Measurement-Based Quantum Computer in the Gapped Ground State of a Two-Body Hamiltonian

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We propose a scheme for a ground-code measurement-based quantum computer, which enjoys two major advantages. First, every logical qubit is encoded in the *gapped* degenerate ground subspace of a spin-1 chain with nearest-neighbor *two-body* interactions, so that it equips built-in robustness against noise. Second, computation is processed by single-spin measurements along multiple chains dynamically coupled on demand, so as to keep teleporting only logical information into a gap-protected ground state of the residual chains after the interactions with spins to be measured are turned off. We describe implementations using trapped atoms or polar molecules in an optical lattice, where the gap is expected to be as large as 0.2 or 4.8 kHz, respectively.

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Introduction.—Reliable quantum computers require hardware with low error rates and sufficient resources to perform software-based error correction. One appealing approach to reduce the massive overhead for error correction is to process quantum information in the *gapped* ground states of some many-body interaction. This is the tactic used in topological quantum computation and adiabatic quantum computation. Yet, the hardware demands for the former are substantial, and the fault tolerance of the latter, especially when restricted to two-body interactions, is unclear [1]. On the other hand, measurement-based quantum computation (MQC), in particular one-way computation on the 2D cluster state [2], runs by beginning with a highly entangled state dynamically generated from nearest-neighbor two-body interactions and performing computation by only single-qubit measurements and feed forward of their outcomes. However, its bare implementation may suffer decoherence of physical qubits waiting for their round of measurements in the far future, and that severely damages a prominent capability to parallelize computation. Although its fault-tolerant method by error correction has been well established [3], it is clearly advantageous if some gap protection is provided on the hardware level.

In this Letter, we propose a ground-code measurement-based quantum computer (GMQC), which enjoys the two aforementioned advantages. GMQC is a conceptual advance, since measurements generally create excitations in the system so that two desired properties, keeping the information in the ground state and processing the information by measurements, are not seemingly compatible. We demand three properties of the ground state for GMQC. (i) There should be an energy gap to penalize errors moving outside the computational ground subspace, and operators connecting logical states should be highly nonlocal. These should persist in the thermodynamic limit to be scalable. (ii) The interactions should be preferably two-body. It is possible that in some ground subspace of H the effective interaction is K-local, but a demerit is the significantly reduced magnitude of the perturbative coupling. (iii) The interactions should be frustration-free, so that when every single spin is measured through computation, the remaining entangled spins to contain logical information can be set in the ground state of their Hamiltonian.

We briefly refer to entangled resource states universal for MQC in the literature. First, the idea to use a ground state of the two-body Hamiltonian for universal MQC is seen in Ref. [4], where the effective five-body Hamiltonian for the 2D cluster state is perturbatively approximated in a low energy sector using ancillas. We do not resort to perturbation, so that not only is the gap much larger in practice, but also the resource state is exact in that we could approach unit fidelity as close as possible by improving accuracy of analog simulation of our Hamiltonian and its preparation to the ground state. Second, the novel use of tensor network states beyond the 2D cluster state [5,6] has been quite motivating for us. But, our key idea of processing the logical information in degenerate ground states while maintaining the Hamiltonian on is incorporated for the first time in our Letter.

Scheme of a ground-code MQC.—In our GMQC, sketched in Fig. 1, we adopt a hybrid approach where the logical two-qubit gates are implemented via dynamical couplings on demand as in the quantum circuit model, in addition to the standard MQC for the time evolution of each logical qubit. This is partly because there has not been known any exact gapped ground state of a two-body Hamiltonian which *per se* serves as a universal resource for the standard MQC. We utilize space-time resources in such a joint way that a "spatially" entangled resource is used to simulate the logical time evolution and "temporal" interaction is used to simulate the logical spatial interactions. Consequently, our GMQC exhibits some new features in contrast to the conventional MQC.

