Fission Decay Widths for Heavy-Ion Fusion-Fission Reactions

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Cross-section and neutron-emission data from heavy-ion fusion-fission reactions are consistent with a Kramers-modified statistical model which takes into account the collective motion of the system about the ground state, the temperature dependence of the location of fission transition points, and the orientation degree of freedom. The strong increase in the nuclear viscosity above a temperature of ~1 MeV deduced by others is an artifact generated by an inadequate fission model.

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For more than 20 years it has been known that the ''standard'' statistical theory of fission leads to an underestimation of the number of measured prescission neutrons emitted in heavy-ion reactions [1–5]. It is generally accepted that the main cause of this discrepancy is associated with the viscosity of hot nuclear matter [6]. Giant dipole resonance γ-ray emission has also been used to infer inadequacies in our models of nuclear fission decay widths [7–10]. These inadequacies have been compensated for by adjusting the viscosity of hot nuclear matter to reproduce experimental data. A consensus appears to have emerged that strong dissipation sets in rather rapidly above a nuclear temperature of ~1.3 MeV [7].

The Bohr-Wheeler fission decay width is often expressed as [8]

$$\Gamma_f^{\mathrm{BW}} = \frac{1}{2\pi\rho_{\mathrm{gs}}(E - V_{\mathrm{gs}})} \times \int_0^{E-V_{\mathrm{gs}}-B_f} \rho_{\mathrm{sp}}(E - V_{\mathrm{gs}} - B_f - \varepsilon)d\varepsilon, \quad (1)$$

where ρ_{gs} and ρ_{sp} are the level densities at the ground state and the fission saddle point, respectively, E is the total excitation energy, V_{gs} is the potential energy at the ground state, and B_f is the fission-barrier height. The level densities are approximated as [8,11]

$$\rho(U) \propto \exp(2\sqrt{aU})/U^2, \quad (2)$$

where U is the thermal excitation energy. The slowing effects of nuclear viscosity are included by using the Kramers-modified [12] Bohr-Wheeler model

$$\Gamma_f = (\sqrt{1 + \gamma^2} - \gamma)\Gamma_f^{\mathrm{BW}}, \quad (3)$$

where γ is the nuclear viscosity given by $\beta/(2\omega_{\mathrm{sp}})$, β is the dissipation coefficient, and ω_{sp} is a measure of the potential curvature at the fission saddle point. Transient delays in the build up of the fission decay width to its asymptotic value are typically only a few 10^{-21} s [13–15]. Even at the highest excitation energies for the reactions considered here, the mean fission lifetimes are longer than 10^{-19} s and thus transient delays can be neglected. Simple arguments show that several other pieces of physics are missing from Eqs. (1) and (2). These equations contain no terms that allow the fission decay width to change with the width of the ground-state well. This problem was overcome by Strutinsky [16] who pointed out that the total level density of the system must be calculated taking into account the collective motion. This effect increases fission lifetimes by a factor of $T/(\hbar\omega_{\mathrm{gs}})$ and has been addressed by some [17]. However, many authors continue to ignore this correction.

The way Eq. (1) is commonly used becomes invalid at high excitation energy because the locations of the equilibrium points are a function of excitation energy and should be defined as the equilibrium points in the level density (or entropy) as a function of deformation, and not as the equilibrium points in the $T = 0$ potential energy, $V(q)$. Searching for the equilibrium points in the entropy is the same as searching for the equilibrium points in a temperature-dependent effective potential energy [17,18]

$$V_{\mathrm{eff}}(q, T) = V(q) - a(q)T^2. \quad (4)$$

The shape dependence of the level-density parameter can be approximated by the expression [19–21]

$$a(q) \sim c_V A + c_S A^{2/3} B_S(q), \quad (5)$$

where c_V and c_S are constants, and $B_S(q)$ is the ratio of the surface energy relative to that of the spherical system. Assuming Eq. (5), the effective potential can be obtained using a $(1 - \alpha T^2)$ dependence of the surface energy, where

