ac Magnetization Transport and Power Absorption in Nonitinerant Spin Chains

Björn Trauzettel,[1,2] Pascal Simon,[2,3] and Daniel Loss[2]

[1]*Institute for Theoretical Physics and Astrophysics, University of Würzburg, D-97074 Würzburg, Germany*

[2]*Department of Physics, University of Basel, CH-4056 Basel, Switzerland*

[3]*Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, B.P. 166, F-38042 Grenoble, France*

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We investigate the ac transport of magnetization in nonitinerant quantum systems such as spin chains described by the *XXZ* Hamiltonian. Using linear response theory, we calculate the ac magnetization current and the power absorption of such magnetic systems. Remarkably, the difference in the exchange interaction of the spin chain itself and the bulk magnets (i.e., the magnetization reservoirs), to which the spin chain is coupled, strongly influences the absorbed power of the system. This feature can be used in future spintronic devices to control power dissipation. Our analysis allows us to make quantitative predictions about the power absorption, and we show that magnetic systems are superior to their electronic counterparts.

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Power dissipation is one of the most important limitations of state-of-the-art electronic systems. The same is true for spintronic devices in which spin transport is accompanied by charge transport. In nonitinerant quantum systems, the dissipation problem is reduced, since true magnetization transport generates typically much less power than charge currents [1,2]. This is one of the main reasons for putting so much hope and effort into spin-based devices for future applications [3–5]. Here we analyze nonitinerant quantum systems described by a spin Hamiltonian in which ac magnetization transport occurs via magnons or spinons (without the transport of charge). In Ref. [6], the spin conductance of such a device has been derived. We generalize this theory to the response to an ac magnetization source. This allows us to directly calculate (and thus estimate) the power absorption of such magnetic systems at a given driving frequency ω using linear response theory. In general, the exchange coupling J in the spin chain and in the reservoirs will be different; see Fig. 1. It turns out that the difference of the exchange coupling plays a crucial role in the dependence of the absorbed power as a function of ω. The larger the difference, the stronger will be the suppression of power dissipation at finite frequencies. At low frequencies, however, the dissipative power is independent of the difference of the exchange couplings and takes a universal value determined by J in the reservoirs.

We analyze the ac transport problem in quantum spin chains by a mapping of the spin Hamiltonian coupled to magnetization reservoirs to the so-called inhomogeneous Luttinger liquid (LL) Hamiltonian [7–9]. Interestingly, the absorbed power that is derived in this Letter has an astonishingly simple dependence on the interaction parameters of the LL model; see Eq. (11) below. In order to describe the system shown in Fig. 1, we consider a one-dimensional *XXZ* spin chain in the presence of a time-dependent magnetic field $\mathbf{B}(\mathbf{x}_i, t) = B_i(t)\mathbf{e}_z$ which can be described by the Hamiltonian $H = H_{XXZ} + H_B(t)$, where

$$H_{XXZ} = J\sum_{\langle i,j\rangle}(s_{i,x}s_{j,x} + s_{i,y}s_{j,y} + \Delta s_{i,z}s_{j,z}), \quad (1)$$

$$H_B(t) = g_e\mu_B\sum_i B_i(t)s_{i,z}. \quad (2)$$

Here $s_{i,\alpha}$ is the α component of the spin operator at $\mathbf{x}_i$, $\langle i, j\rangle$ denotes nearest-neighbor sites, g_e is the g factor, μ_B is Bohr's magneton, and we assume antiferromagnetic coupling with $J, \Delta > 0$. A possible realization of spin chains described by H_{XXZ} is, for instance, a bulk structure of $KCuF_3$ or Sr_2CuO_3, where the exchange among different chains in the crystal is much weaker than the intrachain exchange [10–12]. The Hamiltonian H_{XXZ} can be mapped onto a LL of spinless fermions [13–15]

$$H_{\mathrm{LL}} = \frac{\hbar v}{2}\int dx\left[g[\Pi(x)]^2 + \frac{1}{g}[\partial_x\varphi(x)]^2\right], \quad (3)$$