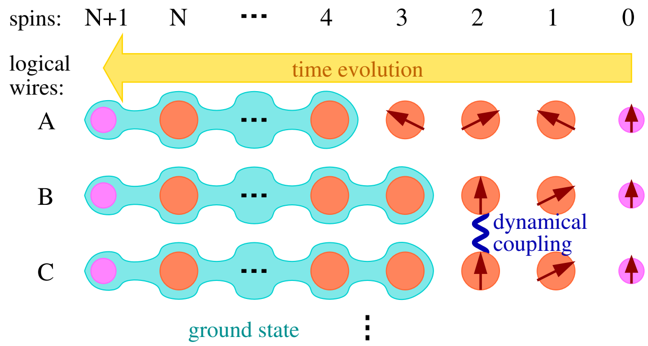

FIG. 1 (color online). A schematic of a ground-code measurement-based quantum computer. Each logical qubit is encoded in every AKLT chain of N spin-1 particles with spin-$1/2$ at the boundaries 0 and $N+1$. Computation is carried by measurements right to left, in processing the logical information in the twofold degenerate, gapped ground state of each residual chain.

The basic Hamiltonian we consider is the 1D Affleck-Kennedy-Lieb-Tasaki (AKLT) model [7], the chain of nearest-neighbor two-body interacting spins, $H = J[\sum_{j=1}^{N-1} P^2_{j,j+1} + P^{3/2}_{0,1} + P^{3/2}_{N,N+1}]$, with $J > 0$ and $P^S_{j,j+1}$ the projector onto the spin-S irreducible representation of the total spin for particles j and $j+1$. Namely, $P^2_{j,j+1} = \frac{1}{2}(\mathbf{S}_j \cdot \mathbf{S}_{j+1} + \frac{1}{3}(\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2) + \frac{\mathbf{1}_9}{3}$ and $P^{3/2}_{j,j'} = \frac{2}{3} \times (\mathbf{1}_6 + \mathbf{s}_j \cdot \mathbf{S}_{j'})$, where $\mathbf{S}, \mathbf{s}$ are spin-1, $1/2$ representations of $\mathfrak{su}(2)$. Without the boundary spins, the finite AKLT spin chain has a fourfold degeneracy corresponding to a total spin-0 (singlet) state and a triplet of spin-1 states. The boundary terms $P^{3/2}$ project out the spin-1 components yielding a unique ground state such that $H|\mathcal{G}\rangle = 0$. The gap ΔE persists in the thermodynamic limit, and has been estimated to be $\Delta E \approx 0.350J$ [8]. A key feature of $|\mathcal{G}\rangle$ is that it can serve, using *single-spin* measurements only, as a logical quantum wire which is capable of performing not only deterministic teleportation as already remarked in Ref. [9] but also an arbitrary logical single-qubit operation similarly to one-way computation as shown below.

The ground state $|\mathcal{G}\rangle$ has a convenient matrix product state (MPS) representation [9,10]. Let us define $|1_j\rangle = \frac{-1}{\sqrt{2}} \times (|S^z_j = 1\rangle - |S^z_j = -1\rangle)$, $|2_j\rangle = \frac{1}{\sqrt{2}}(|S^z_j = 1\rangle + |S^z_j = -1\rangle)$, and $|3_j\rangle = |S^z_j = 0\rangle$, in terms of the three eigenstates of S^z_j,

$$|\mathcal{G}\rangle = \sum_{\{\alpha_j\}=1}^{3} \frac{|\alpha_1\rangle \ldots |\alpha_N\rangle}{\sqrt{3^N}} \left[\mathbf{1}_2 \otimes \prod_{j=N}^{1} \langle \alpha_j | M_j \rangle \right] |\Psi^-_{0,N+1}\rangle, \tag{1}$$

where $|M_j\rangle = X|1_j\rangle - iY|2_j\rangle + Z|3_j\rangle$ [X, Y, Z are the Pauli matrices $\sigma^\mu (\mu = x, y, z)$], and $|\Psi^-\rangle$ is the singlet ($S = 0$) located on the 0th and $(N+1)$th sites. This representation is helpful to see the action of local measurements to the "relative" state of unmeasured parts in simulating unitary evolution of MQC [5,6,11]. Note that, in Refs. [5,6], the ground state of a modified AKLT chain, different from the original, was considered to construct a resource state. However, their extension of the so-called by-product operators into a non-Pauli finite group is less convenient, and it is unclear whether it still possesses a counterpart of the nonlocal string order utilized later.

