Fast-Projectile Stopping Power of Quantal Multicomponent Strongly Coupled Plasmas

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(Received 18 December 2007; published 12 August 2008)

The Bethe-Larkin formula for the fast-projectile stopping power is extended to multicomponent plasmas. The results are to contribute to the correct interpretation of the experimental data, which could permit us to test existing and future models of thermodynamic, static, and dynamic characteristics of strongly coupled Coulomb systems.

DOI: 10.1103/PhysRevLett.101.075002 PACS numbers: 52.40.Mj, 52.27.Gr

Stopping power is a characteristic of primary interest for different areas of physics such as nuclear physics, condensed matter physics, and plasma physics, as it arises when studying the interaction of charged particles with matter. In 1930, Bethe derived his seminal formula for the fast-projectile energy losses assuming that the atoms of the medium behave as quantum-mechanical oscillators [1]. Later, Larkin [2] showed that when fast ions permeate an electron gas, an analogous formula is applicable but with the mean excitation frequency replaced by the plasma frequency ω_p:

$$-\frac{dE}{dx} \simeq_{v \gg v_F} \frac{4\pi Z_p^2 e^4 \rho N}{mAv^2} \ln\Lambda, \tag{1}$$

where $\ln\Lambda = \ln 2mv^2/\hbar\omega_p$ is the quantal Coulomb logarithm, $Z_p e$ and v stand for the charge and velocity of the projectile, respectively, ρ is the target density, A is the mass of the target atoms, N is the Avogadro number, v_F is the electron Fermi velocity, and $\omega_p = (4\pi ne^2/m)^{1/2}$, m and n being the electron mass and density, respectively. This formula is usually employed to determine experimentally n in a charged particle system. Particularly, its applicability seems to be more promising in the field of plasma physics [3–5] for two reasons: First, in an ionized medium the energy loss is mainly caused by the free electrons, leading to an enhancement of the stopping power compared to the cold target [3–5]; second, this technique appears as the only suitable candidate for the diagnosis of hot and dense ($n \gtrsim 10^{19}$ cm^{-3}) plasmas, because most of the other methods fail under these conditions [5].

Usually, it is believed that the electronic subsystem of a plasma provides the main contribution to the stopping power process, especially for fast projectiles. Our first aim in this Letter is to show that, in a multicomponent completely ionized hydrogen plasma with a weakly damped Langmuir mode of dispersion $\omega_L(k)$, the plasma frequency in the Coulomb logarithm of (1) should be substituted by the long-wavelength limiting value of $\omega_L(k)$, $\omega_L(0) = \omega_p\sqrt{1+H}$, with $H = h_{ei}(0)/3 = [g_{ei}(0) - 1]/3$, $g_{ei}(r)$ being the electron-ion radial distribution function. The generalization to partially ionized plasmas or plasmas with complex ions and more species is straightforward. This correction may have further practical implications, in particular, after the experiments reported in Ref. [4], where it was possible to measure separately the enhancement of the stopping power of fast ions due to the increase in the Coulomb logarithm $\ln\Lambda$. Thus, this method will permit us to probe directly strong coupling effects which are relevant to plasmas within the high density energy regime. This includes plasmas arising in astrophysics and space science, planetary interiors, inertial confinement fusion, matter under extreme conditions, metals, and condensed matter plasmas.

Leaving the ionization losses aside, for calculating the stopping power for a fast-projectile passing through a Coulomb fluid we will adopt the polarizational picture, which becomes more accurate as the kinetic energy of the projectile increases. In 1954, Lindhard obtained an expression relating the polarizational stopping power with the medium (longitudinal) dielectric function [6]. This expression can be generalized further by applying the Fermi golden rule to obtain [7–9]

$$-\frac{dE}{dx} = \frac{2(Z_p e)^2}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_{\alpha_-(k)}^{\alpha_+(k)} \omega n_B(\omega) \times [-\mathrm{Im}\epsilon^{-1}(k,\omega)] d\omega, \tag{2}$$

$\alpha_\pm(k) = \pm kv + \hbar k^2/2M$, where M is the mass of the projectile (here we will work with heavy-ion projectiles $M \gg m$), and $n_B = [1 - \exp(-\beta\hbar\omega)]^{-1}$, β^{-1} being the temperature in energy units. In addition, unmagnetized Coulomb fluids are considered, and, hence, the dielectric function effectively depends only on the wave vector modulus. Expression (2) is valid only if the interaction between the projectile and the plasma is so weak that it can be treated as a linear effect and no relativistic effects need to be taken into account, i.e., when the energy lost by a projectile is much less than its kinetic energy, which, in

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turn, is assumed to be much smaller than its rest energy [10].

