Spin Wave and Vortex Excitations of Superfluid ^{3}He-A in Parallel-Plate Geometry

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Quantized vortices with half-integer circulation, which are forbidden from existing in a conventional superfluid because of the single valueness of the wave function, are theoretically predicted to exist in superfluid ^{3}He-A if the order parameters $\hat{\ell}$ and $\hat{\mathbf{d}}$ form $\hat{\ell} \perp \hat{\mathbf{d}}$ texture. To form the $\hat{\ell} \perp \hat{\mathbf{d}}$ texture, we confined the superfluid between parallel plates with a 12.5 μm gap and applied a magnetic field of $H = 26.7$ mT perpendicular to the plates to take NMR and orient $\hat{\mathbf{d}}$ perpendicular to $\hat{\ell}$. NMR spectra exhibit a negative-shift peak which probes that the uniform $\hat{\ell} \perp \hat{\mathbf{d}}$ texture is realized in our cell and show a new satellite signal under rotation. The rotation dependence of the satellite signal is interpreted that a Fréedericksz transition of $\hat{\ell}$ texture is induced by rotation above 1.0 rad/s and vortices start to appear above 1.8 rad/s.

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The superfluid ^{3}He-A phase in narrow channels [1,2] has been widely studied, especially with regard to its configuration of the order-parameter field, called "*texture*." Since its energy gap structure is anisotropic, the orbital vector of the order parameter, $\hat{\ell}$, must be perpendicular to the walls [3]. On the other hand, its spin vector, $\hat{\mathbf{d}}$, can also be directed by a magnetic field, $\mathbf{H}$, as $\hat{\mathbf{d}} \perp \mathbf{H}$. These anisotropic features provide a rare opportunity to control texture by external conditions, such as the symmetry of the container, flow, and magnetic fields. For example, in bulk liquid, there is a uniform $\hat{\ell} \parallel \hat{\mathbf{d}}$ texture owing to the dipole interaction and a variety of vortices has been found in the $\hat{\ell} \parallel \hat{\mathbf{d}}$ texture [4]. In a cylinder [1], the boundary condition forces $\hat{\ell}$ to form a radial distribution and 3 radial $\hat{\ell}$ configurations have been found. Between parallel plates, however, it has been unclear what types of vortices are formed. In this geometry, we can obtain a $\hat{\ell} \perp \hat{\mathbf{d}}$ texture if we confine ^{3}He between parallel plates whose gap is as narrow as the dipole coherence length, $\xi_D \sim 10$ μm, and apply $H > 3$ mT perpendicular to the plates. This $\hat{\ell} \perp \hat{\mathbf{d}}$ texture is quite different from that in bulk and an exotic vortex with a half-quantum circulation is predicted to exist in the parallel-plate geometry [5,6]. The Helsinki group tried to make parallel plates with a gap of 19 μm [2]; however, their gaps were widely varied and the texture was not uniform.

Our sample cell [7] was made of polyimide films of 12.5 and 25 μm thicknesses. All 12.5 μm films have a hole of the radius $R = 1.5$ mm and a channel of 0.3 mm in width that connects the hole to a bulk space. We stacked the 12.5 μm film and the 25 μm film one after the other to form 110 spaces in the 12.5 μm film. These films were packed inside a stycast box which has a filling line for introducing ^{3}He via an Ag-sintered heat exchanger to cool down the liquid. This sample cell was set in a rotating cryostat at ISSP [8], which can be rotated up to a rotation speed $|\Omega| = 6.28$ rad/s. Liquid ^{3}He was confined into the sample cell at a pressure 3.05 MPa and cw-NMR measurements were done at the Larmor frequency $f_L = 869$ kHz (the magnetic field $H = 26.7$ mT). Both axes of the magnetic field and the rotation were perpendicular to these films. Also, we took NMR of the same liquid in a bulk space for a reference.

Temperature dependence of the resonance frequency [7] of both liquids was measured down to 1.6 mK as shown in Fig. 1. Phase transition to the A phase of both liquids can be clearly seen by frequency shift from f_L below the transition temperature $T_c = 2.46$ mK. The NMR spectrum of

