α-Particle Condensation in ^{16}O Studied with a Full Four-Body Orthogonality Condition Model Calculation

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To explore the four-α-particle condensate state in ^{16}O, we solve a full four-body equation of motion based on the four-α-particle orthogonality condition model in a large four-α-particle model space spanned by Gaussian basis functions. A full spectrum up to the 0_6^+ state is reproduced consistently with the lowest six 0^+ states of the experimental spectrum. The 0_6^+ state is obtained at about 2 MeV above the four-α-particle breakup threshold and has a dilute density structure, with a radius of about 5 fm. The state has an appreciably large α condensate fraction of 61%, and a large component of $\alpha + {}^{12}\mathrm{C}(0_2^+)$ configuration, both features being reliable evidence for this state to be of four-α-particle condensate nature.

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It is well established that α-particle clustering plays a very important role for the structure of lighter nuclei [1,2]. The importance of α-particle cluster formation also has been discussed in infinite nuclear matter, where α-particle type condensation is expected at low density [3], quite in analogy to the recently realized Bose-Einstein condensation of bosonic atoms in magneto-optical traps [4]. On the other hand, for trapped fermions, quartet condensation also is an emerging subject, discussed, so far, only theoretically [5]. In nuclei the bosonic constituents always are only very few in number, nevertheless possibly giving rise to clear condensation characteristics, as is well known from nuclear pairing [6]. Concerning α-particle condensation, only the Hoyle state, i.e., the 0_2^+ state in ^{12}C, has clearly been established so far. Several papers of the past [7–10] and also more recently [11–13] have by now established beyond any doubt that the Hoyle state, only having about one third of saturation density, can be described, to good approximation, as a product state of three-α particles, condensed, with their center-of-mass motion, into the lowest mean field $0S$ orbit [14,15]. This shall be the definition of a Bose-condensed state in finite nuclei, clearly reflecting the situation found in infinite matter in [3]. Occasionally, we also shall call it a gaslike state.

The establishment of this novel aspect of the Hoyle state naturally leads us to the speculation about four-α-particle condensation in ^{16}O, which is the focus of this work. The 0^+ spectrum of ^{16}O has, in the past, very well been reproduced up to about 13 MeV excitation energy, including the ground state, with a semimicroscopic cluster model, i.e., the $\alpha + {}^{12}$C orthogonality condition model (OCM) [16]. In particular, this model calculation, as well as that of an α + ^{12}C generator-coordinate-method one [17], demonstrates that the 0_2^+ state at 6.05 MeV and the 0_3^+ state at 12.05 MeV have $\alpha + {}^{12}$C structures [18] where the α particle orbits around the $^{12}\mathrm{C}(0_1^+)$ core in an S wave and around the $^{12}\mathrm{C}(2_1^+)$ core in a D wave, respectively. Consistent results were later obtained by the four-α-particle OCM calculation within the harmonic oscillator basis [19]. However, the model space adopted in Refs. [16,17,19] is not sufficient to account simultaneously for the $\alpha + {}^{12}$C and the four-α-particle gaslike configurations. On the other hand, the four-α-particle condensate state was first investigated in Ref. [11] and its existence was predicted around the four-α-particle threshold with a new type of microscopic wave function of α-particle condensate character. While that so-called THSR wave function [20] can well describe the dilute α cluster states as well as shell-model-like ground states, other structures such as $\alpha + {}^{12}$C clustering are smeared out and only incorporated in an average way. Since there exists no calculation, so far, which reproduces both the four-α-particle gas and $\alpha + {}^{12}$C cluster structures simultaneously, it is crucial to perform an extended calculation for the simultaneous reproduction of both kinds of structures, which will give a decisive benchmark for the existence of the four-α-particle condensate state from a theoretical point of view.

