Enhancement of Ohmic and Stochastic Heating by Resonance Effects in Capacitive Radio Frequency Discharges: A Theoretical Approach

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In low-pressure capacitive radio frequency discharges, two mechanisms of electron heating are dominant: (i) Ohmic heating due to collisions of electrons with neutrals of the background gas and (ii) stochastic heating due to momentum transfer from the oscillating boundary sheath. In this work we show by means of a nonlinear global model that the self-excitation of the plasma series resonance which arises in asymmetric capacitive discharges due to nonlinear interaction of plasma bulk and sheath significantly affects both Ohmic heating and stochastic heating. We observe that the series resonance effect increases the dissipation by factors of 2–5. We conclude that the nonlinear plasma dynamics should be taken into account in order to describe quantitatively correct electron heating in asymmetric capacitive radio frequency discharges.

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In low-pressure capacitive radio frequency discharges, two mechanisms of electron heating play a major role: (i) Ohmic heating due to collisions of electrons with neutrals and (ii) stochastic heating—often referred to as Fermi heating—due to momentum transfer from the oscillating sheath. Various models have been proposed in order to study electron heating phenomena in capacitive discharges [1–11]. None of these models account for resonance effects for a number of reasons: Some models do not prescribe the voltage at the electrode but the total current through the plasma. Other models study a highly idealized, symmetric situation by means of a one-dimensional discharge. Other models neglect electron inertia from the outset. Other models study discharges at relatively high pressures. The various models do not account for nonsinusoidal radio frequency currents due to self-excitation of the plasma series resonance which arises in asymmetric rf discharges [12–14]. An experimental study of such rf discharges observed that harmonics were excited at the series resonance and can enhance the electron heating [15,16]. Recently, analytic calculations of enhanced Ohmic electron heating due to nonlinear series resonance excitation have been described [17–21]. Nonlinear effects on stochastic heating, however, have not been studied yet.

In this work we observe, by means of the nonlinear global model, significant enhancement of both Ohmic and stochastic heating of electrons due to the series resonance effect (see [22] for a more complete exposition as well as particle-in-cell simulations that show the evidence of the effect). We use a model, which self-consistently takes into account the dc bias voltage across the sheath, to calculate Ohmic and stochastic heating for varying gas pressure. In order to study the effect of the plasma series resonance on the heating, we distinguish between two cases: (i) the "fast" dynamics where the plasma series resonance is excited and (ii) the "slow" (or quasistationary) dynamics, where the self-excitation of the plasma series resonance is switched off, by neglecting the inertia of the electrons. We incorporate stochastic heating by using the "hard wall" model proposed first by Godyak [1].

We argue that the nonlinear plasma dynamics (i.e., resonance effects) should be taken into account in order to describe quantitatively correct the heating mechanisms of electrons in asymmetric capacitive radio frequency discharges. The integration of nonlinear heating effects into established rf discharge models should help to remedy the problem of all simulations of rf discharges to arrive at the wrong plasma density (which is always too low). Although our investigation is geared towards capacitively coupled plasmas, it may also find use in the theory of inductively coupled plasma (which are the second type of rf plasmas that dominate the field): Any inductively coupled plasma, except an idealized one, also has a residual capacitive coupling. In fact, one may speculate that the E-to-H transition in inductively coupled plasmas is influenced by nonlinear resonance effects [23–25].

In order to formulate an effective nonlinear model of the high-frequency behavior of an asymmetric capacitive radio frequency discharge, we assume that the plasma can be strictly separated into bulk and sheath regions. While the bulk size (of size l_P) fills most of the discharge volume, the sheath forms a thin layer of average thickness l_S at the driven electrode. We thus adopt the ordering $\lambda_D \ll l_S \ll l_P$, with λ_D being the Debye length.

We further assume the driving angular frequency ω_{RF} to lie between the ion plasma frequency ω_{Pi} and the electron plasma frequency ω_{Pe}, such that the ions are not affected by the rf modulation and react only on the phase-averaged electrical field. For the electrons, we distinguish between small and large structures. A small structure in this sense is the sheath, where the electrons follow the electric field instantaneously. In the discharge as a whole—the large structure—dynamical effects can occur. In particular, the plasma series resonance (or the geometrical resonance frequency) at the frequency $\omega_{\mathrm{PSR}} = [l_S/(l_P - l_S)]^{1/2}\omega_{\mathrm{Pe}}$ can be excited. Altogether, we assume for the time scales the ordering $\omega_{\mathrm{Pi}} \ll \omega_{\mathrm{RF}} < \omega_{\mathrm{PSR}} \ll \omega_{\mathrm{Pe}}$.

