ac Vortex-Dependent Torsional Oscillation Response and Onset Temperature T_0 in Solid ^{4}He

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Detailed studies of ac velocity V_{ac} and T dependence of torsional oscillator responses of solid ^{4}He are reported. A characteristic onset temperature $T_0 \sim 0.5$ K is found, below which a significant V_{ac}-dependent change occurs in the energy dissipation for the samples at ~32 bar and for one at 49 bar. A V_{ac} dependence of the so-called "nonclassical rotational inertia" fraction also appears below $\sim T_0$. The $\log(V_{ac})$ linear dependence, which suggests involvement of quantized vorticies, was examined in the nonclassical rotational inertia fraction. We find a common $1/T^2$ dependence for this linear slope change in all of the samples for $30 < V_{ac} < 300$ μm/s. We discuss that our observation is consistent with nonlinear rotational susceptibility of the vortex fluid, proposed by Anderson above T_c below T_0.

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Since the first report of "nonclassical rotational inertia" (NCRI) in solid ^{4}He samples by Kim and Chan [1], confirmation has come from several torsional oscillator (TO) experiments [1–5], including by the present authors. This finding has been discussed in connection to the NCRI of a supersolid as originally proposed by Leggett [6]. A review paper by Prokof'ev [7] is valuable for understanding recent work up to December 2006. An important conclusion is that the observed phenomena seem to be more complicated than the original proposal of a Bose-Einstein condensation (BEC) of vacancies or other imperfections. Much excitement has been generated by the recent observation of a remarkably large NCRI fraction (NCRIF) under appropriate experimental conditions. Rittner and Reppy [4] found the NCRIF increased in quench-cooled samples as the distance between closely spaced, concentric walls confining the helium was made smaller. These authors attribute the increase of NCRIF to increased disorder in the sample. NCRIF greater than 15% of the total mass could be achieved [4], indicating that simple mechanisms involving only a small fraction of the solid helium are not adequate as explanations of the observed new phase.

According to a recent proposal by Anderson [8], the results previously attributed to NCRI might be caused by "nonlinear rotational susceptibility" (NLRS) [9] on account of features shared with nonlinear magnetization seen in some underdoped (UD) cuprate high-T_c superconductor [10] below an onset temperature T_0 but above T_c, where the resistivity is nonzero. He discusses the $\log(V_{ac})$ linear dependence of NLRS as evidence for a vortex fluid (VF) [8]. A fundamental background for the VF state is as follows. The 100 mK or higher T for the reported occurrence of NCRI is way too high T for the appearance of BEC of any of the known excitations in solid He from the known concentrations, whereas a VF state can appear with the help of vortex excitations in lower-dimensional subsystems in the solid He, where quantized vortices have much lower energies and are possibly thermally excited as in 2D Kosterlitz-Thouless (KT) systems. The VF state is without 3D macroscopic coherence and does not support macroscopic superflow. More recently, Kojima's group reports [11] a significant change occurring below 40 mK in the TO response time when excitation V_{ac} is changed. They also report hysteresis below about this temperature, possibly an indication of a real T_c. Reppy [12] claims that his group observes similar hysteresis below a similar T, although their NCRIF is orders of magnitude larger. Clark, West, and Chan [13] find NCRIF appearing at much lower T in either ultrapure ^{4}He or in ^{4}He single crystals in comparison to that seen in samples prepared by the usual blocked capillary method using the usual commercial grade of ^{4}He, which typically contains about 0.3 ppm ^{3}He impurity. Their saturation NCRIF has been from 0.03% to 0.4% [13].

An interesting and significant observation is that the NCRIF for a ^{3}He concentration of 0.3 ppm may differ by more than 2 orders of magnitude among samples prepared under different conditions [4,11,13], while the characteristic temperatures for the phenomena change by no more than a factor of 2–3. For example, the temperature for the energy dissipation peak T_p is below or around 100 mK. The onset temperature T_0 below which the NCRI fraction begins to appear has been reported to be 250–300 mK [1–4], except for Refs. [5,13,14]. This implies that some low-dimension subsystem exists in solid ^{4}He and the characteristic temperatures are determined primarily by the subsystem local density while the number density of the subsystems determines the overall NCRIF. The latter may be increased by externally induced disorder [4]. All of these observations seem to imply that the conditions for the VF state are satisfied and T_0 would imply the appearance of the low-dimension condensate and quantized vortices, as also discussed for UD cuprates [10].

