Fast Equilibration of Hadrons in an Expanding Fireball

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Because of long chemical equilibration times for standard hadronic reactions in a hadron gas in relativistic heavy ion collisions, it was suggested that hadrons are born into equilibrium after the quark gluon plasma is formed. We develop a dynamical scheme, using master equations, in which Hagedorn states contribute to fast chemical equilibration times of baryons and kaons, just below the critical temperature, estimates of which are derived analytically. The hadrons quickly equilibrate for an initial over- or underpopulation of Hagedorn states. Our particle ratios compared to BNL Relativistic Heavy Ion Collider show a close match.

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(Anti-)strangeness enhancement was first observed at CERN-SPS energies by comparing antihyperons, multistrange baryons, and kaons to pp-data. It was considered a signature for quark gluon plasma (QGP) because, using binary strangeness production and exchange reactions, chemical equilibrium could not be reached within the hadron gas phase [1]. It was then proposed that there exists a strong hint for QGP at SPS because strange quarks can be produced more abundantly by gluon fusion, which would account for strangeness enhancement following hadronization and rescattering of strange quarks. Later, multimesonic reactions were used to explain secondary production of antiprotons and antihyperons [2,3]. At SPS, they give a typical chemical equilibration time $\tau_{\bar{Y}} \approx 1 - 3\,\frac{\text{fm}}{c}$ using an annihilation cross section of $\sigma_{\rho\bar{Y}} \approx \sigma_{\rho\bar{p}} \approx 50$ mb and a baryon density of $\rho_B \approx \rho_0$ to $2\rho_0$, which is typical for SPS. Therefore, the time scale is short enough to account for chemical equilibration within a cooling hadronic fireball at SPS.

A problem arises when the same multimesonic reactions were employed in the hadron gas phase at RHIC temperatures where experiments show that the particle abundances reach chemical equilibration close to the phase transition [4]. At RHIC at $T = 170$ MeV, where $\sigma \approx 30$ mb and $\rho_B^{\text{eq}} \approx \rho_{\bar{B}}^{\text{eq}} \approx 0.04\ \text{fm}^{-3}$, the equilibrium rate for (anti-)baryon production is $\tau \approx 10\,\frac{\text{fm}}{c}$, which is considerably longer than the fireball's lifetime in the hadronic stage of $\tau < 5\,\frac{\text{fm}}{c}$. Moreover, $\tau \approx 10\,\frac{\text{fm}}{c}$ was also obtained in Ref. [5] using a fluctuation-dissipation theorem, and a significant deviation was found in the population number of various (anti-)baryons from experimental data in the 5% most central Au-Au collisions [6]. These discrepancies suggest that hadrons are "born" into equilibrium; i.e., the system is already in a chemically frozen out state at the end of the phase transition [7,8].

In order to circumvent such long time scales, it was suggested that near T_c there exists an extra large particle density overpopulated with pions and kaons, which drive the baryons or antibaryons into equilibrium [9]. But it is not clear how this overpopulation should appear, and how the subsequent population of (anti-)baryons would follow. Moreover, the overpopulated (anti-)baryons do not later disappear [10]. Therefore, it was conjectured that Hagedorn resonances (heavy resonances near T_c with an exponential mass spectrum) could account for the extra (anti-)baryons [10]. Baryon-antibaryon [10,11] and kaon-antikaon production develop according to

$$n\pi \leftrightarrow HS \leftrightarrow n'\pi + B\bar{B}, \quad n\pi \leftrightarrow HS \leftrightarrow n'\pi + K\bar{K}, \tag{1}$$

which provide an efficient method for producing baryons and kaons because of the large decay widths of the Hagedorn states. In Eq. (1), n and n', which can vary, are the number of pions for the decays $HS \leftrightarrow n\pi$ and $HS \leftrightarrow n'\pi + B\bar{B}$ (or $K\bar{K}$), respectively. Since Hagedorn resonances are highly unstable, the phase space for multiparticle decays drastically increases when the mass is increased. Therefore, the resonances catalyze rapid equilibration of baryons and kaons near T_c where the Hagedorn states show up. Here, we use a Bjorken expansion within a cooling fireball in order to see at which temperature the chemical equilibrium values are reached or maintained. In this Letter, we also briefly discuss an analytical solution of the chemical equilibration time, which is valid at a constant temperature near T_c. Moreover, our numerical results for the baryon-antibaryon pairs and kaon-anitkaon pairs suggest that the hadrons can, indeed, be born *out* of equilibrium.