The plasma bulk is assumed to be quasineutral and (for simplicity) homogeneous, $n_e = n_i = n = \mathrm{const}$. The radio frequency current through the discharge is carried by electron conduction alone. The dependence of the current density on the electric field can thus be modeled by a generalized Ohm's law. It takes into account the acceleration of the electrons by the electric field and their momentum loss due to elastic collisions with the neutrals of the background gas. The collision rate is given by momentum transfer collision frequency ν_m. Since we are interested in both Ohmic and stochastic heating, we incorporate stochastic heating by using the hard wall model and apply thus an effective collision rate,

$$\nu_{\mathrm{eff}} = \nu_m + \bar{v}_e/l_p \tag{1}$$

where $\bar{v}_e = (8eT_e/\pi m)^{1/2}$ is the mean thermal speed, with T_{e} the electron temperature (in equivalent voltage units) and m the electron mass [7]. The factor l_p, the bulk size, helps to "transform" stochastic heating, which is a surface effect, into a volume effect in order to allow for comparison with Ohmic heating, which is a volume effect.

Neglecting warm plasma effects, i.e., finite electron temperature, we obtain a generalized Ohm's law of the form

$$\frac{\partial \vec{j}}{\partial t} = \frac{e^2 n}{m}\vec{E} - \nu_{\mathrm{eff}}\vec{j}. \tag{2}$$

We also neglect ionization and recombination. The current density then obeys an equation of continuity,

$$\nabla \cdot \vec{j} = 0. \tag{3}$$

For plasmas which are not too large and have at the same time an electron density that is not too high, electromagnetic effects can be neglected. Hence, we adopt the electrostatic approximation of Maxwell's equations, $\nabla \times \vec{E} = 0$. A detailed discussion of electromagnetic effects in capacitive discharges and the justification of the electrostatic approximation are given in [26–29].

We assume that the "ground" electrode area is much larger than the "powered" electrode area. We thus ignore the impedance of the larger ground electrode sheath. The voltage drop across the sheath in front of the driven electrode can be modeled by a nonlinear voltage charge relation. Since we assume the ion density within the sheath to be homogeneous, the voltage drop V_S is related to the sheath charge Q_S by $V_S = Q_S^2/2e\epsilon_0 n A_E^2$, such that the sheath capacitance is given by

$$C_S = (2en\epsilon_0 A_E^2/V_S)^{1/2} \tag{4}$$

A_E denotes the effective electrode area.

The ion and electron conduction current densities are taken to be $j_{i0} = en v_{\mathrm{Bohm}}$ and $j_e = j_{e0}\exp(-V_S/T_e)$, with $v_{\mathrm{Bohm}} = (eT_e/M)^{1/2}$ the Bohm speed, M the ion mass, and $j_{e0} = en\bar{v}_e$. (This type of sheath model was first used in [30] but without including the effect of the bulk plasma and the resulting series resonance excitation.) The bias capacitance of the electrode sheath is taken to be $C_B = \epsilon_0 A_E/l_B$, where l_B is taken to be smaller than the minimum sheath width, such that the oscillatory voltage across C_B is small.

After averaging over space and applying Kirchoff's laws we obtain a model that allows for nonlinear excitation of the series resonance in an asymmetric capacitive discharge [see Fig. 1]. The model consists of a system of first order nonlinear differential equations for the voltage drop across the sheath V_S, the bias sheath capacitor voltage V_B, and the radio frequency current I_P,

$$\frac{dV_S}{dt} = -C_S^{-1}(I_P + j_{i0}A_E - j_{e0}A_E e^{-V_S/T_e}), \tag{5}$$

$$\frac{dV_B}{dt} = -C_B^{-1} I_P, \tag{6}$$

$$\frac{dI_P}{dt} = L_P^{-1}(V_S + \hat{V}_{\mathrm{RF}}\cos\omega_{\mathrm{RF}}t + V_B) - \nu_{\mathrm{eff}} I_P. \tag{7}$$

$L_P = l_P m/e^2 n A_E$ denotes the "inductance" (due to electron inertia) of the plasma bulk.

