Proton Electromagnetic-Form-Factor Ratios at Low Q^2

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We study the ratio $R \equiv \mu G_E(Q^2)/G_M(Q^2)$ of the proton at very small values of Q^2. Radii commonly associated with these form factors are not moments of charge or magnetization densities. We show that the form factor F_2 is correctly interpretable as the two-dimensional Fourier transformation of a magnetization density. A relationship between the measurable ratio and moments of true charge and magnetization densities is derived and used to show that the magnetization density extends further than the charge density, in contrast with expectations based on the measured reduction of R as Q^2 increases.

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Electromagnetic form factors of the proton and neutron (nucleon) are probability amplitudes that the nucleon can absorb momentum and remain in the ground state, and therefore should determine the nucleon charge and magnetization densities. Much experimental technique, effort, and ingenuity have been used recently to measure these quantities [1,2].

The textbook interpretation of electromagnetic form factors, G_E, G_M, explained in [2], is that their Fourier transforms are measurements of charge and magnetization densities, and conventional wisdom relates the charge and magnetization mean square radii to the slopes of $G_{E,M}$ at $Q^2 = 0$. However, this interpretation is not correct because the wave functions of the initial and final nucleons have different momenta and therefore differ, invalidating a probability or density interpretation. A proper charge density is related to the matrix element of an absolute square of a field operator.

Here we show that the magnetization density is the two-dimensional Fourier transform of the F_2 form factor. This, and the result that the charge density is the two-dimensional Fourier transform of the F_1 form factor [3–6], is used to show that the magnetization density of the proton extends significantly further than its charge density. This is surprising because the observed rapid decrease of the ratio of electric to magnetic form factors with increasing values of Q^2 [1,2] might lead one to conclude that the charge radius is larger than the magnetization radius.

Form factors are matrix elements of the electromagnetic current operator $J^\mu(x^\nu)$,

$$\langle p', \lambda' | J^\mu(0) | p, \lambda\rangle = \bar{u}(p', \lambda')\Big[\gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\alpha}}{2M} q_\alpha F_2(Q^2)\Big] u(p, \lambda), \quad (1)$$

in units of the proton charge, where the momentum transfer $q_\alpha = p'_\alpha - p_\alpha$, and $Q^2 \equiv -q^2 > 0$. The nucleon polarization states are of definite light-cone helicities λ, λ' [7]. The normalization is such that $F_1(0) = 1$, and $F_2(0)$ is the proton anomalous magnetic moment $\mu_a = 1.79$. The Sachs form factors are $G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$, $G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$. The quantities G_E, G_M are experimentally accessible so we define the usual *effective* (*) square radii R_E^{*2}, R_M^{*2} such that for small values of Q^2 the accurately measurable [8] ratio is

$$\mu G_E(Q^2)/G_M(Q^2) \approx 1 + \frac{Q^2}{6}(R_M^{*2} - R_E^{*2}), \quad (2)$$

where $\mu = 2.79$.

In contrast, the form factor F_1 is a two-dimensional Fourier transform of the true charge density $\rho(b)$, where b is the distance from the transverse center of mass position irrespective of the longitudinal momentum [3–6]. At small values of $\mathbf{q}^2$, $F_1(Q^2 = \mathbf{q}^2) \approx 1 - \frac{Q^2}{4}\langle b^2\rangle_{Ch}$ where $\langle b^2\rangle_{Ch}$ is the second moment of $\rho(b)$.

We now interpret F_2 in terms of a magnetization density. The starting point is the relation that $\boldsymbol{\mu}\cdot\mathbf{B}$ is the matrix element of $\mathbf{J}\cdot\mathbf{A}$ in a definite state, $|X\rangle$. We note that the magnetic dipole density, or magnetization in the language of classical E and M texts, is a vector density and therefore contains a direction as well as a magnitude.

