Fluctuations and Correlations of Pure Quantum Turbulence in Superfluid ^{3}He-B

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We describe the first measurements of line-density fluctuations and spatial correlations of quantum turbulence in superfluid ^{3}He-B. All of the measurements are performed in the low-temperature regime, where the normal-fluid density is negligible. The quantum turbulence is generated by a vibrating grid. The vortex-line density is found to have large length-scale correlations, indicating large-scale collective motion of vortices. Furthermore, we find that the power spectrum of fluctuations versus frequency obeys a $-5/3$ power law which verifies recent speculations that this behavior is a generic feature of fully developed quantum turbulence, reminiscent of the Kolmogorov spectrum for velocity fluctuations in classical turbulence.

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Superfluid turbulence has received a great deal of experimental and theoretical interest in recent years [1]. While much is known about superfluid turbulence in ^{4}He at relatively high temperatures [2], it is particularly interesting to investigate what happens in the $T = 0$ limit where there is no normal fluid. A pure superfluid approximates well to an ideal incompressible, inviscid, irrotational fluid, and quantum turbulence is a tangle of quantized vortex lines which defines superfluid flow. Given access to such a simplified system, one can start to address fundamental questions concerning the nature of superfluid turbulence, the decay processes, links to classical turbulence, etc., with a hope that quantum turbulence may teach us more about turbulence in general. In recent years, we have developed the experimental techniques to address these questions in superfluid ^{3}He-B [3–8]. To date, most work on superfluid turbulence has concentrated on time-averaged quantities such as the vortex-line density. However, as with classical turbulence, far more detailed information may be revealed by investigating fluctuations. Here we describe the first experimental study of turbulent fluctuations in a pure superfluid, in the low-temperature limit.

Most of our current understanding of superfluid turbulence has derived from experiments in ^{4}He at relatively high temperatures. In this case, mutual friction strongly couples the flow fields of the normal and superfluid components [2]. The ensuing combined normal-superfluid turbulence has close links with classical turbulence. In classical turbulence, energy flows from large length scales, which carry most of the turbulent energy, to smaller length scales (or large wave numbers k), where viscous dissipation occurs, a process known as the Richardson cascade. The resulting energy spectrum is described by the Kolmogorov law $E(k) \simeq \epsilon^{2/3} k^{-5/3}$, where $\epsilon = \nu\omega^2$ is the dissipation rate per unit mass, ν the kinematic viscosity, and ω the vorticity. This spectrum has been confirmed for superfluid turbulence in ^{4}He at relatively high temperatures by measurements of local pressure fluctuations in the turbulent flow between counterrotating disks [9]. The measured spectra were virtually identical for the normal and superfluid phases. Further evidence for the Richardson cascade in superfluid ^{4}He turbulence is obtained from its decay. This has been measured for a towed grid using second-sound techniques [10,11] and, more recently, it was measured over a wide temperature range in a rotating cryostat by ion trapping techniques [12]. Assuming that the inferred vortex-line density L relates to the vorticity via $\omega = \kappa L$, where κ is the circulation quantum, the decay can be quantitatively explained by a Richardson cascade taking an effective value for the kinematic viscosity [10–12].

In contrast to ^{4}He, the viscosity of normal ^{3}He is so high that turbulence develops in the superfluid only at temperatures low enough that the falling mutual friction begins to decouple the normal-fluid component [13]. At these intermediate temperatures, since the normal fluid is effectively clamped to the cell walls and mutual friction operates over all length scales, we might expect the energy spectrum of superfluid turbulence to differ substantially from the Kolmogorov law [14]. However, at the very low temperatures of this work, the excitation gas is so tenuous that quasiparticle-quasiparticle scattering is completely absent. Here the concept of a normal-fluid component breaks down, and the turbulence becomes a function of the superfluid alone.

Paradoxically, the few remaining ballistic excitations provide an excellent probe of the superfluid flow field. The quasiparticle dispersion curve $E(\mathbf{p})$ is tied to the reference frame of the superfluid and thus becomes tilted by the Galilean transformation $E(\mathbf{p}) \rightarrow E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}_S$ in a superfluid moving with velocity $\mathbf{v}_S$. Consequently, the flow acts as a potential. Thus quasiparticles incident on a region of superflow may experience a potential energy barrier and will be Andreev-reflected if they have insufficient energy to proceed [15]. The Andreev process converts a quasiparticle into a quasihole and vice versa, reversing the group velocity but with negligible momentum transfer. The cross section for Andreev reflection of quasiparticle excitations from the flow field around vortex lines is very large [6,16], so the complex flow field associated with superfluid turbulence will Andreev-reflect a significant fraction of the incident excitations.