The purpose of this Letter is to explore the four-α-particle condensate state by solving a full OCM four-body equation of motion without any assumption with respect to the structure of the four-α-particle system. Here we take the four-α-particle OCM with Gaussian basis functions, the model space of which is large enough to cover the four-α-particle gas, the $\alpha + {}^{12}$C cluster, as well as the shell-model configurations. The OCM is extensively described in Ref. [21]. Many successful applications of OCM are reported in Ref. [2]. The four-α-particle OCM Hamiltonian is given as follows:

$$\mathcal{H} = \sum_{i}^{4} T_i - T_{\rm cm} + \sum_{i<j}^{4} [[V_{2\alpha}^{(\rm N)}(i,j) + V_{2\alpha}^{(\rm C)}(i,j) + V_{2\alpha}^{(\rm P)}(i,j)] + \sum_{i<j<k}^{4} V_{3\alpha}(i,j,k) + V_{4\alpha}(1,2,3,4), \quad (1)$$

where T_i, $V_{2\alpha}^{(\rm N)}(i,j)$, $V_{2\alpha}^{(\rm C)}(i,j)$, $V_{3\alpha}(i,j,k)$, and $V_{4\alpha}(1,2,3,4)$ stand for the operators of kinetic energy for the ith α particle, two-body, Coulomb, three-body, and four-body forces between α particles, respectively. The center-of-mass kinetic energy $T_{\rm cm}$ is subtracted from the Hamiltonian. $V_{2\alpha}^{(\rm P)}(i,j)$ is the Pauli exclusion operator [22], by which the Pauli forbidden states between two α particles in $0S$, $0D$, and $1S$ states are eliminated, so that the ground state with the shell-model-like configuration can be described correctly. The effective α-α interaction $V_{2\alpha}^{(\rm N)}$ is constructed by the folding procedure from two kinds of effective two-nucleon forces. One is the modified Hasegawa-Nagata (MHN) force [23] and the other is the Schmidt-Wildermuth (SW) force [24], see Refs. [15,25] for applications, respectively. We should note that the folded α-α potentials reproduce the α-α scattering phase shifts and energies of the ^{8}Be ground state and of the Hoyle state. The three-body force $V_{3\alpha}$ is as in Refs. [15,25] where it was phenomenologically introduced, so as to fit the ground state energy of ^{12}C. In addition, the phenomenological four-body force $V_{4\alpha}$ which is taken to be a Gaussian is adjusted to the ground state energy of ^{16}O, where the range is simply chosen to be the same as that of the three-body force. The origin of the three-body and four-body forces is considered to derive from the state dependence of the effective nucleon-nucleon interaction and the additional Pauli repulsion between more than two α particles. However, they are short range, and hence only act in compact configurations. The expectation values of those forces do not exceed 7% of that of the corresponding two-body term, even for the ground state with the most compact structure, i.e., being the most sensitive to those forces.

Employing the Gaussian expansion method [26] for the choice of variational basis functions, the total wave function Ψ of the four-α-particle system is expanded in terms of Gaussian basis functions as follows:

$$\Psi(0_n^+) = \sum_{c,\nu} A_c^n(\nu)\Phi_c(\nu), \quad (2)$$

$$\Phi_c(\nu) = \hat{\mathcal{S}}[[\varphi_{l_1}(\boldsymbol{r}_1,\nu_1)\varphi_{l_2}(\boldsymbol{r}_2,\nu_2)]_{l_{12}}\varphi_{l_3}(\boldsymbol{r}_3,\nu_3)]_J, \quad (3)$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ are the Jacobi coordinates describing internal motions of the four-α-particle system. $\hat{\mathcal{S}}$ stands for the symmetrization operator acting on all α particles obeying Bose statistics. ν denotes the set of size parameters ν_1, ν_2, and ν_3 of the normalized Gaussian function, $\varphi_l(\boldsymbol{r},\nu_i) = N_{l,\nu_i} r^l \exp(-\nu_i r^2) Y_{\rm lm}(\hat{\boldsymbol{r}})$, and c the set of relative orbital angular momentum channels $[[l_1,l_2]_{l_{12}},l_3]_J$ depending on either of the coordinate type of K or H [26], where l_1, l_2, and l_3 are the orbital angular momenta with respect to the corresponding Jacobi coordinates. The coefficients $A_c^n(\nu)$ are determined according to the Rayleigh-Ritz variational principle.