Universal computation by measurements.—Computation follows by measuring right to left the single spins along each chain, where spins are indexed by increasing value moving right to left, in accordance with the order of unitaries we simulate in the bra-ket notation. First we need to prepare the unique ground state $|\mathcal{G}\rangle^{\otimes n}$ of the n parallel decoupled 1D AKLT chains. This can be done efficiently and deterministically along each chain, by either turning on H immersing the system in a reservoir and cooling it, or by making use of its MPS description to produce it via sequential unitaries (which scale linearly in N) [12] before turning on H.

The initialization of every quantum logical wire is done by first turning off the coupling $P^{3/2}_{0,1}$ and measuring the rightmost 0th spin-$1/2$ in the $\hat{z}$ basis. Because of the singlet configuration $|\Psi^-_{0,N+1}\rangle$, the $-1/2$ and $1/2$ outcome, denoted by $|1_0\rangle$ and $|0_0\rangle$, respectively, at the 0th site $\hat{z}$-basis measurement induces the initialization of the wire [identified effectively with the preparation of the state at the $(N+1)$th site, since we will see that unitary actions accumulate on this degree of freedom] to $|0^L\rangle$ and $|1^L\rangle$, respectively. With one boundary spin-$1/2$, the ground state is twofold degenerate spanned by $\langle 1_0|\mathcal{G}\rangle$ and $\langle 0_0|\mathcal{G}\rangle$, and computation takes place in this ground subspace after this initialization.

Every logical single-qubit unitary operation is implemented by the single-spin measurement. The interaction $P^2_{j,j+1}$ is turned off before the local measurement of the jth spin, to guarantee that the remaining system stays in the ground state. Since an arbitrary single-qubit operation is decomposed into three rotations around the logical Z and X axes with three Euler angles, we show their measurement directions. The rotation $R^Z(\theta) = |0^L\rangle\langle 0^L| + e^{i\theta}|1^L\rangle\langle 1^L|$ along the Z axis is applied by the single-spin measurement in an orthogonal basis,

$$\{|\gamma^Z_j(\theta)\rangle\} = \{\tfrac{1}{2}[(1 \pm e^{-i\theta})|1_j\rangle + (1 \mp e^{-i\theta})|2_j\rangle], |3_j\rangle\}. \tag{2}$$

If the outcome is either the first or the second (which turns out to occur always with probability $1/3$ for each), we can apply $R^Z(\theta)$ newly on the logical qubit with a by-product operator X or XZ, respectively. If the outcome is the third, we interpret we have applied the "logical identity" with a by-product Z. On the other hand, the rotation $R^X(\theta) = |+^L\rangle\langle +^L| + e^{i\theta}|-^L\rangle\langle -^L|$ along the X axis, where $|\pm^L\rangle = \frac{1}{\sqrt{2}}(|0^L\rangle \pm |1^L\rangle)$, is applied by the single-spin measurement in another orthogonal basis,

$$\{|\gamma^X_j(\theta)\rangle\} = \{\tfrac{1}{2}[(1 \pm e^{i\theta})|2_j\rangle + (1 \mp e^{i\theta})|3_j\rangle], |1_j\rangle\}. \tag{3}$$

If the outcome is either the first or the second, we apply

$R^X(\theta)$ with a by-product XZ or Z, respectively, and otherwise the logical identity with a by-product X.