We use a truncated Hagedorn mass spectrum [12]

$$g(m) = \int_{M_0}^{M} \frac{A}{[m^2 + (m_0)^2]^{(5/4)}} e^{(m/T_H)} dm \tag{2}$$

where the Hagedorn temperature is set to $T_H = 180$ MeV, which lies within the present range of Lattice QCD predictions [13], the normalization factor is $A = 0.5\,\text{MeV}^{(3/2)}$, and $m_0 = 0.5$ GeV. We consider only mesonic, nonstrange resonances and discretize the spectrum into mass bins of

100 MeV that range from the mass $M_0 = 2$ GeV to $M = 7$ GeV.

The abundances' evolution of the Hagedorn states, pions, and baryon-antibaryon pairs due to the reactions in Eq. (1) are described by the following rate equations

$$\begin{aligned}\dot{\lambda}_i &= \Gamma_{i,\pi}\Big(\sum_{n=2}^{\infty} B_{i,n}\lambda_\pi^n - \lambda_i\Big) + \Gamma_{i,B\bar{B}}\big(\lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2 - \lambda_i\big),\\ \dot{\lambda}_\pi &= \sum_i \Gamma_{i,\pi}\frac{N_i^{\rm eq}}{N_\pi^{\rm eq}}\Big(\lambda_i\langle n_i\rangle - \sum_{n=2}^{\infty} B_{i,n} n\lambda_\pi^n\Big)\\ &\quad + \sum_i \Gamma_{i,B\bar{B}}\langle n_{i,b}\rangle\frac{N_i^{\rm eq}}{N_\pi^{\rm eq}}\big(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\big),\\ \dot{\lambda}_{B\bar{B}} &= \sum_i \Gamma_{i,B\bar{B}}\frac{N_i^{\rm eq}}{N_{B\bar{B}}^{\rm eq}}\big(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\big),\end{aligned} \tag{3}$$

where the fugacity is $\lambda = \frac{N}{N^{\rm eq}}$, N is the total number of each particle, and its respective equilibrium value is $N^{\rm eq}$. The summation over i represents the $i^{\rm th}$ Hagedorn resonance bin. The structure of the rate equations for the kaon-antikaon pairs is the same as in Eq. (3); however, $K\bar{K}$ is substituted for $B\bar{B}$.

The branching ratios for $HS \leftrightarrow n\pi$ are described by a Gaussian distribution $B_{i,n} \approx \frac{1}{\sigma_i\sqrt{2\pi}} e^{-[(n-\langle n_i\rangle)^2/2\sigma_i^2]}$ where $\langle n_i\rangle = 0.9 + 1.2\frac{m_i}{m_p}$ is the average pion number each Hagedorn state decays into, found in a microcanonical model [14], $\sigma_i^2 = (0.5\frac{m_i}{m_p})^2$ is the chosen width of the distribution, and $n \geq 2$ is the cutoff for the pion number. Moreover, the branching ratios are normalized such that $\sum_{n=2}^{\infty} B_{i,n} = 1$, which gives $\langle n_i\rangle \approx 3$ to 9 and $\sigma_i^2 \approx 0.8$ to 11. The total decay width, $\Gamma_i \approx 0.15 m_i - 58$ MeV, which ranges from $\Gamma_i = 250$ to 1000 MeV, was found using the mass and decay widths in [15] and fitting them linearly similarly to what was done in Ref. [16]. The decay widths for the baryon-antibaryon decay are $\Gamma_{i,B\bar{B}} = \langle B\rangle\Gamma_i$ and $\Gamma_{i,\pi} = \Gamma_i - \Gamma_{i,B\bar{B}}$. The average baryon number $\langle B\rangle$ per unit decay of Hagedorn resonances within a microcanonical model ranges from 0.06 to 0.4, so $\Gamma_{i,B\bar{B}} = 15$ to 400 MeV [10]. We use only the average values in Eq. (1) so that $\langle n_{i,b}\rangle = \langle n_{i,k}\rangle = 2$ to 4 [10,14] is used for both the baryons and kaons. For the kaons, $\Gamma_{i,K\bar{K}} = \langle K\rangle\Gamma_i$ where $\langle K\rangle = 0.4$ to 0.5 [10,14]. Thus, heavier resonances equilibrate more quickly because of large decay widths.