Figure 1 shows the energy spectrum with $J^\pi = 0^+$, which is obtained by diagonalizing the Hamiltonian, Eq. (1), in a model space as large as given by 5120 Gaussian basis functions, Eq. (3) (the other multipolarities, needing larger basis sets, are more difficult and shall be studied in future work). It is confirmed that all levels are well converged. With the above mentioned effective α-α forces, we can reproduce the full spectrum of 0^+ states, and tentatively make a one-to-one correspondence of those states with the six lowest 0^+ states of the experimental spectrum. In view of the complexity of the situation, the agreement is considered to be very satisfactory.

We show in Table I the calculated rms radii and monopole matrix elements to the ground state, together with the corresponding experimental values. The M(E0) values for the 0_2^+ and 0_5^+ states are consistent with the corresponding experimental values. The consistency for the 0_3^+ state is

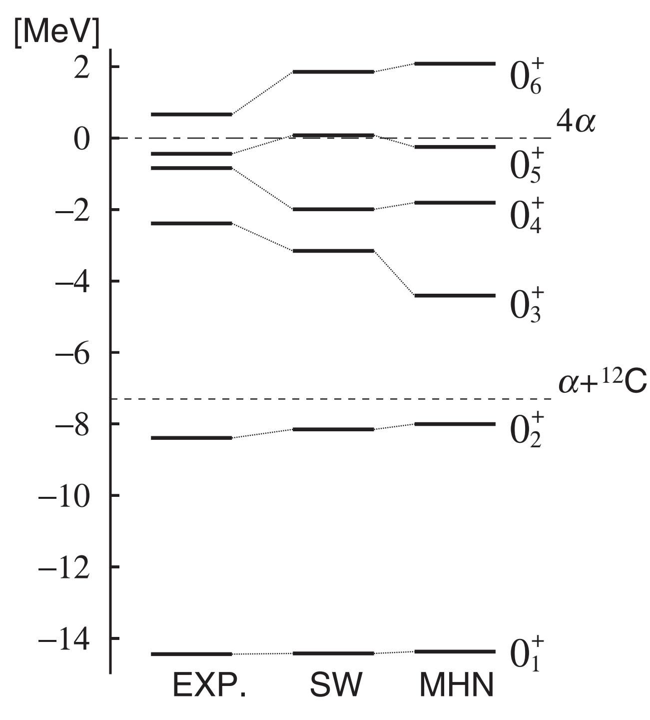

FIG. 1. Comparison of energy spectra between experiment and the present calculation. Two kinds of effective two-body nucleon-nucleon forces MHN and SW are adopted (see text). Dotted and dash-dotted lines denote the $\alpha + {}^{12}$C and four-α-particle thresholds, respectively. Experimental data are taken from Ref. [34], and from Ref. [35] for the 0_4^+ state. The assignments with experiment are tentative; see, however, the detailed discussion in the text.

TABLE I. The rms radii R and monopole transition matrix elements to the ground state $M(\mathrm{E0})$ in units of fm and fm^2, respectively. R_{expt} and $M(\mathrm{E0})_{\mathrm{expt}}$ are the corresponding experimental data. The finite-size effect of α particle is taken into account in R and $M(\mathrm{E0})$ (see Ref. [15] for details).

	R		$M(\mathrm{E0})$		R_{expt}	$M(\mathrm{E0})_{\mathrm{expt}}$
	SW	MHN	SW	MHN		
0_1^+	2.7	2.7			2.71 ± 0.02	
0_2^+	3.0	3.0	4.1	3.9		3.55 ± 0.21
0_3^+	2.9	3.1	2.6	2.4		4.03 ± 0.09
0_4^+	4.0	4.0	3.0	2.4		No data
0_5^+	3.1	3.1	3.0	2.6		3.3 ± 0.7
0_6^+	5.0	5.6	0.5	1.0		No data

within a factor of 2. As mentioned above, the structures of the 0_2^+ and 0_3^+ states are well established as having the $\alpha + {}^{12}\mathrm{C}(0_1^+)$ and $\alpha + {}^{12}\mathrm{C}(2_1^+)$ cluster structures, respectively. These structures of the 0_2^+ and 0_3^+ states are confirmed in the present calculation. We also mention that the ground state is described as having a shell-model configuration within the present framework, the calculated rms value agreeing with the observed one (2.71 fm).