We can readily detect quasiparticle excitations in ^{3}He-B at low temperatures by vibrating-wire techniques [15,17,18]. The thermal damping of a vibrating wire arises from the normal scattering of excitations at the wire surface [18–21]. A wire immersed in turbulence thus experiences a reduction in damping proportional to the fraction of incident excitations which are Andreev-reflected by the turbulent flow. This effect has been exploited to observe turbulence generated by vibrating wires [3,5,6] and vibrating grids [7,8].

The experimental arrangement is shown in the inset in Fig. 2 and is identical to that used for the measurements reported previously [4,7,8]. Vortices generated by the grid are detected by the two facing vibrating-wire resonators: a ''near'' wire, 1 mm from the grid, and a ''far'' wire, 2 mm from the grid. There is also a remote ''thermometer'' wire, not shown in the figure. Briefly, the damping of the two detectors and the thermometer wire are continuously monitored, while the grid is driven to a certain velocity amplitude, generating vortex lines. At low grid velocities, ballistic vortex rings are emitted, and turbulence forms only above a certain critical grid velocity $\sim 3.5\ \mathrm{mm\,s^{-1}}$, from ring collisions and recombination [7]. The vortices Andreev-reflect some of the quasiparticles approaching the detector wires, giving rise to a reduced damping from which the vortex-line density L can be inferred [8]. In the following, we discuss fluctuations $\delta L(t) = L(t) - \langle L \rangle$. For the data presented here, the time-averaged line density $\langle L \rangle$ lies between 10^7 and $10^8\ \mathrm{m^{-2}}$. All of the data were taken at 12 bar and at temperatures below $0.2T_c$, where the normal-fluid fraction is negligible.

In Fig. 1, we show the measured fluctuations in the line density $\delta L(t)$ for the near and far wires for quantum turbulence generated by the grid driven at $5.6\ \mathrm{mm\,s^{-1}}$ (curves A and B in the figure) and the equivalent noise level (curve C) obtained in the absence of turbulence while the grid is stationary. The turbulent fluctuations are easily resolved, and it is clear from Fig. 1 that the dominant fluctuations have time scales of the order of a few seconds or more.

We can define the correlation between the signals from the two detector wires as a function of the delay time Δt as $R_{1,2} = \langle \delta L_1(t) \delta L_2(t - \Delta t) \rangle / \sqrt{\langle \delta L_1^2 \rangle} \sqrt{\langle \delta L_2^2 \rangle}$, where δL_1 and δL_2 are the fluctuations in the vortex-line densities inferred from the responses of the near and far detector wires, respectively. The correlation corresponding to the data of Fig. 1 is shown in Fig. 2 (note that Fig. 1 shows only a small fraction $\sim 3\%$ of the full data sample used to generate the curve in Fig. 2). There are two main points to mention. First, there is a clear correlation peak, indicating that some part of the fluctuations is observed by both wires. This is expected since the two wires do not measure a local line density but rather an angular averaged projection of the vortex lines onto the wire surfaces, so that a local fluctuation in the line density can, in principle, affect both detector wires but by differing amounts depending on the relative locations. Second, the correlation peak maximum lies at a nonzero value of Δt; the fluctuations on the far wire are delayed on average by ~ 3 seconds from the corresponding fluctuations on the near wire. Since the wires are ~ 1 mm apart, this suggests that (i) there is a mean flow of the turbulence away from the grid with a velocity of $\sim 0.3\ \mathrm{mm\,s^{-1}}$ and (ii) there must be some fluctuations that are long-lived on the scale of seconds

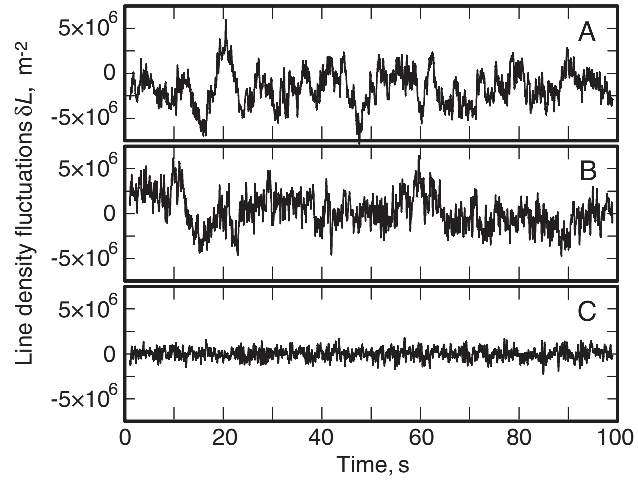

FIG. 1. The fluctuations in the line density for quantum turbulence produced by the grid when driven at a constant velocity amplitude of $5.6\ \mathrm{mm\,s^{-1}}$ producing a mean line density $\langle L \rangle \sim 5 \times 10^7\ \mathrm{m^{-2}}$. Plots A and B show the fluctuations measured with the near and far detector wires, respectively. Plot C shows the corresponding noise level in the absence of turbulence, when the grid is stationary.