We investigated T_0 also on account of a claim that the observed phenomena might not be an intrinsic property of the solid [14] but instead could be caused by superfluid liquid at the grain boundaries. However, experimentally, T_0 has not been established, and it is not known what changes at T_0 because NCRI appears very gradually. This Letter describes experiments on rather stable ^{4}He samples for which NCRIF extrapolated to $T = 0$ K is quite small, that is, NCRIF(0) $< 0.08\%$. We report our observations and discuss the determination of T_0 and describe detailed ac velocity-dependent behavior below T_0 and above T_p.

The samples studied were at pressures between 32 and 35.5 bar and at 49 bar, and all showed similar behavior except for the absolute values of signal. All samples remained quite stable as long as we kept them colder than about 700 mK; with this stability, we hoped to study the most fundamental properties of solid ^{4}He. Most of the presented data are for two samples, which remained reproducible throughout 45 and 200 days of experiments, respectively. The measurements were performed on the ISSP fast rotating cryostat [15]. This provided much more reliable and reproducible data compared to our previous supersolid experiments [5], because this cryostat is far more rigid while also having much more mass, about 10 metric tons, with superior vibration isolation. In addition, the ability to rotate the samples is now available, and we plan presentation of results for dc rotation in future publications. The BeCu TO has a 15 mm long torsion rod with 2.2 mm outside diameter and a 0.8 mm coaxial hole serving as the filling line. The cylindrical sample cell made of brass is mounted on a BeCu base integral with the torsion rod, with a threaded fitting and sealed with Wood's alloy. The interior sample space is 4 mm high and has a 10 mm diameter. Below 4.2 K, the resonant frequency of the TO is approximately 1002 Hz with $Q \approx 1.7 \times 10^6$ as determined from the free decay time constant. The samples were prepared by the blocked capillary method from ^{4}He gas of commercial purity ($\approx$0.3 ppm ^{3}He) with cooling along the melting curve at the rate $\approx$2–5 mK/min. No special annealing was attempted, but the samples were cooled slowly, over a period of a few hours, from the melting curve to 1 K. The final pressure of solid was estimated from a sharp drop in TO amplitude at the melting temperature measured during slow ($\approx$0.55 mK/min) heating after completion of the measurements [5]. The change of period caused by the solidification of the samples is $\Delta p_{\mathrm{load}} \approx 2.4$ μs for all of the samples studied.

In order to discuss solid ^{4}He internal friction separately from empty BeCu TO properties, we have chosen the quantities associated with solid ^{4}He as below, to facilitate comparison with results from other types of experiments on solid ^{4}He [16]; namely, energy dissipation (internal friction) in the solid ^{4}He sample δ is evaluated from TO measurements taking similar considerations of the composite TO [17]. Using additivity of dissipated energy $\Delta\varepsilon$ and the stored energy ε for the composite TO per cycle of oscillation, the definition of internal friction $Q^{-1} = \Delta\varepsilon/2\pi\varepsilon$ gives

$$\Delta\varepsilon_{\mathrm{total}} = \Delta\varepsilon_{\mathrm{empty}} + \Delta\varepsilon_{\mathrm{solid}}, \qquad \varepsilon_{\mathrm{total}} = \varepsilon_{\mathrm{empty}} + \varepsilon_{\mathrm{solid}},$$
$$\frac{\Delta\varepsilon_{\mathrm{total}}}{\varepsilon_{\mathrm{total}}} = \frac{\Delta\varepsilon_{\mathrm{empty}}}{\varepsilon_{\mathrm{empty}} + \varepsilon_{\mathrm{solid}}} + \frac{\Delta\varepsilon_{\mathrm{solid}}}{\varepsilon_{\mathrm{empty}} + \varepsilon_{\mathrm{solid}}}. \tag{1}$$