Suppose we initialize the logical wire $|0^L\rangle$ by the 0th site $-1/2$ outcome (otherwise, we consider the wire is initialized $|0^L\rangle$ with the by-product X from the beginning). According to Eq. (1), before the jth spin is measured, we have a state $|\psi(j)\rangle = \langle 1_0|\prod_{k'=j-1}^{1}\langle\gamma_{k'}|\mathcal{G}\rangle$ given by

$$\sum_{\{\alpha_k\}=1}^{3}\frac{|\alpha_j\rangle\ldots|\alpha_N\rangle}{\sqrt{3^{N-j+1}}}\Big[\prod_{k=N}^{j}\langle\alpha_k|M_k\rangle\Upsilon\prod_{k'=j-1}^{1}R(\theta_{k'})\Big]|0^L\rangle, \quad (4)$$

where the measurement directions of $|\gamma_{k'}(\theta_{k'})\rangle$ must be adapted from $\theta_{k'}$ to $-\theta_{k'}$ when noncommuting by-products from previous measurements are propagated left through the current one, resulting in an accumulated by-product operator Υ.

We describe important properties of the residual Hamiltonian $H(j) = J[\sum_{k=j}^{N-1}P^2_{k,k+1} + P^{3/2}_{N,N+1}]$ through the measurement stage of the jth spin. First, $H(j)$ is gapped as before and is twofold degenerate. Defining the string operators $\Sigma^\mu(j) = e^{i\pi\sum_{k=j}^{N}S_k^\mu}\otimes\sigma^\mu_{N+1}$, we find that $[\Sigma^\mu(j), H(j)] = 0$ whereas $\{\Sigma^x(j), \Sigma^z(j)\}_+ = 0$. At each stage j, the pair $\Sigma^{x,z}(j)$ forms a representation of $\mathfrak{su}(2)$, and the degenerate ground states are only connected by nonlocal operators. For every single quantum wire, we utilize "time-dependent" logical encoding such that $\langle\psi(j)|\Sigma^\mu(j)|\psi(j)\rangle = \langle 0_{N+1}|u^\dagger\Upsilon^\dagger\sigma^\mu_{N+1}\Upsilon u|0_{N+1}\rangle$, where u is the total single-qubit rotation until the $(j-1)$th gate.

Second, the logical state is not disturbed when turning off the interaction coupling the bulk to the jth spin. We can decouple with the time-dependent Hamiltonian $H(j;t) = J(1-c(t))P^2_{j,j+1} + H(j+1)$, where $c(t)$ is monotonically increasing in $t\in[0,1]$ with $c(0)=0$, $c(1)=1$. Now $P^2_{j,j+1}$ does not commute with $H(j+1)$. However, the AKLT Hamiltonian has the property that the ground states also minimize its positive summands; i.e., it is frustration-free [7]. Hence, $P^{\rm gr}(0) = P^{\rm gr}(t)\ \ \forall\, t<1$ and $\frac{\partial^\nu H(j;t)}{\partial t^\nu}P^{\rm gr}(t) = 0\ \forall\ \nu, t$, where $P^{\rm gr}(t) = |\mathcal{G}(t)\rangle\langle\mathcal{G}(t)|$ are projectors onto the ground subspaces of $H(j;t)$. Thus, turning off the end interaction term does not couple to excited states, and can be done in a constant time independent of the system size. Even if there are some unwanted initial perturbations on $H(j;t)$, the gap provides robustness if performed adiabatically. The same argument applies to turning off the boundary terms.

Third, notice that $|\psi(j+1)\rangle$ of Eq. (4) can be written for the general outcome $|r_0\rangle$ $(r = 0, 1)$ of the 0th site measurement as $\frac{1}{\sqrt{3^{N-j}}}\sum_{\{\alpha_k\}=1}^{3}|\alpha_{j+1}\rangle\ldots|\alpha_N\rangle\langle r_0|[V^\dagger\otimes\prod_{k=N}^{j+1}\langle\alpha_k|M_k\rangle]|\Psi^-_{0,N+1}\rangle$, where V is the form of $\tilde{\Upsilon}\prod_{k'=j}^{1}R(\tilde{\theta}_{k'})$ due to the invariance of $|\Psi^-\rangle$ under the bilateral unitaries. But this is equivalent to the state obtained by beginning in the unique ground state of $H(j+1) + P^{3/2}_{0,j+1}$, turning off $P^{3/2}_{0,j+1}$, measuring the 0th spin in the basis $\{V|r_0\rangle, VX|r_0\rangle\}$, with the result $V|r_0\rangle$. This state is in the ground subspace of $H(j+1)$.