It is advantageous to write the equations in dimensionless form. We set the radio frequency period equal to 2π, which implies that the phase is $\phi = \omega_{\mathrm{RF}}t$. To compare the

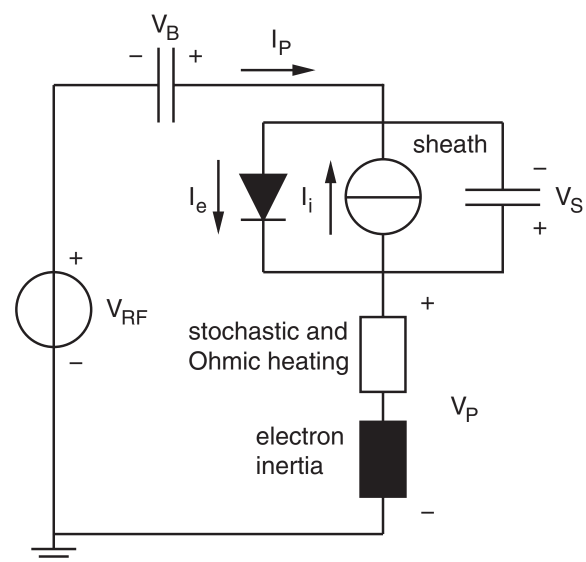

FIG. 1. Model used for nonlinear excitation of the series resonance in an asymmetric capacitive discharge.

nonlinear resonance effects—the fast dynamics—to those without resonance, we determine the slow dynamics by setting the plasma impedance to zero [17].

The equations can be solved by means of numerical standard methods. We use an adaptive Runge-Kutta scheme for solving the equations. The solutions are obtained for a set of generic plasma parameters: $\hat{V}_{\mathrm{RF}} = 400$ V, $\omega_{\mathrm{RF}} = 2\pi \times 13.56$ MHz, $n = 10^{15}$ m^{-3}, $T_e = 3$ V, $l_P = 5.7$ cm, $l_B = 0.0027$ cm, $A_E = 5.7^2\pi$ cm^2, $M =$ argon ion mass, and $\nu_m = K_m n_g$, the electron-neutral momentum transfer collision frequency, $K_m = 10^{-13}$ m^3 s^{-1} the rate coefficient, and $n_g(m^{-3}) = 3.3 \times 10^{22}$ p(Torr) the neutral gas density.

Figure 2 shows the time variations of the sheath charge Q_S and plasma current I_P for both cases, the fast dynamics (solid lines) and the slow dynamics (dashed lines) where resonance effects do not occur. The gas pressure p is set to 5 mTorr. The sheath charge Q_S in the first plot shows significant oscillations around the 5th harmonic. The oscillations of Q_S are amplified in the plasma current variation shown in the second plot, as expected from the frequency dependence $I_P \propto \omega_{\mathrm{RF}} Q_S$. The harmonic content of I_P and $I_{P\mathrm{slow}}$ was determined by calculating the frequency spectrum of the dynamics using a fast Fourier transform. This is shown in the third plot, where the amplitude of the Fourier transform is given for the fast (squares) and the slow dynamics (stars). We see a significant harmonic content for the total current I_P, with the largest harmonics at 5 and 6, comparable to the height of the fundamental.

The self-excitation of rf current harmonics in the low-pressure regime leads to an increase in rms value of the rf current and thus to significantly enhanced Ohmic dissipation [18]. The Ohmic dissipation can be calculated from the plasma current I_P by evaluating

$$\eta_{\mathrm{Ohm}} = \langle \nu_{\mathrm{m}} L_P I_P^2 \rangle \tag{8}$$

The angle brackets denote the mean square value of the normalized dissipation. In the case of slow dynamics where resonances do not occur, the Ohmic dissipation decrease nearly linearly with decreasing gas pressure [Fig. 3 (top panel), dashed line]. For the fast dynamics, the Ohmic dissipation experiences a significant enhancement [Fig. 3 (top panel), solid line]. However, in the limit $p \to 0$ the Ohmic dissipation tends to zero.

The stochastic dissipation can be calculated similarly to (8). After substituting of ν_m by $\bar{v}_e/l_P$ we obtain

$$\eta_{\mathrm{stoch}} = \left\langle \frac{\bar{v}_e}{l_P} L_P I_P^2 \right\rangle \tag{9}$$

In the slow dynamics case a constant stochastic dissipation occurs [Fig. 3 (bottom panel), dashed line]. This is clear since (linear) stochastic heating does not depend on the gas pressure. For the fast dynamics case the increased rms value of the rf current leads to a significant increase in