On the contrary, the structures of the observed 0_4^+, 0_5^+, and 0_6^+ states in Fig. 1 have, in the past, not clearly been understood, since they have never been discussed with the previous cluster model calculations [16,17,19]. Although Ref. [11] predicts the four-α-particle condensate state around the four-α-particle threshold, it is not clear to which of those states it corresponds to. We will analyze the situation with the THSR wave function of [11] in a future publication [27].

As shown in Fig. 1, the present calculation succeeded, for the first time, to reproduce the 0_4^+, 0_5^+, and 0_6^+ states, together with the 0_1^+, 0_2^+, and 0_3^+ states. This puts us in a favorable position to discuss the four-α-particle condensate state, expected to exist around the four-α-particle threshold.

In Table I, the largest rms value of about 5 fm is found for the 0_6^+ state. Compared with the relatively smaller rms radii of the 0_4^+ and 0_5^+ states, this large size suggests that the 0_6^+ state may be composed of a weakly interacting gas of α particles [28] of the condensate type.

While a large size is generally necessary for forming an α condensate, the best way for its identification is to investigate the single-α-particle orbit and its occupation probability, which can be obtained by diagonalizing the one-body (α) density matrix as defined in [14,15,29,30]. As a result of the calculation of the $L = 0$ case, a large occupation probability of 61% of the lowest $0S$ orbit is found for the 0_6^+ state, whereas the other five 0^+ states all have appreciably smaller values, at most 25% (0_2^+). The corresponding single-α-particle S orbit is shown in Fig. 2. It has a strong spatially extended behavior without any node ($0S$). This indicates that α particles are condensed

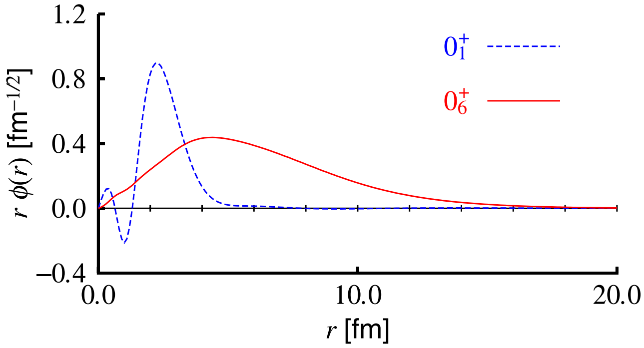

FIG. 2 (color online). The radial parts of single-α-particle orbits with $L = 0$ belonging to the largest occupation number, for the ground and 0_6^+ states with MHN force.

into the very dilute $0S$ single-α-particle orbit, see also Ref. [31]. Thus, the 0_6^+ state clearly has four-α-particle condensate character. We should note that the orbit is very similar to the single-α-particle orbit of the Hoyle state [14,15]. We also show in Fig. 2 the single-α-particle orbit for the ground state. It has maximum amplitude at around 3 fm and oscillations in the interior with two nodal ($2S$) behavior, due to the Pauli principle and reflecting the shell-model configuration.

To further analyze the obtained wave functions, we calculate an overlap amplitude, which is defined as follows:

$$\mathcal{Y}(r) = \left\langle \left[\frac{\delta(r' - r)}{r'^2} Y_L(\hat{\boldsymbol{r}}') \Phi_L({}^{12}\mathrm{C}) \right]_0 \middle| \Psi(0_6^+) \right\rangle. \qquad (4)$$

Here, $\Phi_L({}^{12}\mathrm{C})$ is the wave function of ${}^{12}\mathrm{C}$, given by the three-α-particle OCM calculation [15], and r is the relative distance between the center-of-mass of ${}^{12}\mathrm{C}$ and the α particle. From this quantity we can see how large is the component in a certain $\alpha + {}^{12}\mathrm{C}$ channel which is contained in our wave function (2) for 0_6^+. The amplitudes for the 0_6^+ state are shown in Fig. 3. It only has a large amplitude in the $\alpha + {}^{12}\mathrm{C}(0_2^+)$ channel, whereas the amplitudes in other channels are much suppressed. The amplitude in the Hoyle-state channel has no oscillations and a long tail stretches out to ~20 fm. This behavior is very