$$\alpha = c_S A^{2/3}/E_S^o = c_S \times 0.059\ \mathrm{MeV}^{-1} \quad \text{for} \quad A \sim 200 \quad (6)$$

and E_S^o is the surface energy of the spherical system. For a particular model, Töke and Swiatecki [21] obtain $c_S \sim 0.27\ \mathrm{MeV}^{-1}$ and thus $\alpha \sim 0.016\ \mathrm{MeV}^{-2}$. c_S is known to be very sensitive to the assumed properties of nuclear matter and to other approximations [22]. Other estimates of c_S [19–24] give values of α that range from 0.007 to $0.022\ \mathrm{MeV}^{-2}$. Initially, we shall assume $\alpha = 0.016\ \mathrm{MeV}^{-2}$.

The locations of fission transition points do not change much up to a temperature of ~1 MeV. However, there is a dramatic change in the locations of the transition points

above $T \sim 1$ MeV (see Fig. 1). The dashed vertical lines in Fig. 1 show that the equilibrium points in the effective potential correspond to equilibrium points in the entropy. If the transition point is incorrectly assumed to equal the $T = 0$ value (independent of temperature) then the entropy at the transition point will be increasingly overestimated with increasing temperature. This causes the mean fission lifetime to be increasingly underestimated as the temperature increases above ~1 MeV.

The statistical model of the fission of rotating systems must determine the total level density and the number of fission transition states, taking into account the level density associated with both the shape and orientation degrees of freedom. In the limit of a small deviation from the spherical shape, the effective moment of inertia is large and the rotational energy becomes independent of the orientation of the symmetry axis relative to the total spin. In this case, including the orientation degree of freedom increases the level density by $2J + 1$ [25]. For an arbitrary deformation, this multiplication factor associated with the orientation degree of freedom is less than $2J + 1$ [25]. This decrease in the level density with increasing deformation slows fission at high spins relative to widths obtained using Eqs. (1) and (2).

Including these three effects, the statistical model fission decay width for a rotating system is [26]

$$\Gamma_f = \sum_K P(K)\Gamma_f(K) \Big/ \sum_K P(K), \tag{7}$$

where $P(K)$ is the probability that the system is in a given K state,

$$P(K) \propto \frac{T_{\rm gs}(K)}{\hbar\omega_{\rm gs}(K,T)} \rho_{\rm gs}(E - V_{\rm gs}(K,T)), \tag{8}$$

where K is the spin about the fission axis. $\Gamma_f(K)$ is the fission decay width if the system were restricted to a single K. To include the effect of the nuclear viscosity, $\Gamma_f(K)$ should be determined including the Kramers' reduction factor using

$$\Gamma_f(K) = (\sqrt{1+\gamma^2} - \gamma)\frac{\hbar\omega_{\rm gs}}{2\pi} \exp\left(-\frac{2B_f}{T_{\rm gs} + T_{\rm sp}}\right), \tag{9}$$

where $\omega_{\rm sp}$, $\omega_{\rm gs}$, and B_f are all functions of T and K, and determined using an effective potential with a $(1 - \alpha T^2)$ dependence of the surface energy. $T_{\rm gs}$ and $T_{\rm sp}$ are the temperatures at the equilibrium points. The statistical model code JOANNE4 [26] includes the three effects discussed here.

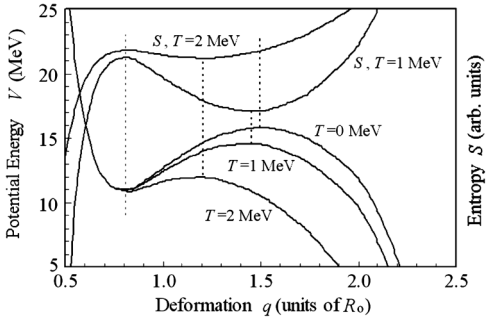

FIG. 1. The modified liquid-drop model (MLDM) [33] potential energy, $V(q)$, as a function of deformation for ^{210}Po with $J = 50$, along with the corresponding effective potential energies, $V_{\rm eff}(q,T)$, at $T = 1$ and 2 MeV assuming $\alpha = 0.016$ MeV^{-2}. Also shown is the deformation dependence of the corresponding entropies, $S(q,E)$. The dashed vertical lines are to guide the eye.