In our case $\varepsilon_{\mathrm{empty}} \gg \varepsilon_{\mathrm{solid}}$, because the stored energy $\sim I$ and $I_{\mathrm{empty}} \gg I_{\mathrm{solid}}$. Therefore δ is given as

$$\delta = \frac{\varepsilon_{\mathrm{empty}}}{\varepsilon_{\mathrm{solid}}}(Q^{-1}_{\mathrm{total}} - Q^{-1}_{\mathrm{empty}}), \tag{2}$$

with

$$\frac{\varepsilon_{\mathrm{empty}}}{\varepsilon_{\mathrm{solid}}} \approx \frac{I_{\mathrm{empty}}}{I_{\mathrm{solid}}} \approx \frac{p_{\mathrm{empty}}}{2\Delta p_{\mathrm{load}}} \approx 210 \quad \text{(for our cell)}, \tag{3}$$

where I_{empty} and I_{solid} are the moment of inertia of the empty BeCu TO and the solid sample, respectively.

The upper graph in Fig. 1(a) shows δ in the 32 bar solid ^{4}He sample, while the lower graph in Fig. 1(b) gives the relative shift of the period $\Delta p/\Delta p_{\mathrm{load}}$ corresponding to the NCRIF or NLRS of the sample solid ^{4}He as a function of T for various ac cell rim velocities V_{ac}. The inset in Fig. 1(a) shows an example of the peak appearing in the data at T_p for samples for approximately the same pressure. The peak is asymmetric as compared with a Gaussian curve fitted to

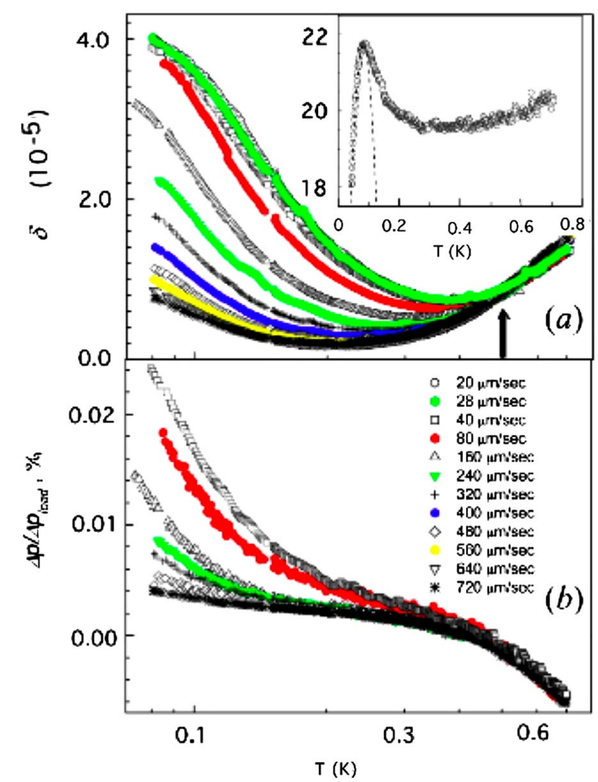

FIG. 1 (color online). T dependence of energy dissipation δ (a) and $\Delta p/\Delta p_{\mathrm{load}}$ (b) at various V_{ac} in μm/s for a 32 bar sample. The values of δ are presented without any artificial shift. Some data are omitted for clarity [all of the data on V_{ac} dependence are plotted in Fig. 2(a) and 2(b)]. An arrow indicates T_0, across which V_{ac} dependence changes. The inset in (a) indicates a typical energy dissipation peak with somewhat higher T_p. The low T part of the peak was fitted with a Gaussian: dashed line. The zero for $\Delta p/\Delta p_{\mathrm{load}}$ in (b) is taken provisionally where V_{ac} dependence goes away.

the data on the low T side. All of the data in the main graphs in Fig. 1 are for $T > T_p$ for a 32 bar sample.