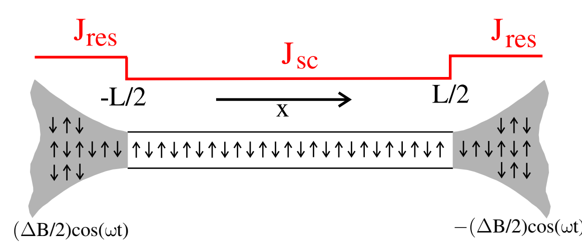

FIG. 1 (color online). Schematic of a quantum spin chain coupled to magnetization reservoirs. The magnetic field bias ΔB changes periodically in time. In the upper part of the figure, we illustrate that the exchange coupling in the spin chain J_{sc} (for $|x| < L/2$) can, in general, be different from the exchange coupling in the gray-shaded magnetization reservoirs J_{res}. As suggested in Ref. [6], this setup can be realized by a bulk material with an intrachain exchange much stronger than the interchain exchange, where the material is heated to a temperature $T > T_N$ in the central part and cooled to $T \ll T_N$ in the reservoir parts. (T_N is the 3D Néel ordering temperature.)

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where we have ignored umklapp scattering [16] and made the identifications $v = v_B/g$, $v_B = Ja\sin(k_B a)/\hbar$, and $g = (1 + 4\Delta/\pi)^{-1/2}$ (a is the lattice constant). In Eq. (3), $\varphi(x)$ is the standard Bose field operator in bosonization associated with spinon excitations here, $\Pi(x)$ its conjugate momentum density, v the spinon velocity, v_B the bare spinon velocity (at $\Delta = 0$), k_B the bare spinon wave vector, and g the interaction parameter ($g = 1$ corresponding to a noninteracting system, i.e., $\Delta = 0$, and, in general for a H_{XXZ} spin chain, $1/2 \leq g \leq 1$) [17,18].

In order to be able to properly describe the effect of reservoirs, we modify the Hamiltonian $H_{\rm LL}$ in the spirit of the inhomogeneous LL model [7–9] described by a Hamiltonian $H_{\rm ILL}$, where we assign a spatial dependence to v and g such that $v(x) = v_l$ and $g(x) = g_l$ are the spinon velocity and the interaction parameter in the reservoirs (for $|x| > L/2$), respectively, and $v(x) = v_w$ and $g(x) = g_w$ are the corresponding quantities in the spin-chain region (for $|x| < L/2$). Within this model, nonequilibrium transport phenomena such as the nonlinear $I - V$ characteristics and the current noise in the presence of impurities have been analyzed extensively [19–23]. Here we are interested in ac magnetization transport which should be seen complementary to the electric ac response analyzed in Refs. [24,25]. The Hamiltonian $H_B(t)$ describes a spatially varying and time-dependent magnetic field $\delta B(x)\cos(\omega t)\mathbf{e}_z$, with $\delta B(x) = -\Delta B/2$ ($\Delta B/2$) for $x < -L/2$ ($x > L/2$). For $|x| < L/2$, $\delta B(x)$ interpolates smoothly between the values $\pm\Delta B/2$ in the reservoirs [26]. The dc ($\omega = 0$) magnetization transport of such a system has been analyzed in Ref. [6], and a spin conductance $G_s = g_l(g_e\mu_B)^2/h$ has been predicted.

The magnetization current in linear response to an oscillating magnetic field can be evaluated using the following expression:

$$I_m(x,t) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dy \sigma_0(x,y,\tau)\partial_y \delta B(y, t-\tau) \quad (4)$$

with the spin conductivity

$$\sigma_0(x,y,\tau) = 2i\frac{(g_e\mu_B)^2}{h}\Theta(\tau)\partial_\tau\langle[\varphi(x,\tau),\varphi(y,0)]\rangle, \quad (5)$$

and the expectation value is taken with respect to $H_{\rm ILL}$. For $x > L/2$ and $|y| \leq L/2$, we obtain

$$\sigma_0(x,y,\tau) = g_l\frac{(g\mu_B)^2}{h}(1-\gamma)\Theta(\tau)\sum_{p=0}^{\infty}\sum_{\alpha=\pm}\bigg\{\gamma^{2p}\delta\bigg[\tau + \alpha\bigg(\frac{x-L/2}{v_l} + \frac{L/2-y}{v_w} + \frac{2pL}{v_w}\bigg)\bigg] + \gamma^{2p+1}\delta\bigg[\tau + \alpha\bigg(\frac{x-L/2}{v_l} + \frac{L/2+y}{v_w} + \frac{(2p+1)L}{v_w}\bigg)\bigg]\bigg\}, \quad (6)$$