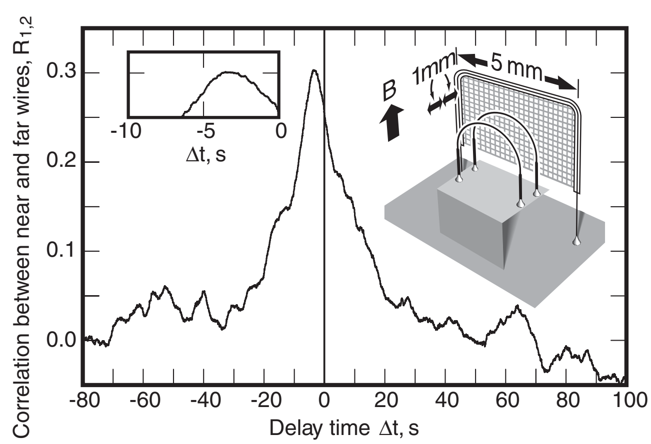

FIG. 2. The correlation between the fluctuations in the line density measured by the two wire resonators, 1 mm apart, while the grid is driven at a velocity amplitude of $5.6\ \mathrm{mm\,s^{-1}}$. The top right inset shows the experimental arrangement. The top left inset shows a time-expanded view of the correlations at short time intervals.

(this is verified by the power spectra discussed below). The correlation peak is very broad which gives a large uncertainty on our measurement of Δt, and we are unable resolve any dependence on the grid velocity. The broad peak is not surprising since the Richardson cascade model, describing the decay behavior [8], predicts a fluctuating large-scale flow velocity of the order of 1 $\mathrm{mm\,s^{-1}}$, somewhat larger than the mean flow. We further note that the mean flow implies some polarization of the vortex tangle, with the vortex-line curvature biased towards the plane of the grid face.

In Fig. 3, we show the power spectrum of the fluctuations as a function of frequency f for the near wire, for various grid velocities. The far wire displays similar behavior. The lowest curve corresponds to the electrical noise of the measurement devices, obtained when the grid is stationary. The noise level decreases above ~3 Hz due to the time constant of the measuring system. The wires detect ballistic vortex rings at the lowest grid velocity (2.8 $\mathrm{mm\,s^{-1}}$) and quantum turbulence at the higher grid velocities [7]. The power spectra for the two regimes are markedly different, and there is a sharp transition between them. For quantum turbulence, the power spectra show a $f^{-5/3}$ dependence above about 0.1 Hz (the error bar on the exponent is roughly 10%), flattening off at lower frequencies. For vortex rings, the behavior is closer to a f^{-1} power law.

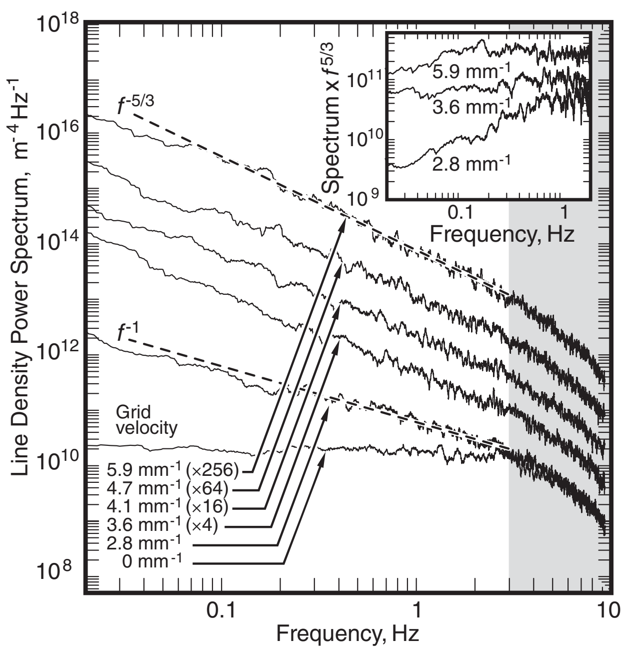

FIG. 3. Frequency power spectrum $|L(f)|^2$ of the line-density fluctuations measured by the near wire for various grid velocities as shown. The spectrum is defined such that its integral over the whole frequency range is equal to the mean square fluctuation. For clarity, the data for the higher grid velocities are displaced by factors of 4 as indicated. In the shaded region, the spectrum is not visible above the noise. The inset shows data scaled by $f^{5/3}$ (after subtracting the background noise level) to indicate more clearly the $-5/3$ power-law behavior for quantum turbulence.