Calculated cross sections and prescission emission are very sensitive to the assumed $T = 0$ potential-energy surface and the assumed deformation dependence of the level-density parameter, but depend only linearly on the nuclear viscosity. Therefore, extreme caution must be applied if attempting to infer the viscosity from cross-section and emission data. We believe the nuclear dissipation has been well constrained by the surface-plus-window dissipation model [27,28] using the mean kinetic energy of fission fragments and the widths of isoscalar giant resonances. For the range of typical fission saddle-point deformations encountered in heavy-ion fusion-fission reactions with compound nuclear mass numbers from $A_{\rm CN} \sim 170$ to 220, the dissipation coefficient in this model is within 10% of 3×10^{21} s^{-1} and independent of temperature.

To confirm the validity of the Kramers-modified statistical model we compare results obtained using Eqs. (7)–(9) to dynamical calculations. We assume the shape degree of freedom is governed by a Langevin equation [29] and couple the orientation degree of freedom (K states) with the heat bath [26]. We assume the same temperature dependence effective potential $V_{\rm eff}(q,T)$, the same dissipation coefficient, and the same inertia [30] for both our statistical and Langevin calculations. The Langevin calculations of others underestimate the fission lifetime because only the $K = 0$ fission barrier is sampled, instead of an equilibrated distribution containing higher $K \neq 0$ barriers. Figure 2 shows estimates of the mean fission lifetime of ^{210}Po systems formed by the reaction ^{18}O + ^{192}Os, as a function of the initial excitation energy. The relationship between excitation energy and average spin of the fissioning systems is determined using measured fusion and evaporation-residue cross sections [31]. The solid curve shows Kramers-modified statistical model calculations using Eqs. (7)–(9) as performed by JOANNE4 [26]. Model parameters are $a = A/8.6$ MeV^{-1}, $\beta = 3 \times 10^{21}$ s^{-1}, and $\alpha = 0.016$ MeV^{-2}. These calculations are consistent with the corresponding two-dimensional (shape and orientation) Langevin calculations shown by the solid circles in Fig. 2. The agreement between these two methods confirms that if the Kramers-modified statistical model is implemented correctly, then the results are in agreement with dynamical calculations.

Dioszegi *et al.* [8] assume $a_{\rm sp}/a_{\rm gs} \sim 1.04$ when inferring the nuclear viscosity of hot rotating ^{224}Th nuclei. The

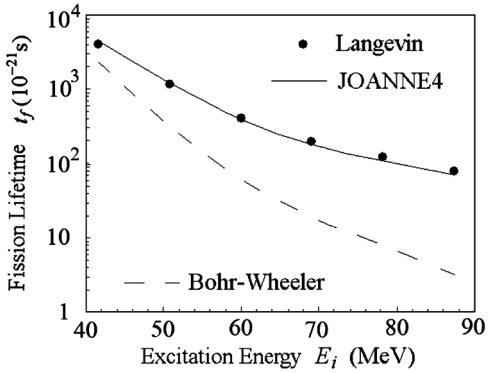

FIG. 2. Various estimates of the mean fission lifetime of ^{210}Po systems formed by the reaction ^{18}O + ^{192}Os, as a function of the initial excitation energy (see text).

dashed curve in Fig. 2 shows estimates of the fission lifetime of ^{210}Po obtained using the standard Bohr-Wheeler model with $a_{sp}/a_{gs} = 1.04$. These calculations are a factor of 2 lower than the more complete calculations shown by the solid curve and circles at $E_i \sim 40$ MeV, and more than a factor of 20 low at $E_i \sim 90$ MeV (see Fig. 2). The solid curve in Fig. 3 shows the nuclear viscosity as a function of excitation energy needed to force the standard Bohr-Wheeler model with $a_{sp}/a_{gs} = 1.04$ to be in agreement with the calculations shown by the solid curve in Fig. 2. This artificial excitation-energy dependence of the nuclear viscosity is similar to the excitation-energy dependence deduced by Dioszegi *et al.* [8]. This result suggests that the rapid onset of the dissipation at nuclear excitation energies above ~40 MeV [7] is an artifact generated by an incomplete model of the fission process.