Logical two-qubit operations are made *dynamically* by coupling two spin-1 particles in adjacent chains, say A and B, followed by their local measurements (equivalently, by a two-spin measurement). First $P^2_{A_j,A_{j+1}}$ and $P^2_{B_j,B_{j+1}}$ are turned off. We introduce the physical interaction $\exp(iH^{\rm int}\pi/\chi)$ between spins A_j and B_j, where $H^{\rm int} = \chi|S^z_{A_j}=1\rangle\langle S^z_{A_j}=1|\otimes|S^z_{B_j}=1\rangle\langle S^z_{B_j}=1|$, and measure both spins A_j and B_j in the standard basis $\{|1\rangle, |2\rangle, |3\rangle\}$. If both outcomes are in either $|1\rangle$ or $|2\rangle$, which occurs with overall probability $(2/3)^2 = 4/9$, we successfully apply the logical controlled-phase gate $\text{CPHASE}_{A,B} = \mathbf{1}_4 - 2|1^L_A1^L_B\rangle\times\langle 1^L_A1^L_B|$. Notice that, in the span by $|1_{A_j}1_{B_j}\rangle$, $|1_{A_j}2_{B_j}\rangle$, $|2_{A_j}1_{B_j}\rangle$, and $|2_{A_j}2_{B_j}\rangle$, $\exp(iH^{\rm int}\pi/\chi)$ acts as $\Gamma = \mathbf{1}_4 - \frac{1}{2}\times(\mathbf{1}_2 - X)\otimes(\mathbf{1}_2 - X)$ and as the identity elsewhere. We see that this induces $\langle\alpha_{A_j}|\otimes\langle\beta_{B_j}|\Gamma|M_{A_j}\rangle\otimes|M_{B_j}\rangle = \Upsilon\text{CPHASE}_{A,B}$ with the by-product $\Upsilon = XZ\otimes XZ$, $XZ\otimes X$, $X\otimes XZ$, or $X\otimes X$ in the aforementioned span. Otherwise (i.e., if at least one outcome is in $|3\rangle$), we end up with applying the logical identity with the by-product $\langle\alpha_{A_j}|M_{A_j}\rangle\otimes\langle\beta_{B_j}|M_{B_j}\rangle$.

To prove that computation is kept in the ground subspace, imagine that we began in a separable state of two chains A and B initialized in $|0^L\rangle$. After CPHASE is successfully applied at the stage j and by-products for A and B are propagated, the joint state involves newly the operators $(\Upsilon_A\otimes\Upsilon_B)\frac{1}{2}(\mathbf{1}_4 + Z_{A_{N+1}} + Z_{B_{N+1}} - Z_{A_{N+1}}Z_{B_{N+1}})$, so that it is nothing but a superposition of logical states each of which is in the kernel of $[H_A(j+1) + H_B(j+1)]$ and hence is in the ground subspace of the two chains.

It can be verified that for any quantum circuit realized with our universal set of gates, the probability for each successful single-, two-qubit gate is constant at 2/3, 4/9, respectively. We can efficiently perform the entire computation, by trying every gate until success, at the same time deterministically teleporting (by the standard basis measurement) other logical qubits to be spatially aligned for subsequent two-qubit gates. A remarkable new feature is that the general single-, two-qubit operations are probabilistic, while the logical identity (teleportation) is essentially deterministic with the adaptive measurements. This variable computational depth originates from the "correlated logical times" due to a two-point spatial correlation of the AKLT chain which decays as $(-1/3)^{|j-j'|}$ between sites j and j'.

At the end of computation, the joint state of leftmost boundary spins is $\Upsilon U|0^L\rangle^{\otimes n}$, where Υ is the total by-product and U the target unitary operator. The final logical measurement, without loss of generality in the computational basis, is simulated by measuring in $\hat{z}$ these spins (after teleporting the logical information if the chain is redundant). There, for example, the logical $|0^L\rangle$ outcome must be considered to occur if either $s^z_{N+1} = 1/2$ and Υ does not contain X or $s^z_{N+1} = -1/2$ and Υ contains X.