Using a Bjorken expansion, we find a relationship between the temperature and the time, $T(t)$, for which the total entropy is held constant

$$\text{const} = s(T)V(t) = \frac{S_\pi}{N_\pi}\int \frac{dN_\pi}{dy}dy \tag{4}$$

where $\int \frac{dN_\pi}{dy}dy = 874$ from Ref. [17] for the 5% most central collisions within one unit of rapidity and the entropy per pion $S_\pi/N_\pi = 5.5$ is larger than that for a gas of massless pions [18]. The volume [3] is

$$V_{\rm eff}(t \geq t_0) = \pi ct[r_0(t_0) + v_0(t-t_0) + \frac{a_0}{2}(t-t_0)^2]^2 \tag{5}$$

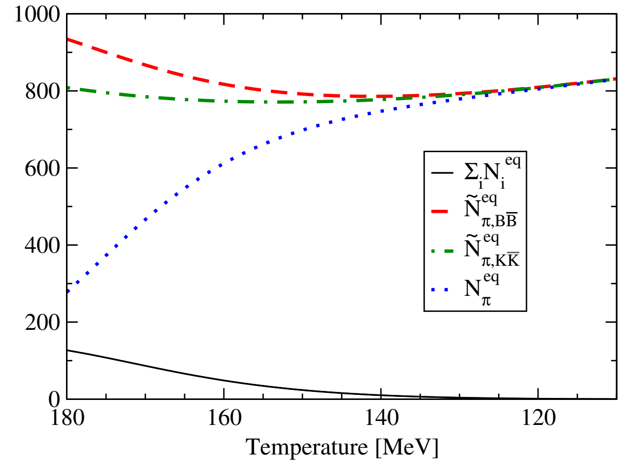

FIG. 1 (color online). Comparison of the total equilibrium number of pions $N_\pi^{\rm eq}$, Hagedorn states $\sum_i N_i^{\rm eq}$, and effective pions $\tilde{N}_{\pi,B\bar{B}}^{\rm eq}$ and $\tilde{N}_{\pi,K\bar{K}}^{\rm eq}$ as defined in Eq. (6).

where the initial radius is $r_0(t_0) = 7.1$ fm. Our chosen average transversal velocity is $v_0 = 0.5c$ with the corresponding acceleration $a_0 = 0.025$.

The equilibrium values of pions, $N_\pi^{\rm eq}$, shown in Fig. 1 are found using a statistical model [19] for the density, and the volume is found from Eq. (5) at the appropriate time according to Eq. (4).

Here, we consider both the direct pions and the indirect pions, which come from resonances such as ρ, ω etc., and both the direct and indirect kaons. In Fig. 1, we see that $N_\pi^{\rm eq}$ increases with decreasing temperature. This occurs because the Hagedorn states dominate the entropy at high temperatures, which affects $N_\pi^{\rm eq}$ due to the entropy constraint in Eq. (4). Therefore, we must consider the number of "effective pions" in the system, i.e., the total number of pions plus the potential number of pions from the Hagedorn resonances, defined as

$$\begin{aligned}\tilde{N}_{\pi,K\bar{K}} &= N_\pi + \sum_i N_i\Big(\frac{\Gamma_{i,\pi}}{\Gamma_i}\langle n_i\rangle + \frac{\Gamma_{i,K\bar{K}}}{\Gamma_i}\langle n_{i,k}\rangle\Big)\\ \tilde{N}_{\pi,B\bar{B}} &= N_\pi + \sum_i N_i\Big(\frac{\Gamma_{i,\pi}}{\Gamma_i}\langle n_i\rangle + \frac{\Gamma_{i,B\bar{B}}}{\Gamma_i}\langle n_{i,b}\rangle\Big)\end{aligned} \tag{6}$$

for the kaons and baryons, respectively. In both cases, $\tilde{N}_\pi^{\rm eq}$ remain roughly constant throughout the Bjorken expansion. Additionally, throughout this Letter, our initial conditions are the various fugacities $\alpha \equiv \lambda_\pi(t_0)$, $\beta_i \equiv \lambda_i(t_0)$, and $\phi \equiv \lambda_{B\bar{B}}(t_0)$ or $\phi \equiv \lambda_{K\bar{K}}(t_0)$, which are chosen by holding the contribution to the total entropy from the Hagedorn states and pions constant, i.e., $s_{Had}(T_0,\alpha)V(t_0) + s_{HS}(T_0,\beta_i)V(t_0) = s_{Had+HS}(T_0)V(t_0) = \text{const}$.