We model the fusion process and adjust the nucleus-nucleus potential and shape of the target nucleus to obtain fits to measured fusion excitation functions [5,31]. The corresponding calculated fusion spin distributions are used as input into the statistical model calculations. All JOANNE4 calculations presented here assume $a = A/8.6$ MeV^{-1} and $\beta = 3 \times 10^{21}$ s^{-1}. In other statistical model codes, the heights of fission barriers are often uniformly scaled by a parameter f_B. In JOANNE4, we scale the MLDM radii from the default values used to calculate the surface and Coulomb energies by the parameter r_S. A scaling of $r_S = 1.0$ gives the default MLDM with fission barriers in agreement with the finite-range liquid-drop model (FRLDM) [32]. Raising r_S above 1 increases the surface energy and decreases the Coulomb energy. This stabilizes the systems and causes the fission barriers to increase. The parameter α controls the shape dependence of the level-density parameter and plays a role similar to a_f/a_n (a_{sp}/a_{gs}) in other models. However, using an effective potential with a $(1 - \alpha T^2)$ dependence of the surface energy is a more complete approach. Within JOANNE4, for each J, K, and T, the fission transition point is found by looking for the unstable equilibrium point in the effective potential energy. For each reaction considered here, the parameters α and r_S are adjusted to reproduce a single fission cross section and a single prescission neutron multiplicity at the same projectile kinetic energy, corresponding to the second lowest prescission neutron multiplicity measurement. Figure 4 shows how the $E_{lab} \sim 103$ MeV ^{18}O + ^{192}Os data constrain the adjustable parameters to $\alpha = 0.017 \pm 0.006$ MeV^{-2} and $r_S = 1.002 \pm 0.002$. Figure 5 shows the parameters α and r_S for five reactions. Figure 6 shows the model predictions for the projectile energy dependence of fission and residue cross sections and pres-

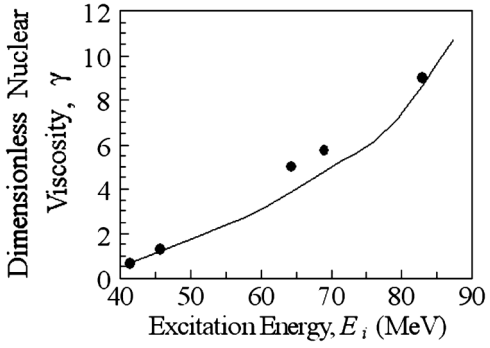

FIG. 3. The solid line shows the nuclear viscosity as a function of excitation energy needed to force the Kramers-modified standard Bohr-Wheeler model with $a_{sp}/a_{gs} = 1.04$ to be in agreement with the calculations shown by the solid curve in Fig. 2. The symbols show the nuclear viscosity inferred by Ref. [8].

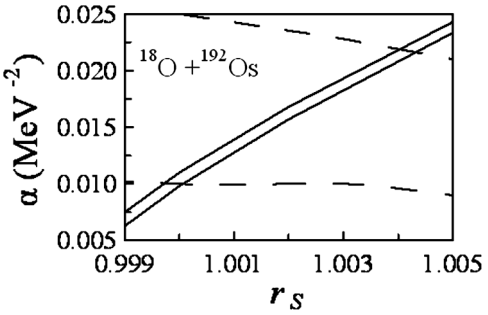

FIG. 4. The $E_{lab} \sim 103$ MeV ^{18}O + ^{192}Os fission cross section [31] and neutron multiplicity [5] constrain the parameters α and r_S to the regions between the solid and dashed curves, respectively.