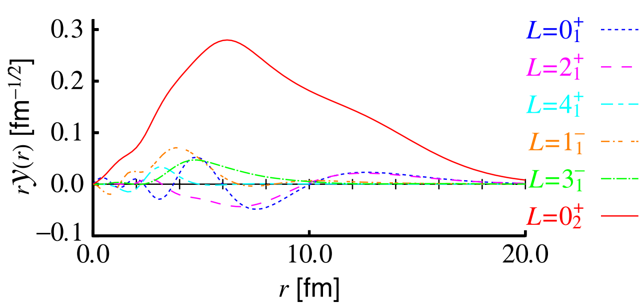

FIG. 3 (color online). $r\mathcal{Y}(r)$ defined by Eq. (4) for the 0_6^+ state with the MHN force.

similar to that of the single-α-particle orbit of the 0_6^+ state discussed above.

The α decay width constitutes a very important information to identify the 0_6^+ state from the experimental point of view. It can be estimated, based on the R-matrix theory, with the overlap amplitude Eq. (4) [32]. We find that the total α decay width of the 0_6^+ state is as small as 50 keV (experimental value: 166 keV). This means that the state can be observed as a quasistable state. Thus, the width, as well as the excitation energy, are consistent with the observed data. All the characteristics found from our OCM calculation, therefore, indicate that the calculated 6th 0^+ state with four-α-particle condensate nature can probably be identified with the experimental 0_6^+ state at 15.1 MeV.

Finally we discuss the structures of the 0_4^+ and 0_5^+ states. Our present calculations show that the 0_4^+ and 0_5^+ states mainly have $\alpha + {}^{12}\mathrm{C}(0_1^+)$ structure with higher nodal behavior and $\alpha + {}^{12}\mathrm{C}(1^-)$ structure, respectively. Further details will be given in a forthcoming extended paper. The calculated width of the 0_4^+ is $\sim$150 keV, which is quite a bit larger than that found for the 0_5^+ state $\sim$50 keV. Both are qualitatively consistent with the corresponding experimental data, 600 and 185 keV, respectively. The reason why the width of the 0_4^+ state is larger than that of the 0_5^+ state, though the 0_4^+ state has lower excitation energy, is due to the fact that the former has a much larger component of the $\alpha + {}^{12}\mathrm{C}(0_1^+)$ decay channel, reflecting the characteristic structure of the 0_4^+ state. The four-α-particle condensate state, thus, should not be assigned to the 0_4^+ or 0_5^+ state [33] but very likely to the 0_6^+ state.

In conclusion, the present four-α-particle OCM calculation, for the first time, succeeded in describing the structure of the full observed 0^+ spectrum up to the 0_6^+ state in ^{16}O. The 0^+ spectrum of ^{16}O up to about 15 MeV is now essentially understood, including the four-α-particle condensate state. This is remarkable improvement concerning our knowledge of the structure of ^{16}O. We found that the 0_6^+ state above the four-α-particle threshold has a very large rms radius of about 5 fm and has a rather large occupation probability of 61% of four α particles sitting in a spatially extended single-α-particle $0S$ orbit. The wave function was shown to have a large $\alpha + {}^{12}$C amplitude only for ${}^{12}\mathrm{C}^*$, i.e., the Hoyle state (the related spectroscopic factor shall be discussed in an extended version of this Letter). These results are strong evidence of the 0_6^+ state, which is a new theoretical prediction, for being the four-α-particle condensate state, i.e., the analog to the Hoyle state in ^{12}C. Further experimental information is very much requested to confirm the structure of this novel state. Also independent theoretical calculations are strongly needed for confirmation of our results.