Take the rest-frame magnetic field to be a constant vector in the 1 (or b_x) [9] direction, and the corresponding vector potential as $\mathbf{A} = Bb_y\hat{\mathbf{z}}$. Then consider the system in a frame in which the plus component of the momentum approaches infinity. The anomalous magnetic moment may be extracted by taking

$$|X\rangle \equiv \frac{1}{\sqrt{2}}[|p^+, \mathbf{R} = \mathbf{0}, +\rangle + |p^+, \mathbf{R} = \mathbf{0}, -\rangle], \quad (3)$$

where $|p^+, \mathbf{R} = \mathbf{0}, +\rangle$ represents a transversely localized state of definite P^+ and light-cone helicity. The state $|X\rangle$ [4,10] may be interpreted as that of a transversely polarized target, up to relativistic corrections caused by the transverse localization of the wave packet [11]. The anomalous magnetic moment μ_a [12] is then computed to be

$$\mu_a = \frac{\langle X| \int dx^- d^2 b b_y \bar{q}(x^-, \mathbf{b})\gamma^+ q(x^-, \mathbf{b})|X\rangle}{\langle X|X\rangle}. \quad (4)$$

Use translational invariance to obtain

$$\mu_a = \langle X| \int d^2 b b_y q_+^\dagger(0, \mathbf{b}) q_+(0, \mathbf{b})|X\rangle, \quad (5)$$

where $q(x^-, \mathbf{b})$ is a quark-field operator, and $q_+ = \gamma^0\gamma^+ q$. This quantity is also an electric dipole moment. This is because we use a frame in which either the nucleon or the observer is moving very fast. Objects with magnetic moments in the rest frame seem to have an electric dipole moment when viewed from a moving frame [13].

This matrix element of a quark density operator is closely related to Burkardt's [4,10] impact parameter generalized parton distributions (GPD):

$$q_X(x, \mathbf{b}) \equiv \langle X| \int \frac{dx^-}{4\pi} q_+^\dagger(0, \mathbf{b}) q_+(x^-, \mathbf{b}) e^{ixp^+x^-}|X\rangle$$
$$= \frac{1}{2p^+}\left(\mathcal{H}_q(x, \xi = 0, b) - \frac{1}{2M}\frac{\partial}{\partial b_y}\mathcal{E}_q(x, \xi = 0, b)\right), \quad (6)$$

where $\mathcal{H}_q$ and $\mathcal{E}_q$ are two-dimensional Fourier transforms of the GPDs H_q, E_q [14]. Integration of Eq. (6) over x sets x^- to zero, so that Eq. (5) can be reexpressed (after integration by parts) as

$$\mu_a = \frac{1}{2p^+} \int d^2 b \int dx \mathcal{E}_q(x, \xi = 0, b). \quad (7)$$

But the integral of $\mathcal{E}_q$ over x is just the two-dimensional Fourier transform of $2p^+ F_2$, so that

$$\mu_a = \int d^2 b \rho_M(b), \quad (8)$$

where

$$\rho_M(b) = \int \frac{d^2 q}{(2\pi)^2} F_2(t = -\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}. \quad (9)$$

The subscript M denotes that this density generates the anomalous magnetic moment. The quantity $\rho_M(b)$ is properly a true magnetization density. The result Eq. (8) is a light-cone version of the usual calculation of magnetization density M, the difference between B and H, obtained by summing the magnetic dipoles in a given small volume.

It is also possible to consider the quantity $-\int dx b_y \frac{\partial}{\partial b_y} \mathcal{E}_q(x, 0, b)$ as the magnetization density. However, this definition would depend on the choice of the x axis as the direction of the magnetic field. A true intrinsic quantity should not depend on such a choice, so we use the form of Eqs. (7)–(9). Note that performing the integral Eq. (9) yields the result that the proton $\rho_M(b)$, constructed from the basic definition of the magnetic moment, is a positive definite quantity. This is in contrast to the transverse density ρ_T of [15].