The literature on the polarizational stopping power is very extensive. The problem has been analyzed within the random-phase approximation (RPA) [7] and beyond, introducing an analytic formula for the local field correction (LFC) factor [11]. In addition, there are also nonlinear polarization effects [12], which are beyond the scope of this work. Whereas we assume that the coupling between the projectile and the target plasma can be treated perturbatively, we do not impose any restriction on the value of the coupling parameter $\Gamma = \beta e^2/a$ $[a = (4\pi n/3)^{-1/3}$ being the Wigner-Seitz radius], with the proviso that the latter remains in the liquid phase [13]. As said before, here we will focus on a completely ionized strongly coupled hydrogen plasma. The modeling of the dielectric properties of this kind of plasmas constitutes a difficult problem, because its characteristic lengths, i.e., Wigner-Seitz radius and Debye radius $\lambda_D = (4\pi n e^2 \beta)^{-1/2}$, are of the same order of magnitude (in a strongly coupled plasma $\Gamma = a^2/3\lambda_D^2 \gtrsim 1$, which makes mean field theories, such as the RPA, and perturbative treatments no longer valid), and, at the same time, its electronic subsystem is degenerate.

The framework.—Our dielectric formalism is based on the method of moments [14,15], which allows us to determine the dielectric function $\epsilon(k, \omega)$ from the first known frequency moments or sum rules. The sum rules that we employ are actually the power frequency moments of the loss function (LF) $\mathcal{L}(k, \omega) = -\omega^{-1}\mathrm{Im}\epsilon^{-1}(k, \omega)$ defined as $C_\nu(k) = \pi^{-1}\int_{-\infty}^{\infty} \omega^\nu \mathcal{L}(k, \omega)d\omega$, $\nu = 0, 1, \ldots$. Because of the parity of the LF, all odd-order frequency moments vanish. The even-order frequency moments are determined by the static characteristics of the system. After a straightforward calculation, one obtains [14–16]: $C_0(k) = [1 - \epsilon^{-1}(k, 0)]$, $C_2(k) = \omega_p^2$, and $C_4(k) = \omega_p^4[1 + K(k) + U(k) + H]$, with $K(k) = [\langle v_e^2\rangle k^2 + \hbar^2 k^4/(2m)^2]/\omega_p^2$, $\langle v_e^2\rangle$ being the average squared characteristic velocity of the plasma electrons. The last two terms in C_4 can be expressed in terms of the partial structure factors $S_{ab}(k)$, $a, b = e, i$: $U(k) = (2\pi^2 n)^{-1} \times \int_0^\infty p^2[S_{ee}(p) - 1]f(p, k)dp$, $H = (6\pi^2 n)^{-1} \times \int_0^\infty p^2 S_{ei}(p)dp$, where we have introduced $f(p,k) = 5/12 - p^2/(4k^2) + (k^2 - p^2)^2 \ln|(p + k)/(p - k)|/(8pk^3)$.

The Nevanlinna formula of the theory of moments expresses the dielectric function which satisfies the known sum rules $\{C_{2\nu}\}_{\nu=0}^2$ [14,17,18]:

$$\epsilon^{-1}(k, z) = 1 + \frac{\omega_p^2(z + q)}{z(z^2 - \omega_2^2) + q(z^2 - \omega_1^2)}, \qquad (3)$$

where $\omega_1^2 = \omega_1^2(k) = C_2/C_0$, $\omega_2^2 = \omega_2^2(k) = C_4/C_2$, in terms of a function $q = q(k, z)$, which is analytic in the upper complex half-plane $\mathrm{Im}z > 0$ and has there a positive imaginary part. It must also satisfy the limiting condition: $[q(k, z)/z] \to 0$ as $z \to \infty$ for $\mathrm{Im}z > 0$. In an electron liquid, this Nevanlinna parameter function plays the role of the dynamic LFC $G(k, \omega)$. In particular, the Ichimaru viscoelastic model expression for $G(k, \omega)$ is equivalent to the Nevanlinna function approximated as i/τ_m, τ_m being the effective relaxation time of the Ichimaru model [19]. In a multicomponent system, the Nevanlinna parameter function stands for the species' dynamic LFCs. In general, we do not have enough phenomenological conditions to determine that function $q(k, \omega)$ which would lead to the exact expression for the LF. One might benefit from the Perel'-Eliashberg (PE) [20] high-frequency asymptotic form [14] $\mathrm{Im}\epsilon[k, \omega \gg (\beta\hbar)^{-1}] \simeq (4/3)^{1/4} r_s^{3/4}/3(\omega_p/\omega)^{9/2}$, where $r_s = ame^2/\hbar^2$ is the Brueckner parameter.