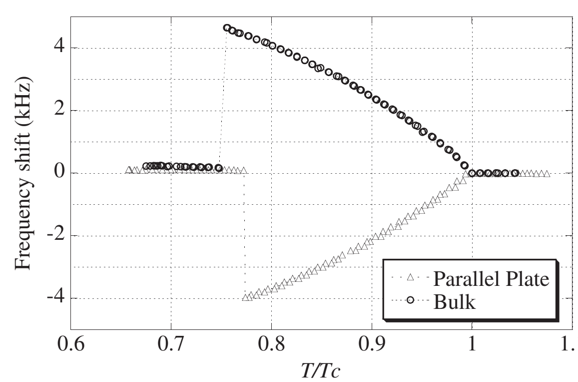

FIG. 1. Frequency shift from the Larmor frequency observed between parallel plates (triangles) and in bulk (circle) as a function of temperature normalized by T_c. Frequency shifts disappeared in lower temperature because of the transition to the superfluid B phase.

the liquid in bulk shows a positive frequency shift while that between the parallel plate shows a negative-frequency shift. In the A phase, the resonance frequency can be written as $f = f_L + f_A(T)\cos 2\theta$, where $f_A(T)$ is the transverse frequency shift of the A phase and θ is the angle between $\hat{\ell}$ and $\hat{\mathbf{d}}$ [3]. In bulk liquid, $\theta = 0$ due to the dipole energy which prefers $\hat{\ell}$ and $\hat{\mathbf{d}}$ to be parallel. The negative-frequency shift which can be fitted as $f = f_L - 0.93 f_A(T)$ probes that $\hat{\ell}$ was oriented almost perpendicular to $\hat{\mathbf{d}}$ in the whole sample. The sharp, negative-shift peak [main peak, shown in [7] or Fig. 2(a)] demonstrates that the films were stacked parallel and the 110 slab spaces are formed quite uniformly [9]. The small peak observed near f_L is due to a solid ^{3}He absorbed on the films because its magnetization showed the Curie law. Also, there is no signal coming from the bulk liquid in the parallel-plate spectrum.

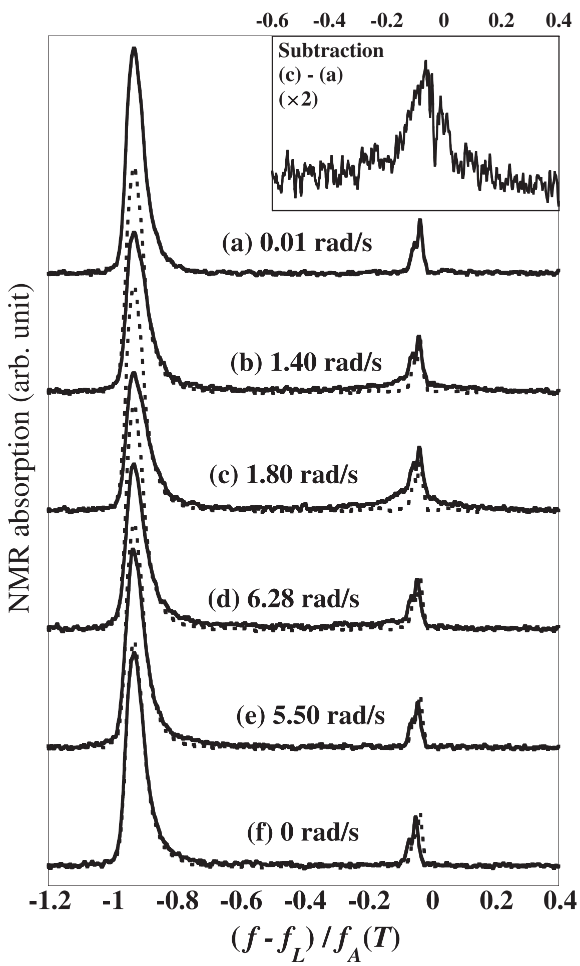

FIG. 2. NMR spectra at 0.01, 1.40, 1.80, 6.28, 5.50. and 0 rad/s. All spectra were measured at $0.81T_c$ in the order of showing up to down. The superfluid was cooled at $H = \Omega = 0$ during the superfluid transition. For the spectra of (b)–(f), the first spectrum (a) is shown as a dotted line for reference. The inset is an enlarged view of the satellite signal obtained from a subtraction between data at 1.8 rad/s and at 0.01 rad/s.

Dependence of the NMR spectrum on rotation was measured at a constant temperature. Some of the specific NMR spectra recorded at $0.81T_c$ were shown in Fig. 2. In acceleration, no change was observed in the NMR spectrum below 1.0 rad/s. Above 1.0 rad/s [Fig. 2(b)], a new satellite signal was observed near f_L. This new satellite signal (almost merged with the solid signal) increased as the rotation speed increased and reached the maximum at 1.8 rad/s [Fig. 2(c)]. This new satellite signal can be clearly seen by subtracting the NMR spectrum taken at 1.8 rad/s [Fig. 2(c)] from that taken at 0.01 rad/s [Fig. 2(a)] as shown in the inset of Fig. 2. Above 1.8 rad/s, the satellite signal started to decrease and became almost constant above ~5.0 rad/s up to 6.28 rad/s [Fig. 2(d)]. In deceleration from 6.28 rad/s, the satellite signal decreased as rotation speed decreased and disappeared below 5.5 rad/s [Figs. 2(e) and 2(f)].