For small values of $\mathbf{q}^2$: $F_2(Q^2 \approx \mathbf{q}^2) \approx \mu_a(1 - \frac{Q^2}{4} \times \langle b^2\rangle_M)$, where $\langle b^2\rangle_M$ is the second moment of $\rho_M(b)$. Use the definitions of the Sachs form factors, Eq. (2), and the low Q^2 expansions to relate the true moments to the effective square radii so that

$$\langle b^2\rangle_M - \langle b^2\rangle_{Ch} = \frac{\mu}{\mu_a}\frac{2}{3}(R_M^{*2} - R_E^{*2}) + \frac{\mu}{M^2}. \quad (10)$$

The low Q^2 measurement of the form factor ratio determines also the difference of true moments $\langle b^2\rangle_M - \langle b^2\rangle_{Ch}$, which is approximately the difference of the effective square radii plus a specific relativistic correction $\frac{\mu}{M^2} \approx 0.1235$ fm^2. This is the consequence of the Foldy term [16], arising from the interaction of the anomalous magnetic moment of the nucleon with the external magnetic field of the electron.

Using the high precision results from polarization transfer experiments (generally accepted to be less sensitive to two-photon exchange effects [17]) we compare the world's data set for $\mu G_E/G_M$ with Eq. (2) in Fig. 1. We fix the value of R_M^* from a new state-of-the-art determination [18], $R_M^* = 0.778(29)$ fm and plot the data as a function of the parameter $\frac{Q^2}{6} R_M^{*2}$. We find

$$\langle b^2\rangle_M - \langle b^2\rangle_{Ch} = 0.109\,60 \pm 0.006\,78 \text{ fm}^2, \quad (11)$$

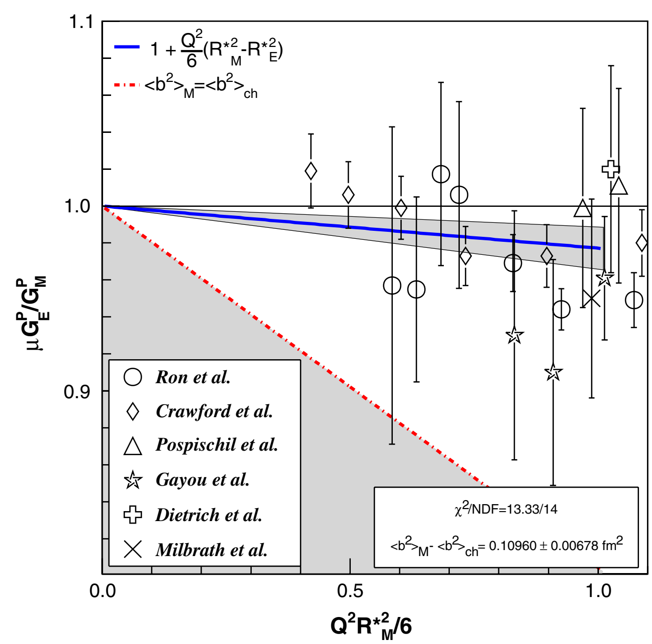

FIG. 1 (color online). Results of a linear fit to world data set of high precision polarization transfer experiments [33–38], shown by the solid (blue) line and error band. The shaded area indicates $\langle b^2\rangle_{Ch} > \langle b^2\rangle_M$. The dashed (red) line shows the critical slope $S_c = -\frac{3}{2}\frac{\kappa}{M^2}$ [Eq. (10)] giving $\langle b^2\rangle_M = \langle b^2\rangle_{Ch}$.