The initial estimate for the Hagedorn state equilibration time is $\tau_i \equiv 1/\Gamma_i$. In order to estimate the chemical equilibration time, we use Eq. (3) in a static environment to find the equilibration time to be in the general ballpark [3,10] of

TABLE I. Equilibration times from analytical estimates where QE is quasiequilibrium and TOT is total equilibrium.

	Time Scale	$T = 180 - 170$ MeV
$\lambda_\pi \approx 0$	$\tau_\pi^0 \equiv \frac{N_\pi^{eq}}{\sum_i \Gamma_i N_i^{eq} \beta_i}$	$0.1 - 0.4 \frac{\text{fm}}{c}$
$\lambda_\pi \approx 1$	$\tau_\pi \equiv \frac{N_\pi^{eq}}{\sum_i \Gamma_i N_i^{eq} \langle n_i^2 \rangle}$	$0.02 - 0.06 \frac{\text{fm}}{c}$
QE	$\tau_\pi^{QE} \equiv \frac{N_\pi^{eq}}{\sum_i \Gamma_i N_i^{eq} \sigma_i^2} + \frac{\sum_{QE} N_i^{eq} \langle n_i^2 \rangle}{\sum_i \Gamma_i N_i^{eq} \sigma_i^2}$	$2.7 - 3.7 \frac{\text{fm}}{c}$
TOT	$\tau^{tot} \equiv \tau_{2\text{ GeV}} + \tau_\pi^{QE}$	$3.5 - 4.5 \frac{\text{fm}}{c}$

$$\tau_{B\bar{B}} \equiv \frac{N_{B\bar{B}}^{eq}}{\sum_i \Gamma_{i,B\bar{B}} N_i^{eq}} = 0.2 - 0.7 \frac{\text{fm}}{c}$$
$$\tau_{K\bar{K}} \equiv \frac{N_{K\bar{K}}^{eq}}{\sum_i \Gamma_{i,K\bar{K}} N_i^{eq}} = 0.1 - 0.3 \frac{\text{fm}}{c} \tag{7}$$

between $T = 180$ to 170 MeV. These time scales are only precise when the pions and Hagedorn states are held in equilibrium [20]. In reality, the chemical equilibration times are more complicated due to nonlinear effects, and the evolution of the equilibration must be divided into separate stages for a sufficient analysis.

To find time scale estimates, we consider the more simplified case near T_c excluding the baryons and kaons, i.e., Eq. (3) without the baryonic terms. The evolution of the rate equations can be divided into three stages as shown in Table I and derived in [20]. Initially, when the pions are far from equilibrium ($\lambda_\pi \approx 0$), the Hagedorn states can be held constant at a constant fugacity β_i. Substituting $\lambda_\pi \approx 0$ and $\lambda_i \approx \beta_i$ into Eq. (3), we obtain τ_π^0. As the pions near equilibrium, we can then use $\lambda_\pi \to 1$ to obtain τ_π. Eventually, the right-hand sides of Eq. (3) become roughly zero before full equilibration (known as quasiequilibrium), which occurs once the lightest resonance reaches quasiequilibrium $\tau_{2\text{ GeV}} = 0.8 \frac{\text{fm}}{c}$. To obtain τ_π^{QE}, we solved Eq. (6) without the baryonic term, assuming that $\lambda_\pi \to 1$ and that the right-hand side of the pion rate equation equals zero. Then, the total equilibration time τ^{tot} is the sum of $\tau_{2\text{ GeV}}$ and τ_π^{QE}. Following the first time scale, τ_π^0, the pions are only 5% off from equilibrium (see Figs. 2 and 3) and after the second time scale, τ_π, the pions are within 2% of equilibrium. The rest of τ^{tot} takes into account nonlinear effects. Thus, even though τ^{tot} is longer, the most significant time scale is τ_π^0. The equilibration times increase directly with N_π^{eq}, $\langle n_i^2 \rangle$ and are shortened by large Γ_i's and wide branching ratio distributions σ_i's. Because N_i^{eq} decreases quickly as the system is cooled, the equilibration time is significantly longer at lower temperatures. In Fig. 2, our analytical fit, based on Table I [20], match our numerical results well and nicely concur with the numerical results in Fig. 3. Additionally, the baryons take slightly longer than predicted in Eq. (7), but still equilibrate quickly (Fig. 3).