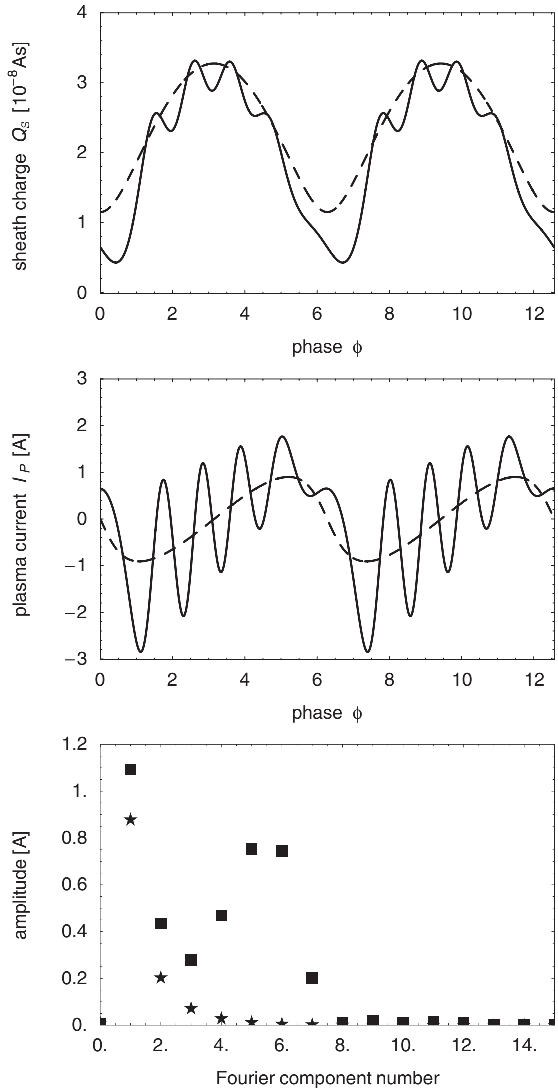

FIG. 2. Time variations of sheath charge Q_S and plasma current I_P (at a gas pressure $p = 5$ mTorr) for both cases, the fast dynamics (solid lines) and the slow dynamics (dashed lines). Bottom panel: Amplitude of the Fourier transform of the plasma current (fast dynamics, squares; slow dynamics, stars).

stochastic dissipation with decreasing pressure [Fig. 3 (middle panel), solid line].

The total dissipation, which is of course the sum of both Ohmic and stochastic dissipation, for varying pressure for both cases (the fast and the slow dynamics case) is shown in Fig. 3 (bottom panel). We can see that, for a range of relatively low pressures, the series resonance effect increases the dissipation by factors of 2–5.

Based on an effective nonlinear global model of a capacitive radio frequency discharge, which self-consistently takes into account the dc bias voltage across the sheath, we have studied plasma series resonance effects on both Ohmic and stochastic heating. We demonstrate that the underlying nonlinear plasma dynamics play a major role—in particular in the low-pressure regime. Consistent

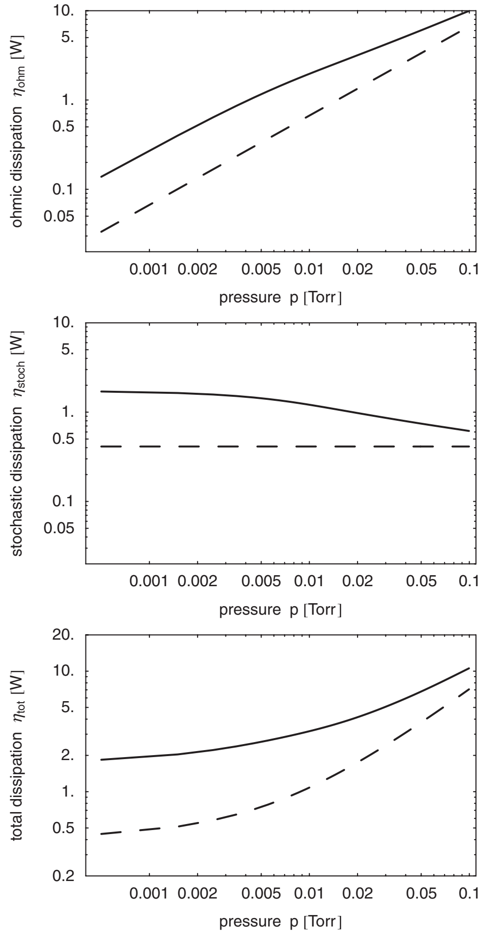

FIG. 3. Ohmic (top panel), stochastic (middle panel), and total dissipation (bottom panel) for cases with (solid line) and without (dashed line) the series resonance effect versus gas pressure p.

with recent experimental results [16] and particle-in-cell simulations [22], we have found that the heating of electrons is enhanced by factors of 2–5 due to self-excitation of the plasma series resonance and the related harmonic content in the radio frequency current. We conclude that nonlinear plasma dynamics should be taken into account in order to describe quantitatively correct the heating mechanisms of electrons in capacitive radio frequency discharges.

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