [12,13,15–19]. Additionally, MS predicted a magnetoacoustic Faraday effect (AFE). Its observation by Lee *et al.* [17] confirmed the existence of TS in liquid $^3$He, the only liquid where transverse sound is known to propagate. In this Letter, we present measurements of the absolute attenuation of transverse sound in superfluid $^3$He-$B$. These measurements show a larger attenuation than expected from Eq. (1), which we suggest arises from surface Andreev bound states.

To measure the attenuation, an acoustic cavity was constructed with one wall as a shear transducer (*AC* cut) and the other as an optically polished quartz reflector. The thickness of the acoustic cavity is $D = 31.6 \pm 0.1\ \mu$m and was filled with liquid $^3$He. This is small enough that standing waves of TS are able to form in the cavity [13,15,18]. Here we use the 13th to the 25th transducer harmonics (76 to 147 MHz). We note that the TS velocity is a sensitive local indicator of the temperature in the acoustic cavity, which was used to ensure that there was no heating from the transducer or other sources. Furthermore, the acoustic cavity walls are guaranteed parallel via a spring loaded setup that maintains the cavity spacing at all times and temperatures. Information on these experimental techniques has been described in detail elsewhere [12,13,15,19]. Throughout we use the weak-coupling-plus (WCP) gap of Ref. [20], tabulated in Ref. [6] with values of $T_c$ given by Greywall [21]. And in $\Lambda_0$ and $\Lambda_{2^-}$ we use the Tsuneto function calculated using the WCP gap and all Fermi liquid parameters up to $l \leq 2$ [15,18,19].

The electrical impedance of the shear transducer is sensitive to the standing TS wave at the surface of the transducer and was monitored with a continuous wave impedance bridge [19]. The output of the bridge is

$$V_Z = a + b\cos\theta \sin\left(\frac{2D\omega}{c_t} + \phi\right), \tag{2}$$

where $\theta$ is the angle of the polarization of the TS wave at the surface of the transducer relative to the intrinsic polarization of the shear transducer, $c_t$ is the phase velocity of TS, and $\phi$ is a fixed phase that depends on the experimental conditions. A smoothly varying background of acoustic impedance [22] is represented by $a$ and the attenuation, $\alpha$ is proportional to $-\ln b$. By varying the temperature or pressure at fixed acoustic frequency we sweep $\hbar\omega/\Delta(T, P)$ (see Fig. 1), changing the acoustic frequency relative to the energy of the ISQ mode and therefore $c_t$, producing oscillations in $V_Z$ [13,15].

In previous reports [12,13,15] for which examples are shown in the insets in Fig. 2, we noted that TS attenuation is inversely related to the amplitude of the acoustic response oscillations, but we could not make a quantitative interpretation. Here, on the other hand, we obtain the absolute value of the TS attenuation, taking advantage of the acoustic Faraday effect. Using the AFE, we rotate the linear polarization of the TS waves in the acoustic cavity [13] by applying a magnetic field along the sound propagation direction. When the polarization is rotated by $\pi/2$, there is a minimum in the envelope of acoustic response oscillations, modeled by the $\cos\theta$ in Eq. (2). The angle $\theta$ is proportional to the path length and consequently, at this minimum, the standing waves to which our transducer is sensitive have an effective path length of $4D$. Under these conditions smaller amplitude oscillations occur twice as frequently in the same interval of a temperature or pressure sweep. Comparing the amplitude of the waves with a path length of $2D$ to the amplitude of the waves with a path of $4D$ we find the absolute attenuation, $\alpha$, for one particular frequency, temperature and pressure:

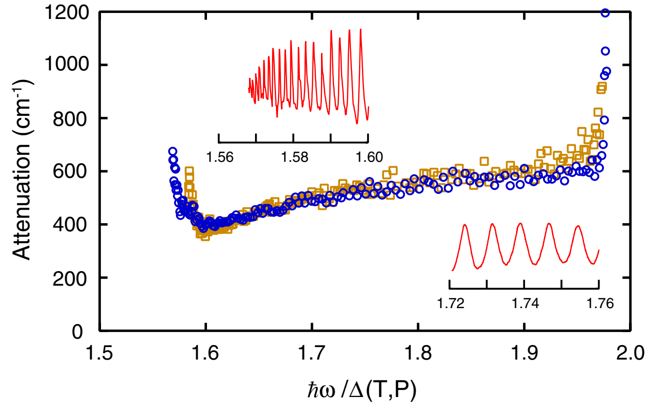


FIG. 2 (color online). Attenuation of transverse sound as a function of energy normalized to the energy gap at constant temperature, $\approx 550\ \mu$K from a pressure sweep. Blue (or dark gray) circles and gold (or gray) squares are for 88 and 111.5 MHz, respectively. In the insets (88 MHz) we show examples of the acoustic interference oscillations in $V_Z$ over small energy ranges for comparison, both on the same energy and amplitude scales.