In Fig. 4, the baryons and kaons are shown for an expanding system where we see that the baryons reach chemical equilibrium by $T = 165$ MeV $(t - t_0 \approx 2 - 3 \frac{\text{fm}}{c})$ and the kaons at $T = 160 - 140$ MeV. As with

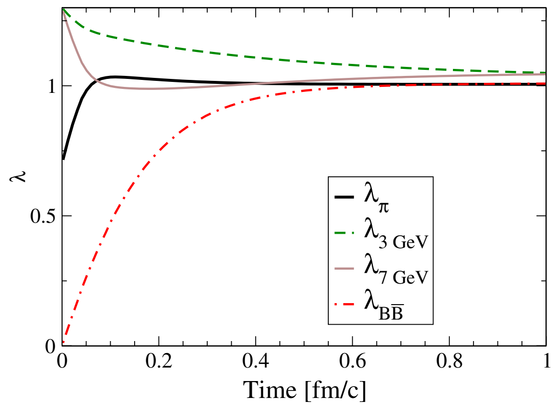

FIG. 3 (color online). Same as Fig. 2 with no initial baryons ($\phi = 0$).

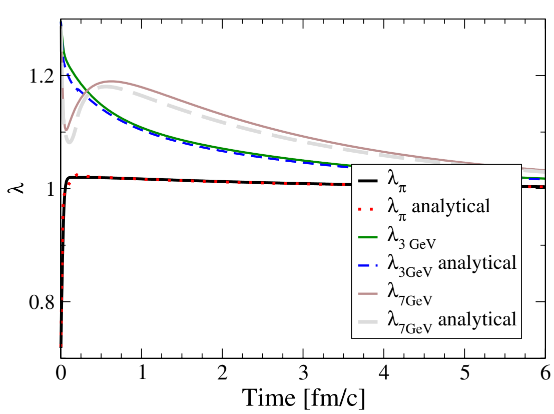

FIG. 2 (color online). Numerical and analytical results in a static environment, i.e., fixed volume and temperature, at $T = 180$ MeV when $\beta_i = 1.3$ and $\alpha = 0.7$.

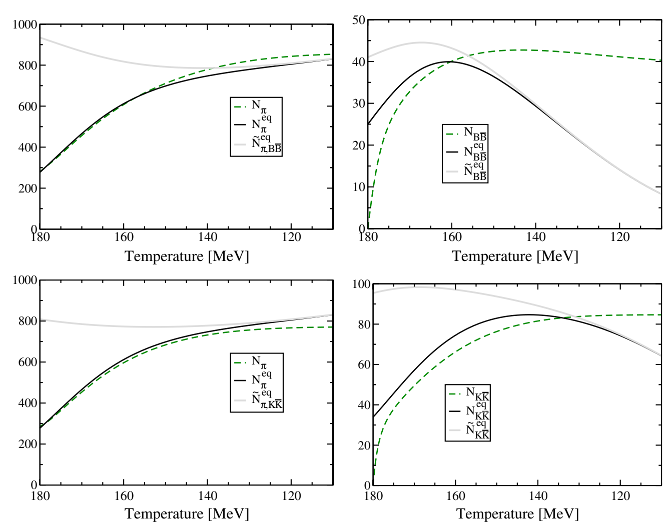

FIG. 4 (color online). Results for an expanding fireball when $\alpha = 1$, $\beta = 1$, and $\phi = 0$. The effective number of baryons $\tilde{N}_{B\bar{B}}^{eq}$, kaons $\tilde{N}_{K\bar{K}}^{eq}$, and pions $\tilde{N}_{\pi,B\bar{B}}^{eq}$ and $\tilde{N}_{\pi,K\bar{K}}^{eq}$ are given.