It is tempting to use the mean flow v inferred from Fig. 1 to convert the frequencies in the turbulence spectra into wave numbers $2\pi f/v$ or wavelengths v/f to give the relevant length scales. If we do this using $v \sim 0.3\ \mathrm{mm\,s^{-1}}$, then the associated length scale varies from a few millimeters at the start of the $f^{-5/3}$ power-law behavior down to about 0.1 mm at the point where the signal merges with the background noise level. Although the correspondence between frequency and length scale is questionable when the mean flow is relatively small, this range of length scales coincides well with the expected inertial range of length scales where the Richardson cascade operates, i.e., from the largest scale given by size of the turbulent region to the mean intervortex spacing.

Recent measurements of fluctuations in the line density for superfluid turbulence in ^{4}He have been made with second-sound techniques at relatively high temperatures [22]. The measured power spectra are remarkably similar with the same $f^{-5/3}$ dependence, despite the comparison of two very different superfluids in opposite temperature regimes (the experiments in ^{4}He were performed with a significant normal-fluid component). This reveals a very important result; the behavior of quantum turbulence in the low-temperature limit (as inferred from large length or time scale fluctuations) is very similar to the combined superfluid–normal-fluid turbulence measured in ^{4}He at high temperatures. Since the existence of a Kolmogorov spectrum is directly confirmed for superfluid turbulence in ^{4}He at high temperatures [9], this constitutes compelling experimental evidence that the Kolmogorov spectrum continues to hold in the $T = 0$ limit.

Further experimental evidence for the Richardson cascade and the Kolmogorov spectrum in the zero-temperature limit is gained from computer simulations [23] and from measurements of the decay of turbulence [8,12]. As we have shown previously [8], the decay of the turbulence in our experiments is in good agreement with the classical model with an effective kinematic viscosity given by $\sim 0.2\kappa$. The effective kinematic viscosity is very similar to that obtained from measurements in superfluid ^{4}He at high temperatures [10–12]. However, the effective kinematic viscosity has recently been found to undergo a sharp transition to a much lower value in superfluid ^{4}He at low temperatures [12]. It has been suggested [24,25] that this behavior could arise from a "bottleneck" in the energy cascade process, linking the classical-like behavior at length scales larger than the intervortex spacing to a Kelvin-wave cascade which transports energy to much smaller length scales where it is ultimately dissipated. It is interesting to investigate why the effective kinematic viscosities at low temperatures appear to be so different in the two superfluids. We note that the ultimate energy dissipation process in superfluid ^{3}He must involve quasiparticle production from the vortex cores. Owing to the much larger core size and lower energy scales, dissipation should set in at much lower frequencies in superfluid ^{3}He compared to phonon emission by Kelvin waves in superfluid ^{4}He. With a little further development, it should be possible to extend measurements of quantum turbulent fluctuations in superfluid ^{3}He to higher frequencies where the dissipation mechanism might be probed directly.

The $f^{-5/3}$ power law in the measured line-density power spectrum is reminiscent of the Kolmogorov law. However, the interpretation is not straightforward, as was noted in the earlier ^{4}He work [22]. In order to link quantum and classical turbulence, it is generally assumed that the line density provides a measure of the mean vorticity via $\omega = \kappa L$. However, in classical turbulence, the mean vorticity has a frequency spectrum which is much flatter than the $-5/3$ power law revealed in Fig. 3. A possible explanation was recently proposed [26], where it is argued that the line density can be decomposed into two components. A small polarized component contains the majority of the turbulent energy and controls the large-scale flow which has the Kolmogorov spectrum. The larger unpolarized component, which dominates the line-density measurement, is swept passively along by this flow, which then leads to the observed $-5/3$ power law. Our measurements confirm that this dependence exists even in the absence of a normal-fluid component and thus supports the idea that the $-5/3$ power-law behavior of line-density fluctuations is a signature of the quantum nature of superfluid turbulence [26].

In conclusion, we describe the first measurements of fluctuations in quantum turbulence in the zero-temperature limit. The turbulence exhibits fluctuations and large-scale correlations consistent with the classical Richardson cascade in which energy flows through an inertial range extending from the largest length scales to the smallest scales where dissipation occurs. The power spectrum of the line-density fluctuations displays the same $-5/3$ power-law behavior as that observed in superfluid ^{4}He at relatively high temperatures, suggesting that this is a universal property of superfluid turbulence. Fluctuations are a key feature of all turbulent systems, and we now have the experimental techniques to study quantum turbulence fluctuations in detail. We should also point out that, since the vortices here are quantized, the line density of the vortex cores which we measure is a very well-defined quantity compared with many other probes of turbulent behavior. The relative simplicity of quantum turbulence combined with its strong similarities to classical turbulence makes it an intriguing system to provide a better understanding of turbulence in general.

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