$$\alpha = -\frac{1}{2D}\ln\left[\frac{b_{4D}}{b_{2D}}\right]. \tag{3}$$

The absolute value of the attenuation at all temperatures and pressures can then be determined, as shown in Fig. 2, with *no* fit parameters. The increased attenuation at the low energy end of Fig. 2 is from the ISQ mode and the increase in the attenuation near $2\Delta$ originates from the $2\Delta$ mode, recently reported [15]. The ISQ-mode frequency has a weak pressure dependent deviation from $\sqrt{12/5}\Delta(T, P)$ [12,13] which is reflected in the offset of the upturns in attenuation at low energy for the two frequencies in Fig. 2. With our technique we are able to observe propagating TS with an attenuation as high as 1000 cm$^{-1}$. As yet, we have not found any indication of propagating TS in the normal state of $^3$He and its observation will require overcoming this higher than expected attenuation [23]. We note in passing that our measurements were performed in the quantum limit of attenuation described by Landau [2], with $\hbar\omega/2\pi k_B T = 1.2$ (1.3) for the 88 (111.5) MHz data.

The contribution to the attenuation from the ISQ mode can be calculated from Eq. (1) with only a single fit parameter: the width of the ISQ mode. We use the form $\Gamma = \Gamma_c e^{-\Delta/k_B T}$, where $\Gamma_c = \Gamma_0 T_c^2$, and $\Gamma_0$ is pressure independent. The ISQ attenuation is shown separately, for the 88 MHz data, by the gray curve in Fig. 3. In order to represent the observed nonmonotonic dependence of attenuation on $\hbar\omega/\Delta$ it is clear that there must be an additional contribution. This unexpected behavior apparently increases smoothly with energy and then saturates, $\hbar\omega/\Delta \approx 1.7$. To obtain a quantitative assessment of this anomalous attenuation we must choose a value for $\Gamma_0$ which, if taken either too large or too small, will introduce an unphysical, sharp kink at $\hbar\omega/\Delta \sim 1.6$. Our final result using $\Gamma_0 = 9.5 \pm 2$ MHz/mK$^2$ is given by the green squares in Fig. 3. Since the ISQ-mode attenuation dominates only near the mode the subtracted result is largely unaffected by our choice of $\Gamma_0$, which we find to be a factor of 3 larger than previously suggested [14], based on a less accurate measurement of the ISQ-mode width [24]. Additionally, we find that the anomalous attenuation approaches the temperature independent value at low temperatures given in Fig. 3, as demonstrated by temperature sweeps in Fig. 4.

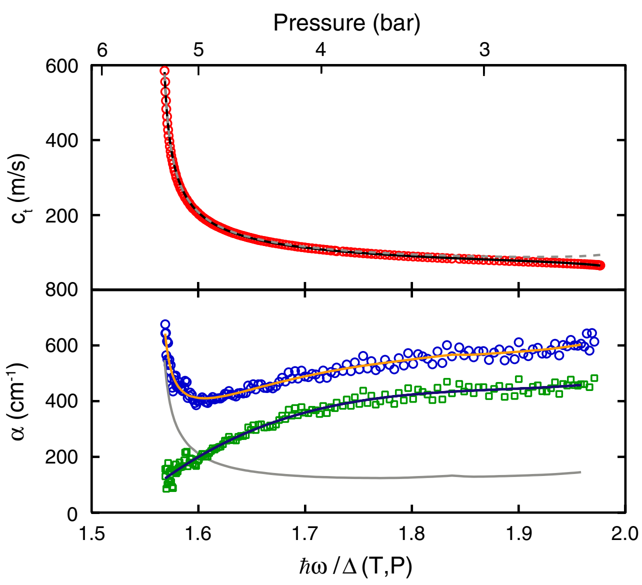


FIG. 3 (color online). Phase velocity of transverse sound (upper panel) and attenuation (lower panel) at 88 MHz at $\approx$550 $\mu$K in zero magnetic field. In the upper panel, the red (or black) circles are the data, the dashed gray curve is calculated from Eq. (1), and the black curve is the dispersion that accounts for the $2\Delta$-mode [15]. In the lower panel, the attenuation data (blue or dark gray circles) have been deconvolved by subtracting the contribution from coupling to the ISQ mode (gray curve) calculated from Eq. (1) leaving an anomalous attenuation given by the green (or gray) squares.

In contrast to the attenuation, we have found that the phase velocity of TS is accurately accounted for by the dispersion relation for the order parameter collective mode, Eq. (1) [15,18], as shown in the upper panel of Fig. 3. We infer that the anomalous attenuation cannot be associated with order parameter collective modes. Furthermore, the data at 88 and 111.5 MHz are nearly identical, shown in Fig. 5, indicating that the attenuation is not explicitly dependent on frequency at the same values of $\hbar\omega/\Delta$, nor does it depend on temperature in the low temperature limit, Fig. 4. On this basis we can rule out quasiparticle-quasiparticle scattering as the source, since this mechanism should decrease to zero exponentially at low temperatures. We have applied magnetic fields up to 300 G along the TS propagation direction and have found that the attenuation does not depend on magnetic field, outside of the regions of field induced birefringence from order parameter collective modes (AFE). We suggest that the anomalous attenuation might be attributed to the interaction of TS waves with surface Andreev bound states (SABS).