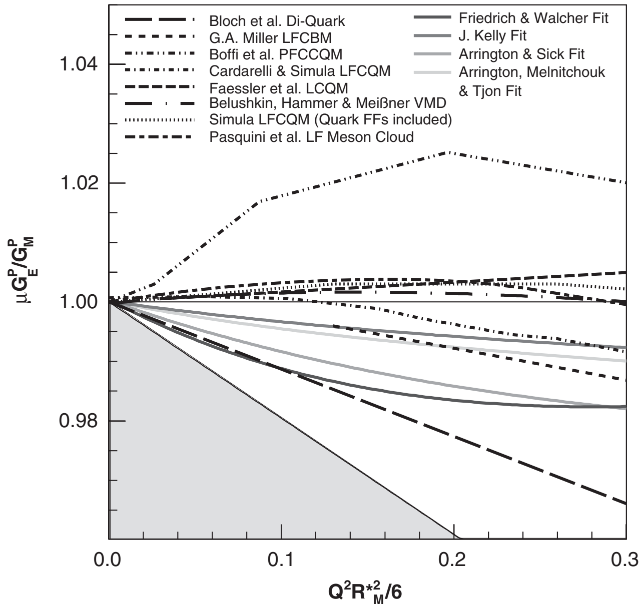

FIG. 2. The proton electromagnetic form factor ratio from several recent calculations and fits. The shaded area indicates $\langle b^2\rangle_{Ch} > \langle b^2\rangle_M$. The fits are shown using solid black curves with different line types, and the calculations as solid lines using different shades of gray.

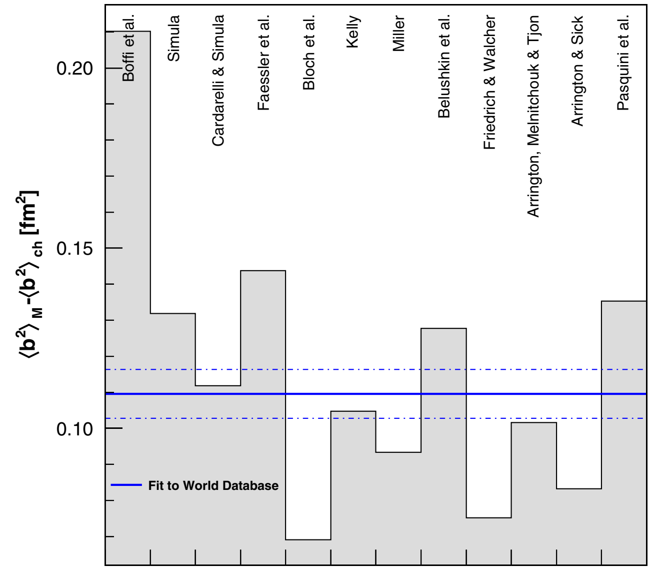

FIG. 3 (color online). $\langle b^2\rangle_M - \langle b^2\rangle_{Ch}$ from recent calculations and fits. All fits and calculations yield a positive value. The solid (blue) line shows the result Eq. (11).

which is about 12% smaller than the contribution of the magnetic moment Foldy term. Thus the difference $\frac{2}{3}\frac{\mu}{\kappa}\times(R_M^{*2} - R_E^{*2}) = -0.0139 \pm 0.00678$ fm^2 presently has the opposite sign of the result for the difference of the true moments of the distribution, indicating the need to base interpretations on the true moments. Note also that $(R_M^{*2} - R_E^{*2})$ is determined to an accuracy of only about 50%. Improving the accuracy can only be achieved by using very small values of Q^2, for which no high precision polarization transfer results exist. However, cross section measurements that have been corrected for the effects of the exchange of two photons are consistent with the ratio $R = 1$ [19], corresponding to $R_M^* = R_E^*$ in rough agreement with our results.

Figure 2 shows several model calculations and fits for the form factor ratio [17,19–29], which vary greatly. Improved experiments [8] would be able to distinguish these diverse approaches, and more fundamentally, better determine the value of $(R_M^{*2} - R_E^{*2})$. We also use a linear fit, at small values of Q^2, to the results of various calculations and some global fits. These are shown in Fig. 3. While there is significant variation, all agree with our result Eq. (11).

Our result that the magnetization density extends further than the charge density is consistent with the failure of the spin of the quarks to account for the angular momentum of the proton [30], and the likely importance of quark orbital angular momentum (OAM). This is because quarks carrying OAM, and therefore much magnetization-generating current, are located away from the center. For example, consider the pion cloud, which dominates the proton's exterior, as a source of OAM. The pion cloud is more influential for magnetic properties than for electric ones (e.g., Refs. [31,32]), and causes a proton magnetization radius that is larger than the charge radius.

To reiterate, our model-independent result is that the magnetization density of the proton extends further than its charge density. A natural interpretation involves the orbital angular momentum carried by quarks. Future experimental measurements of the ratio of the proton's electromagnetic form factors would render the present results more precise.

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