It is seen that both the period and the dissipation δ are changing over the entire T range for the measurements. Above 0.5 K, δ increases, and the relative period decreases as T increases. In addition, a much stronger apparent dependence on V_{ac} begins for T below 0.5 K, especially for the energy dissipation. The change of the V_{ac} dependence allows the assignment of a unique characteristic temperature $T_0 = 0.5$ K as indicated by an arrow. At $T > T_0$ (normal region), the absolute value of δ can be compared with available data obtained by other techniques (sound and elastic deformation). The $\delta < 2 \times 10^{-5}$ that we find is very much smaller than other available data [18] and the resonant dislocation vibration mechanism analysis [19]. Based on the present size of δ, the most probable mechanism for dissipation is thermoelastic internal friction and not dislocation motion [18], as has been proposed for larger excitation experiments. The original data in Fig. 1(b) form a set of parallel curves above ~0.45 K, but for the graph they have been shifted to coincide in this T range.

A striking difference between the properties seen at $T < 0.5$ K in Fig. 1 in comparison with other superfluid systems is that the dissipation as well as NLRS are largest for the smallest excitation velocity. This behavior is opposite to what is seen for a KT transition [20], the superfluid transition in a 3D He film system [21], or bulk liquid ^{4}He in Vycor [22]. In all systems, the dissipation increases when the excitation exceeds some critical value including 0. So far, nobody has tried to explain this experimental observation for solid He. In order to clarify this point, we dared to analyze quantitatively in various ways and found interesting facts as follow. The δ and TO period shift $\Delta p/\Delta p_{load}$ as a function of V_{ac} are analyzed at different T's below 300 mK in Fig. 2. All of the data in Figs. 2(a) and 2(b) are taken from the same data set directly from Fig. 1, and Fig. 2(c) is added to include $\Delta p/\Delta p_{load}$ data for a 49 bar sample. This 49 bar sample was prepared and measured with shifted V_{ac} range in order to clarify the fact that the results are reproducible even with a new sample after a heat cycle to room temperature. If we plot data at higher T's than 300 mK, we obtain almost horizontal displays of data for each T, in the same frame as in Fig. 2(a) for δ, and the same is true for NLRS $= \Delta p/\Delta p_{load}$, but we need to lower the frame bottom to include higher T data.

In Fig. 2, $\Delta p/\Delta p_{load}$ is constant at low V_{ac} and starts to decrease above ~10 μm/s. The most important feature, however, seems to be the linear dependence on $\log(V_{ac})$ seen in Fig. 2(c). This dependence was observed previously in an annular cell [1] and is supposed to support the VF model [8]. We further tried to clarify especially in Fig. 2(c) its velocity range. We observe that linear dependence on $\log(V_{ac})$ is seen for one decade in V_{ac}, 30–300 μm/ sec for all of the T's plotted and then above this range, $\log(V_{ac})$ dependence changes. Further detailed analysis is possible for higher V_{ac}, and we plan to report separately in the near