We estimate rotation dependence of the size of the satellite signal, $I_s(\Omega)$, by integrating differences of the NMR spectrum under rotation around the Larmor frequency as,

$$I_s(\Omega) \equiv \int_{f_L - f_A/3}^{f_L + f_A/3} (I(f,\Omega) - I(f,0))df, \qquad (1)$$

where $I(f,\Omega)$ is the NMR intensity at a frequency f and a rotation speed Ω. This size can be normalized by the whole integration of the NMR spectrum, I_{total}, as $I_s'(\Omega) = I_s(\Omega)/I_{\text{total}}$ (I_{total} is a constant because of the constant spin susceptibility of the A phase [3]) and is plotted in Fig. 3.

We interpret this new satellite signal as a result of a textural transition of $\hat{\ell}$ texture. When the sample is being rotated, the normal component of the superfluid rotates as a solid body of which the velocity is written as $v_n = r\Omega$,

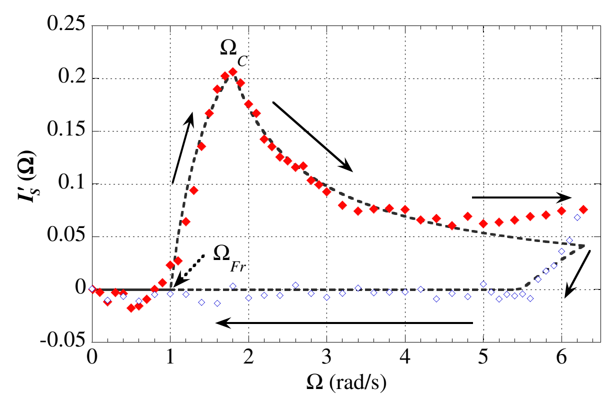

FIG. 3 (color online). Rotation dependence of normalized size of the satellite signal at 0.81 T_c. Data taken in the acceleration (deceleration) process is shown in red filled (blue open) symbol. The dashed line is a model calculation based on that a Fréedericksz transition takes place for $\Omega \geq \Omega_{\text{Fr}}$ and vortices start to appear for $\Omega \geq \Omega_c$, in which $\Omega_{\text{Fr}} = 1.0$ rad/s, $\Omega_c = 1.8$ rad/s and a reduction parameter $\alpha = 0.3$ are used (discussed in text).

where r is the distance from the rotation axis. Because of the anisotropic energy gap structure of the A phase, normal flow tends to orient $\hat{\ell}$ parallel to the flow. Between parallel plates, $\hat{\ell}$ texture under a normal flow is determined by a balance between orientational forces (the boundary condition of $\hat{\ell} \perp$ wall, the flow effect and the dipole interaction) and the bending energy [3]. Under such a competition, it is well known that there is a threshold in the orieantational parameter and, when the parameter exceeds the limit, the system undergoes a Fréedericksz transition (FT) from a uniform texture to a nonuniform one. For example, FT induced by a magnetic field [10] and a rotation [11] have been observed in the A phase by a torsional oscillator. From Fig. 3, the onset velocity for FT in the parallel-plate geometry, v_{Fr}, can be found as $v_{\mathrm{Fr}} = R\Omega_{\mathrm{Fr}} = 1.5$ mm/s, where $\Omega_{\mathrm{Fr}} = 1.0$ rad/s is the onset $I_s'(\Omega)$ started to increase.