Physical implementations.—While the scheme of GMQC works independent of implementations, we sketch two physical realizations. In Refs. [13,14] it was shown how to obtain the AKLT model (without the boundary terms) using tunneling induced collisions between bosonic atoms trapped one per site in a 3D optical lattice. The lattice beams are chosen such that the confinement is very strong along $\hat{x}, \hat{y}$ directions but weaker along $\hat{z}$, so that the system describes noninteracting 1D chains. The spin states correspond to $F = 1$ hyperfine ground states of an alkali atom, and the effective coupling scales like $J \sim t^2/U_S$ where t is the tunneling matrix element and U_S ($S = 0, 2$) is a total spin dependent repulsive s-wave scattering energy in a common well. For realistic lattice parameters $U_S \approx 5$ kHz and for $t/U_S = 0.3$, we expect the gap $\Delta E \approx 0.2$ kHz. A measurement on spin A_j is achievable by adiabatically expanding the lattice along $\hat{z}$ using the accordion technique [15] which decreases t, U_S but also allows the lattice wells within each wire to be better addressed. Then turning on optical tweezers [16] near the site A_j, one can shift the potential depth at A_j so that tunneling to the neighboring wells is effectively zero. State-dependent measurements can be done using spatially resolving microwave or Raman fields. To perform the CPHASE gate between A_j, B_j with wires separated along $\hat{y}$, after turning off the interactions apply Raman pulses resonant at those locations that map $|S^z = 1\rangle$ to the first vibrational state of each well. Introducing a second lattice along $\hat{y}$ with twice the period of the first as in Ref. [14], the intensity can be adjusted so that the first excited vibrational states of nearest-neighbor wells interact to generate a tunneling phase gate [17]. The second lattice intensity can then be tuned to turn off tunneling and the states mapped back to vibrational ground states to commence with measurement. To engineer the boundary terms, one could try to use different species on the boundaries with interactions satisfying $\tilde{P}^{3/2}_{j,j'} = \frac{1}{3}(3\mathbf{1}_9 + 2\tilde{\mathbf{s}}_j \cdot \mathbf{S}_{j'} - S^{z2}_j)$, where $\tilde{s}^\mu$ acts on in the basis $\{|S^z = 1\rangle, |S^z = -1\rangle\}$ and zero elsewhere.

An alternative is trapped spin-1 polar molecules with microwave-induced dipole-dipole interactions instead of tunneling along $\hat{z}$. In Ref. [18], it was shown that the AKLT spin lattice model can be realized with $J \approx 13.7$ kHz ($\Delta E \approx 4.8$ kHz). Boundaries can be loaded with spin-1/2 species molecules using an additional optical lattice, and the necessary interactions could be designed using spectroscopic resolvability for the microwave coupling fields between the different species. Interactions can be turned off by expanding the lattice along $\hat{z}$ while tuning the microwave fields to keep the same interactions but with reduced strength. Then a rightmost molecule can be moved away from the bulk using optical tweezers and measured using state-dependent photoionization. The CPHASE gates can be generated between a pair of molecules in adjacent wires by outcoupling them using optical tweezers and using a microwave field to map $|S^z = 1\rangle \rightarrow |S^z = 0\rangle$. Then $H^{\rm int}$ is available using a single microwave field to induce the interaction $(\mathbf{1}_9 - S^{z2})^{\otimes 2}$ [18], and measuring the spins in this new basis. In both implementations, the initial ground state can be prepared efficiently by adiabatically increasing the interaction of H from a configuration with antiparallel spins induced by a staggered magnetic field as in Ref. [14].

Conclusion.—We have described a scheme to perform MQC entirely within the gapped ground state of quantum many-body system with a two-body interaction. For ground state protection, it is vital that the physical spin states are *degenerate* energy levels [19], meaning that local amplitude and phase errors are equally likely. The constant energy gap ΔE protects against noise whose spectral weight is smaller than it, providing a mechanism for anti-aging of the computational resource.

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