where $\gamma = (g_l - g_w)/(g_l + g_w)$ is the reflection coefficient of spinon excitations at a sharp boundary with different interaction coefficients g_l and g_w [9] and $\Theta(\tau)$ the Heaviside function. The resulting spin current under continuous wave radiation reads

$$I_m^{(\rm cw)}(x,t) = 2(1-\gamma)\frac{g_l v_w}{\omega}\frac{(g_e\mu_B)^2}{h}\frac{\Delta B}{L}\sum_{p=0}^{\infty}\gamma^{2p}\sin\bigg(\frac{\omega L}{2v_w}\bigg)\bigg\{\cos\bigg[\omega\bigg(t - \frac{(2p+1/2)L}{v_w} + \frac{L-2x}{2v_l}\bigg)\bigg] + \gamma\cos\bigg[\omega\bigg(t - \frac{(2p+3/2)L}{v_w} + \frac{L-2x}{2v_l}\bigg)\bigg]\bigg\}. \quad (7)$$

We clearly observe an interaction dependence of the magnetization current in Eq. (7) through g_l and γ. The presence of higher harmonics due to higher order terms in γ^m would be a strong experimental evidence for the spatial inhomogeneity of spin-spin coupling in realizations of XXZ spin chains. The physics behind the result in Eq. (7) is the following one: The system is driven with a continuous wave due to the ac magnetization source; therefore, spinon excitations constantly enter and leave the spin chain from and to the reservoirs. Whenever they experience a boundary in the exchange interaction, they are partly transmitted and partly reflected with a reflection coefficient γ. The resulting expression (7) is the superposition of all possible contributions to the spin current after infinitely many reflection processes.

As a natural consequence, one may wonder whether an initial magnetization signal is actually transmitted through the spin chain. This depends crucially on the value of γ. To answer this question, we look at the magnetization current in linear response to a unit pulse described by $\partial_y\delta B(y,t) = \delta B_p\delta\tau_p\delta(t-t_0)\delta(y-y_0)$, with $y_0 \in [-L/2, L/2]$ (where δB_p corresponds to the height and $\delta\tau_p$ to the duration of the pulse). If we insert this expression into Eq. (4), we obtain for the spin current

$$I_m^{(\rm pul)}(x,t) = g_l\delta B_p\delta\tau_p\frac{(g_e\mu_B)^2}{h}(1-\gamma)\Theta(t-t_0)\sum_{p=0}^{\infty}\sum_{\alpha=\pm}\bigg\{\gamma^{2p}\delta\bigg(\tau + \alpha\bigg[\frac{x-L/2}{v_l} + \frac{L/2-y_0}{v_w} + \frac{2pL}{v_w}\bigg]\bigg) + \gamma^{2p+1}\delta\bigg(\tau + \alpha\bigg[\frac{x-L/2}{v_l} + \frac{L/2+y_0}{v_w} + \frac{(2p+1)L}{v_w}\bigg]\bigg)\bigg\}. \quad (8)$$

The form of $I_m^{(\mathrm{pul})}(x, t)$ shows that the initially sharp δ pulse is decomposed into a sum of infinitely many δ pulses. Importantly, the amplitude of these pulses decreases by a factor γ in a stepwise fashion once in each time interval L/v_w corresponding to the transit time in the wire. So, to answer the question of how much signal has been transmitted, we have to fix x, y_0, t_0, and γ in Eq. (8) and sum up all of the prefactors of the δ functions that can be nonzero in a given time interval between t_0 and t. This analysis implies that all of the dissipation happens in the leads and intrinsic relaxation is absent, which is related to the fact that the LL Hamiltonian describes a free boson [27]. All of our results in the absence of scatterers are temperature-independent and also hold for finite temperatures within the validity regime of our model, which is $k_B T \ll J$. In the presence of impurities, the situation is different. Then, intrinsic dissipation matters and temperature-dependent corrections due to impurity scattering arise; see below.