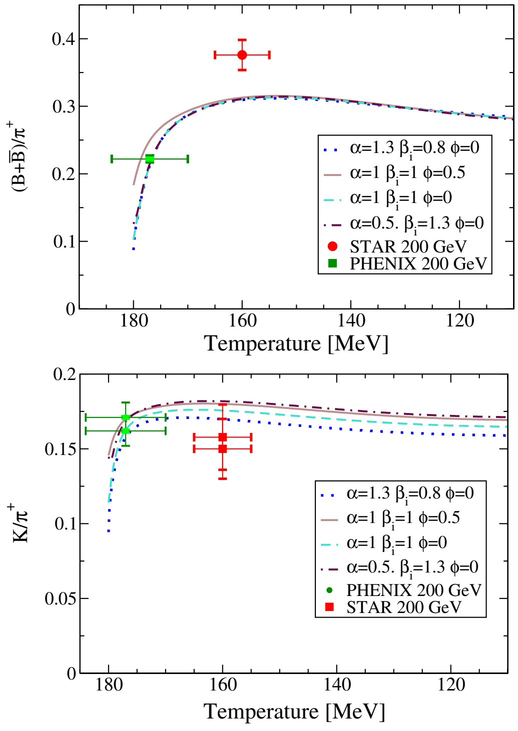

FIG. 5 (color online). Comparison of the ratios obtained employing the expanding fireball picture for various initial conditions to data at $\sqrt{s} = 200$ GeV from PHENIX [21] and STAR [22].

the pions, we consider the effective number of baryons and kaons because of the effects of Hagedorn resonance on the entropy at high temperatures, so

$$\tilde{N}_{B\bar{B}} = N_{B\bar{B}} + \sum_i N_i \frac{\Gamma_{i,B\bar{B}}}{\Gamma_i},$$
$$\tilde{N}_{K\bar{K}} = N_{K\bar{K}} + \sum_i N_i \frac{\Gamma_{i,K\bar{K}}}{\Gamma_i}, \tag{8}$$

as shown in Fig. 4. Not surprisingly, $\tilde{N}^{\rm eq}_{\pi,B\bar{B}}$ and $\tilde{N}^{\rm eq}_{\pi,K\bar{K}}$ remain almost constant due to the constraint in Eq. (4).

In Fig. 5, we compare our total baryon to pion ratio $(B + \bar{B})/\pi^+$ to experimental data from PHENIX [21], and STAR [22]. $(B + \bar{B})/\pi^+$ is calculated by $B + \bar{B} = p + \bar{p} + n + \bar{n} \approx 2(p + \bar{p})$. It should be noted here that in our calculations, we use both the effective number of baryons, in Eq. (8), and pions, in Eq. (6). We obtain $(B + \bar{B})/\pi^+ \approx 0.3$, which matches the experimental data well. Moreover, our results are independent of the chosen initial conditions. Also, in Fig. 5, we compared the kaon to pion ratio to the data at PHENIX [21] and STAR [22] (both K/π^+ and $\bar{K}/\pi^+$ are shown). Again, we use the effective number of kaons (8) and pions (6). Our K/π^+ ratios compare to the experimental data very well, and they level off between 0.16 to 0.17. As with the baryon antibaryon pairs, we do not see a very strong dependence on our initial conditions. In Fig. 5, both figures agree well with experimental data. Moreover, they remain roughly constant after $T = 170 - 160$ MeV. This demonstrates that the potential Hagedorn states can be used to explain dynamically the build up of the known particle yields.

In future work, we will consider strange baryonic degrees of freedom and thoroughly study the effects of our initial conditions and parameters. To conclude, we used Hagedorn resonances as a dynamical mechanism to quickly drive baryons and kaons into equilibrium between temperatures of $T = 165 - 140$ MeV. Once a Bjorken expansion was employed, we found that our calculated K/π^+ and $(B + \bar{B})/\pi^+$ ratios matched experimental data well, which suggests that hadrons do not at all need to start in equilibrium at the onset of the hadron gas phase.

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