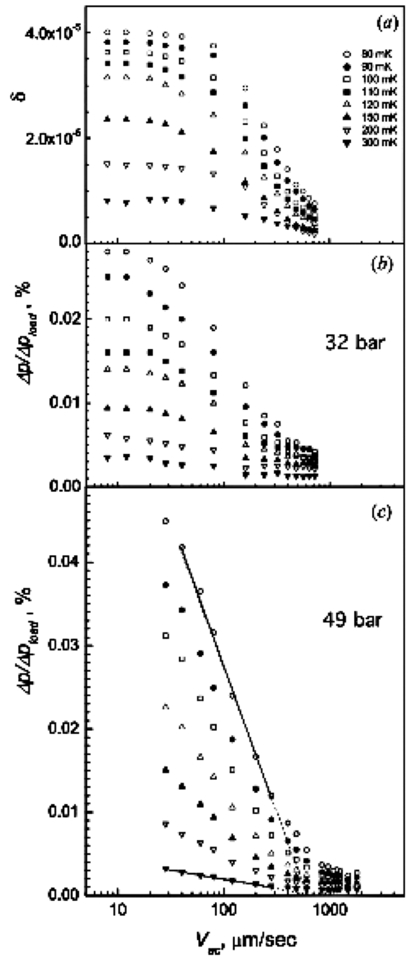

FIG. 2. δ (a) and $\Delta p/\Delta p_{load}$ (b) of 32 bar sample as a function of V_{ac} at $T < 300$ mK. $\Delta p/\Delta p_{load}$ (c) is the data of the new sample at 49 bar. The solid lines in (c) show the linear dependence on $\log(V_{ac})$ for the V_{ac} range; $\sim 30 < V_{ac} < 300$ μm/s and at higher V_{ac} some other dependence appears. We observe practically the same $\log(V_{ac})$ linear dependence as in (c) also for a 32 bar sample (b) by fitting with linear lines. Extrapolated linear lines are found to converge at a point ~600 μm/s for the V_{ac} range. This point of convergence also seems to coincide with the zero in Fig. 1(b).

future. We can also estimate the characteristic V_{ac} corresponding to suppression of the major part of $\Delta p/\Delta p_{load}$ as ~600 μm/s as indicated by dotted lines. Actually similar $\log(V_{ac})$ linear dependence is also seen in Fig. 2(b) when one draws linear lines through the data for almost the same $\log(V_{ac})$ range. It is remarkable that such fundamental features of the measured data are reproduced for completely independent samples at different pressures. The characteristic velocity ~600 μm/s seems T-independent within our experimental accuracy.

While examining the above evidence, we noticed that all of the $\log(V_{ac})$ linear slopes in the region ~ 30 μm/s $< V_{ac} < 300$ μm/s showed a simple T^{-2} dependence as is seen in Fig. 3. We do not know the real origin, but it is interesting to note that it does not include a finite temperature shift like Curie-Weiss behavior as for magnetic sus-

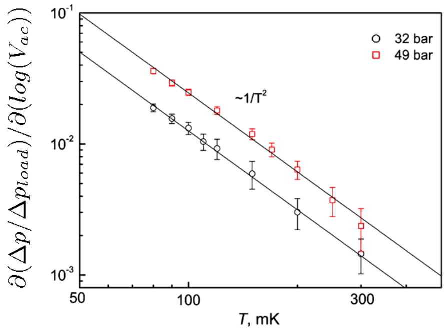

FIG. 3 (color online). T dependence of the slope $d(\Delta p/\Delta p_{\mathrm{load}})/d[\log(V_{\mathrm{ac}})]$. Clear $1/T^2$ dependence is seen for both of the completely independent solid ^{4}He samples at 32 and 49 bar pressure.

ceptibility, but just Curie law-like behavior with zero Weiss temperature. Curie law behavior is observed for metallic spin glasses down to the susceptibility peak. This T dependence suggests to us that $\Delta p/\Delta p_{\mathrm{load}}$ actually is NLRS and not that of an order parameter below T_c. It reminds us also of T^{-2} dependence of polarizability of rotating turbulence [23].

What would be the origin of these unusual features of solid ^{4}He? We discuss below a possible scenario as coming from fluctuations in the VF state, which is regarded as a kind of superfluid turbulent state. In superfluid turbulence, fluctuations are controlled by external rotation [24] and characterized by a distribution over a certain momentum space [25]. The width of this distribution is primarily determined by both the longest straight vortex line length, which is of the order of the system size, and the smallest length scale of the vortex tangle or that of vortex rings. The VF state does not support macroscopic superflow either, which is consistent with reports of absence of superflow. We can quote that smaller excitation or rotational speed supports larger fluctuations, and larger ac excitation or faster rotation suppresses the phase fluctuations. So the ac velocity of ~ 10 μm/s has been often considered as some critical velocity of a supersolid, but it is actually a characteristic velocity of the turbulent vortex fluid state. Our analysis made this point clear for the first time. We expect a real T_c at some lower T than studied here, and we expect the appearance of real critical velocities of a supersolid only below T_c.