The corrected Bethe-Larkin formula.—Let us choose a model function q satisfying the conditions mentioned after the Nevanlinna formula (3) that would permit us to treat the stopping power calculation analytically. If we put simply $q(k, \omega) = i0^+$, then we get the following particular solution of the moment problem:

$$\frac{\mathcal{L}(k, \omega)}{\pi C_0(k)} = \frac{\omega_2^2 - \omega_1^2}{\omega_2^2}\delta(\omega) + \frac{\omega_1^2}{2\omega_2^2}[\delta(\omega - \omega_2) + \delta(\omega + \omega_2)]. \qquad (4)$$

Physically, Eq. (4) describes an undamped collective excitation mode (Feynman approximation) at ω_2 with an additional central peak accounting for hydrodynamic diffusional processes [21]. The applicability of this expression is justified provided that the damping of the collective excitation is small enough, making this mode act as the main energy transfer channel. Thus we can disregard the details of the rest of the excitation spectrum. If we introduce expression (4) into the Lindhard formula (2), it immediately reduces to

$$-\frac{dE}{dx} \simeq_{v \gg v_F} \frac{(Z_p e \omega_p)^2}{v^2} \ln\frac{k_2}{k_1}, \qquad (5)$$

where the "cutoff" wave numbers k_1 and k_2 are such that the inequality $0 < \omega_2(k) < kv$ is satisfied with $v/v_F \to \infty$ and $\omega_2(k)$ understood as the plasma Langmuir mode dispersion law $\omega_L(k)$. For a weakly coupled plasma, the RPA dispersion law is valid which neglects the correlational contributions to $\omega_L(k)$: $\omega_L(k) = [\omega_p^2 + \langle v_e^2\rangle k^2 + \hbar^2 k^4/(2m)^2]^{1/2}$. Then, if v is asymptotically large, we have $k_1 = \omega_p/v$, $k_2 = 2mv/\hbar$, and we recover the Bethe-Larkin (BL) result [1,2]. Notice that, in the above-mentioned inequality for ω_2, we have presumed that $kv \gg \hbar k^2/2M$, which is equivalent to disregarding, at most, terms of the order of m/M. Similar terms were omitted in the above expressions for the moments C_2 and C_4, as well.

To take into account all Coulomb and exchange interactions in the system analytically, we might use for the electron-electron contribution $U(k)$ its long- and short-range asymptotic forms $U(k \to 0) \simeq -v_{ee}^2 k^2/\omega_p^2$ and

$U(k \to \infty) \simeq -h_{ee}(0)/3$, where $v_{ee}^2 = -4E_{ee}/(15\text{ nm})$ is defined by the plasma electron-electron interaction energy density E_{ee} of the plasma [9], $h_{ee}(0)$ being equal to the previous expression for $U(k)$ but with the function $f(p, k)$ replaced by unity. If we interpolate the plasma mode dispersion law as $\omega_L(k) = [\omega_p^2(1 + H) + wk^2 + \hbar^2 k^4/(2m)^2]^{1/2}$, with $w = 2\langle v_e^2\rangle - v_{ee}^2$, then the cutoff wave number k_1 is modified as $k_1' = \omega_p'/v$, with $\omega_p' = \omega_p\sqrt{1 + H}$, for $v/v_F \to \infty$, so that the fast-projectile stopping power becomes

$$-\frac{dE}{dx} \simeq_{v \gg v_F} \left(\frac{Z_p e \omega_p}{v}\right)^2 \ln\frac{2mv^2}{\hbar\omega_p\sqrt{1 + H}}. \tag{6}$$