We now turn to the discussion of the power absorption under continuous wave radiation. We derive the absorbed power of the 1D spin chain using Fermi's golden rule and linear response theory. The resulting expression is

$$W(\omega) = \frac{1}{2}\left\{\int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dy \mathrm{Re}\sigma_0(x, y, \omega)\right\}\left|\frac{\Delta B}{L}\right|^2, \tag{9}$$

where

$$\mathrm{Re}\,\sigma_0(x, y, \omega) = g_w \frac{(g_e \mu_B)^2}{h}\left\{\cos[\tilde{\omega}(\tilde{x} - \tilde{y})] + \frac{2\gamma(1-\gamma^2)\cos(\tilde{\omega})\cos[\tilde{\omega}(\tilde{x}+\tilde{y})]}{1+\gamma^4 - 2\gamma^2\cos(2\tilde{\omega})} + \frac{2\gamma^2\cos[\tilde{\omega}(\tilde{x}-\tilde{y})][\cos(2\tilde{\omega}) - \gamma^2]}{1+\gamma^4-2\gamma^2\cos(2\tilde{\omega})}\right\}, \tag{10}$$

and we have introduced dimensionless variables $\tilde{x} = x/L$, $\tilde{y} = y/L$, and $\tilde{\omega} = \omega/\omega_L$, with $\omega_L = v_w/L$. It is straightforward to do the two remaining integrals in Eq. (9), and the final result reads

$$W(\omega) = g_w \frac{(g_e \mu_B \Delta B)^2}{2h}\left(\frac{\sin(\tilde{\omega}/2)}{\tilde{\omega}/2}\right)^2 \times \frac{1-\gamma^4 + 2\gamma(1-\gamma^2)\cos(\tilde{\omega})}{1+\gamma^4 - 2\gamma^2\cos(2\tilde{\omega})}. \tag{11}$$

This is the main result of our work. It demonstrates that a measurement of the absorbed power due to ac response of the quantum spin chain is a feasible way to measure interaction-dependent coefficients such as g_w and γ. In Fig. 2, we show the interaction dependence of the absorbed power $W(\omega)$. It demonstrates that stronger repulsive interactions inside the wire with respect to the leads suppress the dissipative power.

If we compare Eqs. (7) and (11), we observe an interesting finite-size effect, namely, that $W(\omega)$ vanishes as $[\sin(\tilde{\omega}/2)]^2$ close to $\tilde{\omega} \approx 2\pi$, whereas the leading contribution to $I_m^{(\mathrm{cw})}(x, t)$ vanishes only as $\sin(\tilde{\omega}/2)$ close to that driving frequency. Thus, the power absorption is more strongly suppressed than the magnetization current at frequencies close to $2\pi\omega_L$. This feature can be used in future devices to transfer data at special frequencies with low power dissipation. In the limit $\omega \to 0$, we obtain $W(0) = g_l(g_e\mu_B\Delta B)^2/2h$ corresponding to Joule heating, where $W(0) = I_m^2(\omega = 0)/(2G_s)$.

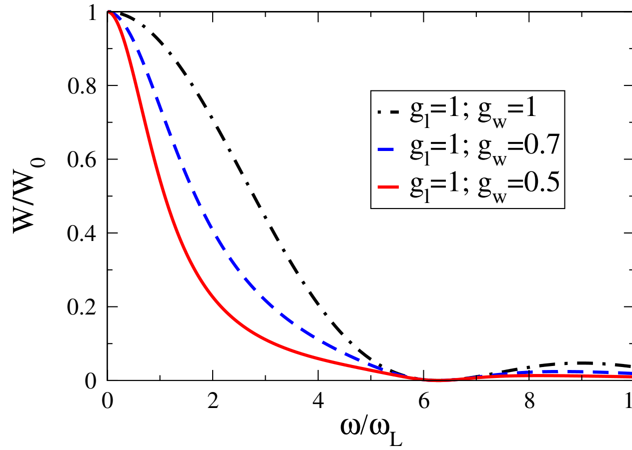

FIG. 2 (color online). The absorbed power is shown in units of $W_0 \equiv (g_e\mu_B\Delta B)^2/2h$ as a function of the ac frequency ω in units of the ballistic frequency $\omega_L = v_w/L$. It is clearly visible that stronger repulsive interactions inside the wire decrease the absorbed power in the system.