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[1] K. Wildermuth and Y. C. Tang, *A Unified Theory of the Nucleus* (Vieweg, Braunschweig, 1977).
[2] K. Ikeda *et al.*, Prog. Theor. Phys. Suppl. **68**, 1 (1980).
[3] G. Röpke *et al.*, Phys. Rev. Lett. **80**, 3177 (1998).
[4] F. Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463 (1999).
[5] A. S. Stepanenko *et al.*, arXiv:cond-mat/9901317; B. Doucot and J. Vidal, Phys. Rev. Lett. **88**, 227005 (2002); H. Kamei *et al.*, J. Phys. Soc. Jpn. **74**, 1911 (2005); S. Capponi *et al.*, Phys. Rev. A **77**, 013624 (2008).
[6] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
[7] H. Horiuchi, Prog. Theor. Phys. **51**, 1266 (1974); **53**, 447 (1975).
[8] Y. Fukushima *et al.*, Suppl. J. Phys. Soc. Jpn. **44**, 225 (1978); M. Kamimura, Nucl. Phys. **A351**, 456 (1981).
[9] E. Uegaki *et al.*, Prog. Theor. Phys. **57**, 1262 (1977); E. Uegaki *et al.*, Prog. Theor. Phys. **59**, 1031 (1978); **62**, 1621 (1979).
[10] P. Descouvemont and D. Baye, Phys. Rev. C **36**, 54 (1987).
[11] A. Tohsaki *et al.*, Phys. Rev. Lett. **87**, 192501 (2001).
[12] Y. Funaki *et al.*, Phys. Rev. C **67**, 051306(R) (2003).
[13] M. Chernykh *et al.*, Phys. Rev. Lett. **98**, 032501 (2007).
[14] H. Matsumura *et al.*, Nucl. Phys. **A739**, 238 (2004).
[15] T. Yamada *et al.*, Eur. Phys. J. A **26**, 185 (2005).
[16] Y. Suzuki, Prog. Theor. Phys. **55**, 1751 (1976); **56**, 111 (1976).
[17] M. Libert-Heinemann *et al.*, Nucl. Phys. **A339**, 429 (1980).
[18] H. Horiuchi *et al.*, Prog. Theor. Phys. **40**, 277 (1968).
[19] K. Fukatsu *et al.*, Prog. Theor. Phys. **87**, 151 (1992).
[20] Y. Funaki *et al.*, Nucl. Phys. News **17**, 11 (2007).
[21] S. Saito, Prog. Theor. Phys. **40**, 893 (1968); **41**, 705 (1969); Prog. Theor. Phys. Suppl. **62**, 11 (1977).
[22] V. I. Kukulin *et al.*, Nucl. Phys. **A417**, 128 (1984).
[23] A. Hasegawa *et al.*, Prog. Theor. Phys. **45**, 1786 (1971); F. Tanabe *et al.*, *ibid.* **53**, 677 (1975).
[24] E. W. Schmid *et al.*, Nucl. Phys. **26**, 463 (1961).
[25] C. Kurokawa and K. Kato, Phys. Rev. C **71**, 021301(R) (2005); Nucl. Phys. **A792**, 87 (2007).
[26] M. Kamimura, Phys. Rev. A **38**, 621 (1988); E. Hiyama *et al.*, Prog. Part. Nucl. Phys. **51**, 223 (2003).
[27] Y. Funaki *et al.* (to be published).
[28] The value of the density of this state may be too low, since the extension of states around and above threshold depend very sensitively on their precise position.
[29] Y. Suzuki and M. Takahashi, Phys. Rev. C **65**, 064318 (2002).
[30] T. Yamada *et al.*, arXiv:0804.1672.
[31] Y. Funaki *et al.*, arXive:nucl/th-0801.3131.
[32] A. M. Lane *et al.*, Rev. Mod. Phys. **30**, 257 (1958).
[33] The reason why we previously [20] assigned the 0_4^+ and 0_5^+ states as being candidates for α condensation will be explained elsewhere.
[34] F. Ajzenberg-Selove, Nucl. Phys. **A460**, 1 (1986).
[35] T. Wakasa *et al.*, Phys. Lett. B **653**, 173 (2007).