Here the correction H stems from the electron-ion correlation contribution to the moment $C_4(k)$ and is also the one responsible for the upshift in the value of the Langmuir frequency predicted in the long-wavelength limit for an electron-ion plasma with an undamped collective mode. Although the accurate calculation of H under realistic conditions is a difficult task [22,23], it is possible to find a simplified analytic expression based on the temperature Green's function technique by a regularized summation over the Matsubara frequencies [14], yielding $H = (4/3)r_s\sqrt{\Gamma}/(2\sqrt{r_s} + \Gamma\sqrt{6})$ (see also Ref. [24] for an alternative approach based on a nontrivial renormalization via pair correlations in liquid metals). Whereas in a weakly coupled plasma ($\Gamma \ll 1$) this correction is negligible, in a strongly coupled Coulomb system it could be possible to retrieve directly H [or $g_{ei}(0)$] by fitting Eq. (6) to some experimental data. For instance, if we take $g_{ei}(0) = 10$ [23] and $\ln\Lambda = 14$ [4], then the stopping power obtained by the BL formula gets modified by $\sim$5%, which indicates to what extent the experimental accuracy needs to be improved.

The damped collective mode.—The collective mode is expected to be damped [22]; this implies that one cannot employ the solution of the moment problem (4) any longer. Here we will determine, on the basis of the Chebyshev-Markov and other model-free inequalities, the bounds for the asymptotic form of the fast-projectile stopping power. Let us consider the contribution

$$\mathcal{S}_1 := \int_0^{k_1'' \le k_1'} \frac{dk}{k} \int_{\alpha_-}^{\alpha+} \omega^2 n_B(\omega) \mathcal{L}(k, \omega) d\omega$$
$$\le \int_0^{k_1''} \frac{4\pi^2 e^2}{\hbar k^3} \alpha_+(k) dk \int_0^{\alpha_+} S(k, \omega) d\omega \tag{7}$$

on account of the fluctuation-dissipation theorem (FDT) [19]. Then, by applying the upper bound obtained in Ref. [25] for the charge-charge static structure factor of a quantal multicomponent plasma under the assumption of perfect screening $\lim_{k\to 0} S(k)/k^2 \le \hbar\omega_p \coth(\hbar\omega_p\beta/2)/(8\pi n e^2)$, we can approximate the previous integral as $\mathcal{S}_1 \lesssim \pi\omega_p \coth(\hbar\omega_p\beta/2) k_1'' v$ for $k_1'' \le k_1' \sim v_F/v$.

This contribution should be compared with those stemming from

$$\mathcal{S}_2 := \int_{k_1'' \le k_1'}^{k_2'' \ge k_2} \frac{dk}{k} \int_{\alpha_-}^{\alpha_+} \omega^2 n_B(\omega) \mathcal{L}(k, \omega) d\omega$$
$$\ge \int_{k_1'}^{k_2} \frac{dk}{k} \int_0^{kv} \omega^2 \mathcal{L}(k, \omega) d\omega. \tag{8}$$

Clearly, we can find an upper bound for $\mathcal{S}_2$ analogous to expression (6). To determine a lower bound, we might apply the Chebyshev-Markov inequalities (CMI) [18]. In particular, if we take the measure $d\sigma = \omega^2 \mathcal{L} d\omega$, then

$$\mathcal{S}_2 \ge \frac{\pi\omega_p^2}{2} \int_{k_1'}^{k_2} \frac{dk}{k} \left(\frac{(kv)^2 - \omega_2^2}{(kv)^2 + \omega_2^2}\right). \tag{9}$$

Since we have assumed that, for all $k \in (k_1', k_2)$, $kv > \omega_2(k)$, then, for some $\xi > 1$ such that $\xi k_1' \sim v_F/v$ as $v/v_F \to \infty$, we have

$$\mathcal{S}_2 \ge \frac{\pi\omega_p^2}{4} \int_{k_1'}^{\xi k_1'} \frac{dk}{k} \left(1 - \frac{\omega_2^2}{(kv)^2}\right)$$
$$= \frac{\pi\omega_p^2}{4}\left(\ln\xi - \frac{\xi^2 - 1}{2\xi^2}\right) + \mathcal{O}\left(\frac{v_F^2}{v^2}\right). \tag{10}$$

Hence, if we want to ensure that the first leading term of the stopping power asymptote is contained in $\mathcal{S}_2$, it is sufficient to choose the lower cutoff as $k_1'' \sim v_F^2/v^2$, to obtain $\mathcal{S}_1 \le \pi\omega_p^2 \coth(\hbar\omega_p\beta/2) v_F/v + \mathcal{O}(v_F^2/v^2)$, which becomes negligible compared to S_2 as $v/v_F \to \infty$.