In our thin slab geometry, $\hat{\ell}$ texture around the center can be bent when the normal flow exceeds a critical velocity, v_{Fr}. On the other hand, the $\hat{\mathbf{d}}$ texture remains parallel to the wall because of the magnetic field applied perpendicular to the wall. Therefore, the dipole energy ($\propto -(\hat{\ell} \cdot \hat{\mathbf{d}})^2$) has a position dependence between the walls like a well. Such a nonuniform dipole field can trap a spin wave and a new satellite signal appears in the NMR spectrum. For example, vortices in the bulk A phase have a dipole-unlocked region around the vortices (so called "*soft core*") and spin waves

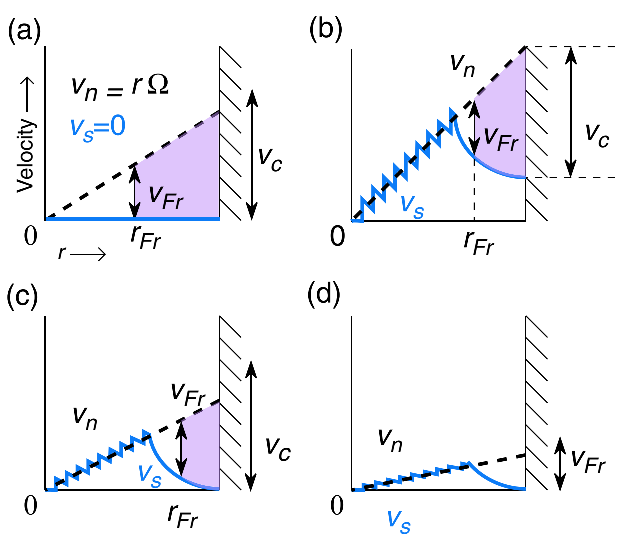

FIG. 4 (color online). An illustration of velocity field. Normal (super) fluid velocity is represented as a black dashed (blue solid) line. The region where $v_n - v_s > v_{\mathrm{Fr}}$ is illustrated as a shaded area (purple). In acceleration: (a) $\Omega > \Omega_{\mathrm{Fr}}$, FT region proceeds inward as rotation speed increases. (b) $\Omega > \Omega_c$, vortices form a vortex cluster which expands as rotation speed increases to keep $v_n - v_s = v_c$ at $r = R$. In deceleration: (c) vortex cluster expands with keeping the number of vortices. The FT region quickly shrinks as rotation speed decreases. (d) There is no FT region.

trapped by the vortices were observed as a new satellite signal in NMR spectrum [12].

To confirm that the FT induced by a normal flow is responsible for the satellite signal, a hydrostatic model in the Ginzburg-Landau region [13] was used to evaluate trapping well in a slab of 12.5 μm thickness under a rotation and the frequency of the trapped spin wave was calculated. At $1.8\Omega_{\mathrm{Fr}}$ [rotation speed at Fig. 2(c)], it was shown that a new satellite signal should appear at $(f - f_L)/f_A \sim 0$, which is very close to the observed value of -0.07.

The curve of $I_s'(\Omega)$ can be explained by this FT as follows. Above Ω_{Fr} in acceleration, FT started from $r = R$ and expanded inward in the acceleration as $1 - (r_{\mathrm{Fr}}/R)^2$ as illustrated in Fig. 4(a), where $r_{\mathrm{Fr}} = R\Omega_{\mathrm{Fr}}/\Omega$ is the inner edge of the FT region. The rotation speed at the peak of $I_s'(\Omega)$, $\Omega_c = 1.8$ rad/s, can be attributed to a critical velocity for appearance of vortices in the parallel-plate geometry. Vortices produce superflow, v_s, around them and reduce all normal flow effect, where counterflow, $v_n - v_s$, plays a role of normal flow before the vortex appearance. A model of vortex distribution in bulk liquid [11] is used to evaluate the superflow effect on $I_s'(\Omega)$ as illustrated in Fig. 4(b). In this model, vortices come into the superfluid so that the counterflow at the wall is maintained as $v_n(R) - v_s(R) = v_c$, where $v_c = R\Omega_c$ is the critical velocity for vortex, and these vortices form a cluster around the rotation center. Inside the cluster, there is no macroscopic counterflow, so no Fréedericksz transition (the $\hat{\ell}$ texture is uniform). While outside, the counterflow is given as $v_n(r) - v_s(r) = r\Omega - \kappa N/2\pi r$, where κ is the quantum of circulation and N is the number of vortices, and the FT region is contracted by the vortices. In deceleration, the vortex cluster expands while keeping the vortex number [Fig. 4(c)], and the FT disappears in further deceleration [Fig. 4(d)]. In Fig. 3, we plot the area where the counterflow exceeds v_{Fr} as a function of rotation speed as a dashed line with a fitting parameter α as $\alpha[1 - (r_{\mathrm{Fr}}/R)^2]$. This line describes $I_s'(\Omega)$ very well by adjusting $\alpha = 0.3$. This good coincidence strongly supports our interpretation that FT is taking place above Ω_{Fr} by normal flow and the normal flow effect is reduced by vortices which appear above Ω_c.