SABS play an important role in the understanding of unconventional superconductors and superfluids. For example, SABS have been studied in tunneling experiments in $Sr_2RuO_4$ [25] and the high $T_c$ superconductors [26,27]. In superfluid $^3$He they have been found to dominate the transverse acoustic impedance [28] and have been observed in the surface specific heat [29]. Moreover, in the absence of excited quasiparticles, there is no coupling between a transverse transducer and $^3$He, for example, when the scattering at the transducer surface is specular

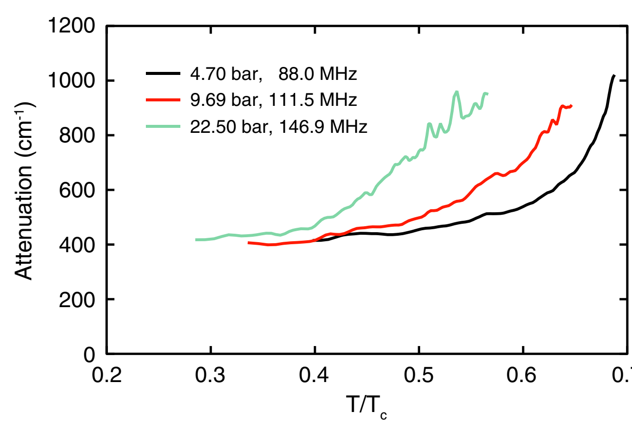


FIG. 4 (color online). The temperature dependence of the TS attenuation. The pressures are chosen for the acoustic frequencies such that $\hbar\omega/\Delta$ is within the shaded region of Fig. 1 and never crosses the ISQ mode. The data have been minimally smoothed and are shown as a line for clarity. Quasiparticle-quasiparticle scattering, seen as an increasing attenuation at high $T/T_c$, is expected to decrease to zero at zero temperature. Instead, there is a crossover from the quasiparticle dominated region at high $T/T_c$ to a temperature independent anomalous region at low $T/T_c$. The low temperature endpoints of the data correspond to $\hbar\omega/\Delta \approx 1.64$.

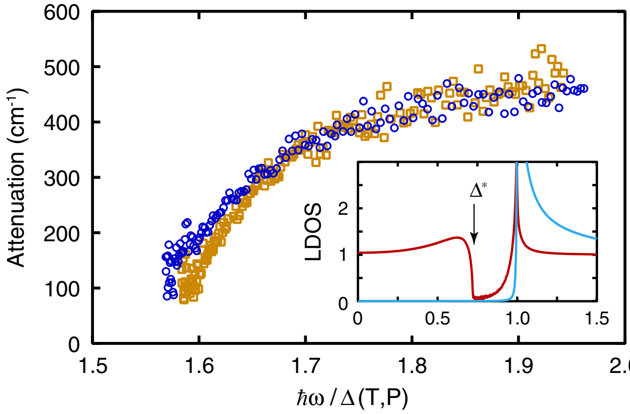


FIG. 5 (color online). The anomalous attenuation, for 88 MHz (blue or dark gray circles) and 111.5 MHz (gold or gray squares). A smooth crossover in the attenuation appears at $\hbar\omega = \Delta + \Delta^\star \approx 1.7\Delta$. The inset shows the local density of states, as a function energy normalized to $\Delta$, at the transducer surface (red or black) and in the bulk $^3$He (light blue or light gray) at $T/T_c = 0.5$ for diffusive boundary conditions.

[30]. However, quasiparticles that scatter diffusely transfer momentum parallel to the transducer surface and couple to transverse currents in the $^3$He [30]. These local excitations are the bound states (SABS). In $^3$He-$B$ they have a characteristic energy given by $\Delta^\star$ [22,31], the upper limit of the density of states band which we show integrated over all trajectories in the inset of Fig. 5 (red or black trace). These midgap states are responsible for structure observed in the temperature dependence of the acoustic impedance [22,28] between the transducer and helium and should also affect the amplitude of a transverse sound wave reflected from a surface. Excitation of SABS will attenuate the wave and we expect this to follow the frequency dependence of the imaginary part of the acoustic impedance [22], increasing with frequency up to $\hbar\omega = \Delta + \Delta^\star$ and then leveling off. This scenario is qualitatively consistent with the attenuation shown in Fig. 5 where we observe a smooth but distinct crossover near $\hbar\omega \approx 1.7\Delta$ to a regime of anomalous attenuation at higher energy. With this interpretation our results are in good agreement with the theoretical value for $\hbar\omega = \Delta + \Delta^\star = 1.75\Delta$, at $T/T_c \sim 0.4$ for diffusive boundary conditions.

In summary, we have measured the attenuation of transverse sound in $^3$He taking advantage of the acoustic Faraday effect to determine absolute values. We found an anomalous contribution to the attenuation which cannot be accounted for in terms of collective modes or quasiparticle scattering in the bulk. We suggest that scattering of transverse sound with surface Andreev bound states is the most likely mechanism. A crossover in the frequency dependence of the attenuation corresponds to the theoretical value of the upper limit of the midgap in the surface density of states of $\Delta^\star/\Delta = 0.7$.

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