The last contribution to the stopping power (2) reads

$$\mathcal{S}_3 := \int_{k_2'' \ge k_2}^{\infty} \frac{dk}{k} \int_{\alpha_-}^{\alpha_+} \omega^2 n_B(\omega) \mathcal{L}(k, \omega) d\omega. \tag{11}$$

In particular, if $k_2'' \gg 2Mv/\hbar = k_2 M/m$, then

$$\mathcal{I} \le \mathcal{S}_3 \le n_B[\alpha_-(k_2'')]\mathcal{I}, \tag{12}$$

$\mathcal{I} = \int_{k_2''}^{\infty} dk/k \int_{\alpha_-}^{\alpha_+} \omega^2 \mathcal{L}(k, \omega) d\omega$. By applying again the CMI, but now with the measure $d\Sigma = \mathcal{L} d\omega$, it is possible to prove that the satisfaction of all three sum rules $\{C_{2\nu}\}_{\nu=0}^2$ alone does not guarantee the convergence of $\mathcal{S}_3$. To this aim, we may introduce an additional condition on the decay of the LF in the interval of interest (α_-, α_+). Precisely, from the inequalities

$$\alpha_-^2 \int_{\alpha_-}^{\alpha_+} d\Sigma \le \int_{\alpha_-}^{\alpha_+} \omega^2 d\Sigma \le \alpha_+^2 \int_{\alpha_-}^{\alpha_+} d\Sigma, \tag{13}$$

we see that $\mathcal{S}_3$ converges if and only if $\int_{\alpha_-}^{\alpha_+} d\Sigma \lesssim (k_F/k)^\gamma$, $\gamma > 4$, which can be achieved by imposing on the distribution $\Sigma(\omega)$ the following Hölder condition:

$$|\Sigma(\alpha_+) - \Sigma(\alpha_-)| \le \left(\frac{\omega_p}{\alpha_+}\right)^\mu \left|\frac{\alpha_+ - \alpha_-}{\omega_p}\right|^\nu, \tag{14}$$

with $0 < \nu \leq 1$, $\mu \geq 3$, for $k \geq k_2''$. Then $\mathcal{S}_3 \lesssim 2\omega_p^2 M k_2/(m k_2'')$. Therefore, it is feasible to choose an upper cutoff as $k_2'' \sim v^2/v_F^2$ to get that $\mathcal{S}_3 \leq 2\omega_p^2 M v_F/(mv) + \mathcal{O}(v_F^2/v^2)$. A Hölder-type condition such as (14) can be satisfied in a number of physical models, namely, in an electron-ion hydrogen plasma, where the above-mentioned PE asymptote [20] is applicable if one assumes the spatial dispersion to be negligible for wavelengths much higher than the maximum impact parameter, which is valid for the range of frequencies and wave numbers considered for $\mathcal{S}_3$. In the case of a uniform electron gas, the asymptotic expression derived in Ref. [26] satisfies a similar condition as well, although one needs to take into account the region of nonanalyticity of the perturbative expansion [27].

With the aforementioned conditions, it follows that the stopping power $-dE/dx$ given in (2) satisfies asymptotically, as $v/v_F \to \infty$,

$$\left(\frac{v_F}{v}\right)^2 \lesssim \left(\frac{v_F}{Z_p e \omega_p}\right)^2 \left(-\frac{dE}{dx}\right) \lesssim \left(\frac{v_F}{v}\right)^2 \ln\frac{v}{v_F}.$$

This theorem provides the bounds for the fast-projectile asymptotic form leading term. These are based on inequalities which do not depend on the particular details of the fluctuation spectrum at low and intermediate frequencies.

Conclusions.—In this Letter, we have studied the modification of the BL expression for the plasma stopping power due to the presence of an ion component, strong coupling, and the decay of the Langmuir mode. We have shown that, for a perfectly defined plasma collective mode with negligible damping, the above-mentioned expression is affected by the electron-ion correlation. In addition, we have derived bounds for the fast-projectile asymptotic, on the basis of well-established results of the linear response theory of Coulomb systems, namely, the zero-frequency sum rule, the f-sum rule, the fourth moment sum rule, and the FDT, together with the compressibility sum rule. This general result constitutes a sum rule for the calculation or numerical estimate of the fast-projectile stopping power for any model dielectric function satisfying the above-mentioned conditions, not only in plasma physics but also in other multicomponent uniform charged particle systems of condensed matter physics.

The authors acknowledge the financial support of the European Social Fund, the Spanish Ministerio de Educación y Ciencia (Project No. ENE2007-67406-C02-02/FTN), and the INTAS (Project No. 06-1000012-8707).

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