The vortices appeared above Ω_c; however, they did not show any direct signals such as vortices in bulk [12] as shown in Fig. 2(e). Since there is no counterflow which exceeds v_{Fr} at this rotation speed, only a vortex contribution is expected in the NMR spectrum. However, the spectrum looks virtually the same with that taken at rest [Fig. 2(a)]. This is a surprising result because, in bulk liquid at 5.5 rad/s, almost 1/3 of the NMR signal is observed as a vortex signal. Moreover, any broadening of the main peak, which is observed for vortices in the bulk ^{3}He-A phase [12], was not observed. This is a very different result from that by the Helsinki group [2]. They reported a broadening of the main peak of $\Delta\Gamma/\Delta\Omega = 110$ in

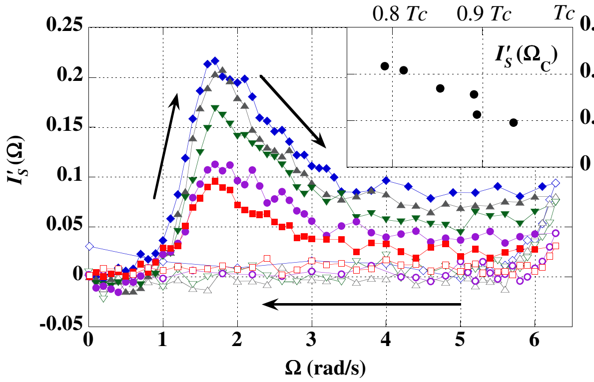

FIG. 5 (color online). The temperature dependence of $I'_s(\Omega)$ at 0.79 (blue diamonds), 0.81 (gray triangles), 0.85 (green inverted triangles), 0.89 (purple circles), and 0.93 T_c (red squares). All measurements were done after warming above T_c and cooling $H = \Omega = 0$ during the superfluid transition. The filled (open) symbols are taken in acceleration (deceleration). The inset shows $I'_s(\Omega_c)$ vs T/T_c.

the absence of macroscopic counterflow and attributed it to a scattering of spin waves from a soft core of vortices. There results demonstrate that vortices in parallel-plate geometry are quite different from vortices that have been detected in the *A* phase so far. The vortices do not have a soft core, and would be phase vortices with single circulation quantum or even half-quantum vortices [5].

One of unexpected results is that we need a strong reduction factor α to explain the size of the satellite signal. Since the length scale of spin waves of the *A* phase is ξ_D, the ratio of the FT's signal should be just given as $1 - (r_{\mathrm{Fr}}/R)^2$. Furthermore, to our surprise, measurements of $I'_s(\Omega)$ at different temperatures revealed that α has a temperature dependence and is seemed to disappear at T_c, while Ω_{Fr} and Ω_c are temperature independent as shown in Fig. 5. This is a quite different result from that of the bulk liquid [12], where the spin waves have a temperature-independent signal size because ξ_D is not varied by temperature. Further studies would be required to reveal the nature of this unknown dissipation and the temperature dependence.

Recently, another challenging interpretation for the satellite signal is given by Kee and Maki [6]. They suggest that half-quantum vortex (HQV) can form a bound pair at some rotation speed, and the pair HQV can trap a spin wave which has a very similar frequency shift as what we observed. They claim that what is happening above Ω_{Fr} in acceleration is not a FT but a pairing of HQVs, and the HQV pair are broken up for $\Omega > \Omega_c$ because they are less energy favorable in higher rotation speeds.

To test if HQV pairs can be formed, we cooled down the sample below T_c with a rotation speed of just below Ω_c to form no-counterflow status at a rotation speed expected to be favored by the HQV pair. An NMR spectrum taken by this way did not show any satellite signal, which supports the FT explanation, because any FT does not take place if there is no counterflow. However, a possibility that a pair of HQV is not formed just below T_c cannot be ruled out. Kee and Maki propose that applying a gradient field is a promising way to identify the bound pair of HQVs, which still remains as a future work.

In conclusion, the superfluid ^{3}He-*A* phase under rotation has been investigated in parallel-plate geometry of 12.5 μm gap. We found a new satellite signal under rotation and investigate the dependence on rotation and temperature. We interpret the satellite signal as a result of FT of $\hat{\ell}$ texture induced by rotation. The rotation dependence of the size of the satellite signal can be explained in such a way that FT takes place above Ω_{Fr} and vortices come in above Ω_c. We speculate that vortices in the parallel-plate geometry were singular vortices which do not have a soft core. Another possibility that we might observe HQV pairs is pointed out by Kee and Maki, which requires further studies of this system.

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