While preparing this Letter, we found an interesting study of the mechanical properties in solid ^{4}He under shear motion by Beamish's group [26]. We have no concrete idea how a solid should behave simultaneously as a superfluid, and it will become an interesting question. What they measure actually would not be a simple shear modulus, especially when the shear stress may not be transmitted across the sample as in a usual solid.

In summary, we studied NLRS as well as the energy dissipation in solid ^{4}He using TO responses of samples at 32 and 49 bar in wide ranges of $T > T_p$ and V_{ac}. NLRS varies nearly linearly with $\log(V_{\mathrm{ac}})$ over a decade of V_{ac} below an onset at $T_o \sim 500$ mK, and this slope changes as $1/T^2$ until a crossover to another T dependence, for all of the samples studied. It seems to support Anderson's VF model.

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[1] E. Kim and M. H. W. Chan, Nature (London) **427**, 225 (2004); Science **305**, 1941 (2004); Phys. Rev. Lett. **97**, 115302 (2006).
[2] M. Kondo, S. Takada, Y. Shibayama, and K. Shirahama, J. Low Temp. Phys. **148**, 695 (2007).
[3] A. S. C. Rittner and J. D. Reppy, Phys. Rev. Lett. **97**, 165301 (2006).
[4] A. S. C. Rittner and J. D. Reppy, Phys. Rev. Lett. **98**, 175302 (2007).
[5] A. Penzev, Y. Yasuta, and M. Kubota, J. Low Temp. Phys. **148**, 677 (2007).
[6] A. J. Leggett, Phys. Rev. Lett. **25**, 1543 (1970).
[7] N. Prokof'ev, Adv. Phys. **56**, 381 (2007).
[8] P. W. Anderson, Nature Phys. **3**, 160 (2007).
[9] P. W. Anderson, arXiv:0705.1174.
[10] Y. Wang *et al.*, Phys. Rev. B **73**, 024510 (2006).
[11] Y. Aoki, J. Graves, and H. Kojima, Phys. Rev. Lett. **99**, 015301 (2007).
[12] J. D. Reppy (private communication).
[13] A. C. Clark, J. T. West, and M. H. W. Chan, Phys. Rev. Lett. **99**, 135302 (2007).
[14] S. Sasaki *et al.*, Science **313**, 1098 (2006).
[15] M. Kubota *et al.*, Physica (Amsterdam) **329B–333B**, 1577 (2003).
[16] V. L. Tsymbalenko, Sov. Phys. JETP **47**, 787 (1978).
[17] Z. S. Li *et al.*, Rev. Sci. Instrum. **74**, 2477 (2003).
[18] M. Paalanen, D. Bishop, and H. Dail, Phys. Rev. Lett. **46**, 664 (1981).
[19] I. Iwasa, K. Araki, and H. Suzuki, J. Phys. Soc. Jpn. **46**, 1119 (1979).
[20] D. Bishop and J. Reppy, Phys. Rev. B **22**, 5171 (1980).
[21] M. Fukuda *et al.*, Phys. Rev. B **71**, 212502 (2005).
[22] J. D. Reppy and A. Tylar, in *Excitations in Two-Dimensional and Three-Dimensional Quantum Fluids*, edited by A. F. G. Wyatt and H. J. Lauter (Plenum Press, New York, 1991), pp. 291–300.
[23] M. Tsubota (private communication); M. Tsubota and C. Barenghi *et al.*, Phys. Rev. B **69**, 134515 (2004).
[24] C. E. Swanson, C. F. Barenghi, and R. J. Donnelly, Phys. Rev. Lett. **50**, 190 (1983); M. Tsubota, T. Araki, and C. Barenghi, Phys. Rev. Lett. **90**, 205301 (2003).
[25] M. Kobayashi and M. Tsubota, J. Phys. Soc. Jpn. **74**, 3248 (2005).
[26] J. Day and J. Beamish, Nature (London) **450**, 853 (2007).