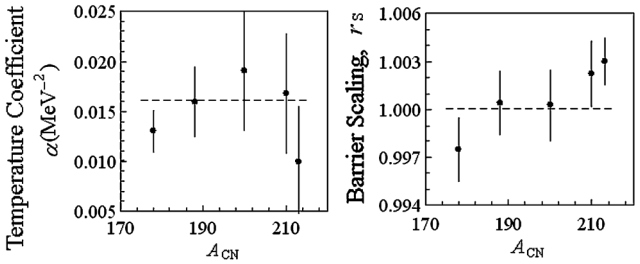

FIG. 5. Fit parameters α and r_S for five reactions. The dashed lines show the values corresponding to the model calculations of Refs. [21,32].

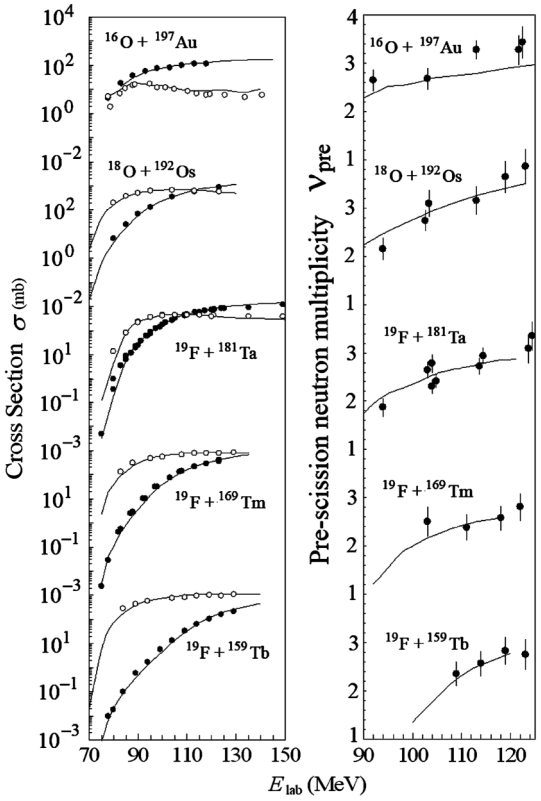

FIG. 6. JOANNE4 model predictions (solid curves) for the projectile energy dependence of cross sections and prescission neutron multiplicities for five reactions. The experimental data are from Refs. [5,31,34–37]. The fission and residue cross sections are shown by solid and open symbols, respectively.

cission neutron multiplicities, using the α and r_S values represented by the circles in Fig. 5. These predictions are consistent with the data. To reproduce this data set, the model calculations of others would require either large fission dynamical delays [5] or a strong temperature dependence of the nuclear viscosity as shown in Fig. 3. It must be emphasized that the results presented here should not be used to support $\beta = 3 \times 10^{21}$ s^{-1} at fission transition points. Equally good reproductions of the data can be obtained by changing α by $\sim$0.0025 MeV^{-2} for each change in β of 10^{21} s^{-1}. For example, if β is reduced to 10^{21} s^{-1} then the required α scatter about $\sim$0.011 MeV^{-2}. The required r_S are very insensitive to changes in β.

Other authors have assumed that their ability to model nuclear fission is complete enough that the properties of a temperature-dependent nuclear dissipation can be extracted from cross-section and prescission emission data. We assume that the reduced nuclear dissipation is independent of temperature. After making several improvements to the standard method used to calculate fission decay widths, we find that fusion-fission cross-section and prescission neutron data for $A_{CN} = 170$–220 are consistent with a Kramers-modified statistical model of fission, the FRLDM [32], a reduced nuclear dissipation coefficient independent of temperature, and a shape dependence of the level-density parameter in the range of theoretical estimates [19–24].

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