We now address the robustness of our main result (11) against impurity scattering. An impurity can be modeled as an altered link in the H_{XXZ} chain, i.e., a local change in J on a nearest-neighbor link [28,29]. Within bosonization, such a scatterer at position x_0 in the system can be written as $H_I = \lambda\cos[\sqrt{4\pi}\varphi(x_0, t) + 2k_B x_0]$. If one of the two energy scales $\hbar v_w/L$ or $\hbar\omega$ is larger than λ (the bare impurity strength), we can treat H_I perturbatively up to lowest nontrivial order (which is second order in λ). In the presence of impurity scattering, the spin conductivity that enters into the calculation of Eq. (9) is subject to a (small) correction $\sigma_I(x, y, \omega)$ given by Eq. (46) of Ref. [22]. For finite frequencies, $\sigma_I(x, y, \omega)$ needs to be evaluated numerically. In the zero frequency limit, we find power-law corrections to the spin conductance (see Refs. [30,31] for the corresponding electric case) resulting in power-law corrections to the absorbed power. As a result, as long as $\hbar v_w/L$ or $\hbar\omega$ is larger than λ, the effect of impurity scattering is negligibly small.

The system which we considered previously consists of a spin chain smoothly connected to reservoirs. One may wonder how the previous result gets modified for isolated finite-size spin chains, to which a time-dependent oscillating magnetic field is applied along the chain [such that $dB(x, t)/dx = \Delta B\cos(\omega t)$]. For long Heisenberg chains, H_{XXZ} still maps onto a LL of spinless fermions as in Eq. (3) but with open boundary conditions (OBC). Using the formalism described in Ref. [32], we find after lengthy but straightforward calculations that the real part of the conductivity for such an isolated system is given by $\mathrm{Re}\sigma_0^{\mathrm{OBC}}(x, y, \omega) = 2g_w \frac{(g_e\mu_B)^2}{h} \sin(\omega x/v_w)\sin(\omega y/v_w)$ for $\omega = \omega_n \equiv \pi n v_w/L$ $[n = 1, \ldots, (L-a)/a]$ and 0 otherwise. From this expression, we obtain the power needed to periodically "shake" the spin-chain excitations, using Eq. (9),

$$W_{\mathrm{OBC}}(\omega) = g_w \frac{(g_e\mu_B\Delta B)^2}{h}\left(\frac{\sin(\tilde{\omega}/2)}{\tilde{\omega}/2}\right)^2 \sin^2(\tilde{\omega}/2) \quad (12)$$

for $\tilde{\omega} = \omega_n/\omega_L$ and 0 otherwise. Note that this power cannot be identified as dissipative power because a disconnected LL does not contain a dissipative term. This is the major difference from the case with leads, i.e., Eq. (11), where dissipation occurs in the reservoirs.

Let us now compare typical values for the absorbed power in electric systems versus magnetic systems. We set $g_l = g_w = 1$ for simplicity but keep in mind how finite interactions change the power absorption according to Eq. (11). The absorbed electric power in the dc limit is given by $W_{\mathrm{el}} = (e\Delta V)^2/h$. For a typical electric bias of $\Delta V = 1$ mV, we obtain $W_{\mathrm{el}} \approx 3.87 \times 10^{-11}$ J s^{-1}, whereas the absorbed magnetic power for a typical magnetic bias of $\Delta B = 0.1$ T is $W \approx 2.59 \times 10^{-15}$ J s^{-1} (assuming $g_e = 2$) which is 4 orders of magnitude smaller. The rule of the thumb is $W_{\mathrm{el}}(\Delta V = 0.1\ \mathrm{mV}) \sim W(\Delta B = 1\ \mathrm{T})$. Thus, we expect substantial advantages of magnetic systems versus electric systems as far as power consumption is concerned.

In summary, we have analyzed magnetization current and power absorption of quantum spin chains coupled to magnetization reservoirs with a time-dependent magnetic field applied to the reservoirs. Both quantities depend crucially on the difference of the exchange interactions within the wire as compared to the magnetization leads. In fact, we envision to use this dependence as a way to control power dissipation in nonitinerant quantum systems in which magnetization transport occurs via spinons.

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