Universal Scaling in Nonequilibrium Transport through a Single Channel Kondo Dot

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Scaling laws and universality play an important role in our understanding of critical phenomena and the Kondo effect. We present measurements of nonequilibrium transport through a single-channel Kondo quantum dot at low temperature and bias. We find that the low-energy Kondo conductance is consistent with universality between temperature and bias and is characterized by a quadratic scaling exponent, as expected for the spin-$\frac{1}{2}$ Kondo effect. We show that the nonequilibrium Kondo transport measurements are well described by a universal scaling function with two scaling parameters.

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Seemingly unrelated physical phenomena can sometimes display strikingly similar behavior. The evolutions of thermodynamic parameters of certain liquid-gas and paramagnetic-ferromagnetic phase transitions, for example, are characterized by identical sets of critical exponents even though the underlying forces (van der Waals and magnetic exchange, respectively) are vastly different [1]. A similar type of universality can also be exhibited in the response of a physical system to different perturbations such as temperature, applied bias, or magnetic field. Here, even though each perturbation affects the system in a qualitatively different manner, the characteristic exponent of the lowest order response is identical for several of the perturbations. While the coefficients of the lowest order response generally depend on system-specific energy scales, these dependences can usually be eliminated by scaling each perturbation relative to a characteristic energy, allowing the whole class of systems to be neatly described by a single universal scaling function.

The universal scaling laws governing perturbations that drive a system out-of-equilibrium have been especially difficult to derive theoretically with high accuracy. One of the best-known phenomena that has eluded a precise theoretical description is the nonequilibrium Kondo effect [2–9]. The Kondo effect is a many-body state that arises from the interaction of a local magnetic moment with a reservoir of conduction electrons [10–12]. The effect has been observed in a variety of systems including mesoscopic quantum dots, where the spin of an unpaired electron on the dot acts as the magnetic impurity [13,14]. For quantum dots, the nonequilibrium Kondo effect can occur whenever a bias voltage is applied between two reservoirs (i.e., two leads) that participate in forming the Kondo state. The applied bias drives the Kondo state out of equilibrium, as neither reservoir is in equilibrium with the quantum dot. While previous measurements have shown that transport at zero bias does indeed follow a universal scaling curve in temperature, the low-energy power-law temperature and nonequilibrium regimes have not been closely examined [15]. All theoretical considerations predict that universal scaling behavior in temperature T and applied bias V is maintained in the nonequilibrium Kondo regime [3,4,11], but they disagree about the coefficient values of the universal scaling function and how many system-specific parameters determine the scaling relations [6]. Experimental work has not elucidated the theoretical situation, as existing measurements have focused on higher energies ($eV \sim kT_K$) from which reliable low-energy scaling behavior cannot be extracted [13–17].

In this Letter we examine low temperature and low bias transport through a spin-$\frac{1}{2}$ Kondo dot in the nonequilibrium regime. We find that the nonequilibrium conductance in the Kondo plateau is consistent with a quadratic power law at low energies [18], as previously predicted for single-channel Kondo transport but not examined experimentally. Furthermore we show that low-energy conductance is well described by a universal scaling function in temperature and bias with two scaling parameters: the Kondo temperature T_K and the zero-temperature conductance G_0. Our extracted universal scaling function coefficients are in general agreement with calculations within the Anderson-model framework. An explanation of the observed deviations will require further study.

Our measurements are performed using a quantum dot formed from a low density ($n_e = 2 \times 10^{11} e^-/\mathrm{cm}^2$), high mobility ($\mu = 2 \times 10^6\ \mathrm{cm}^2/\mathrm{V\,s}$) two dimensional electron gas (2DEG) located 68 nm below the surface of a GaAs/AlGaAs heterostructure. Gate electrodes [Fig. 1(d)] define the dot and finely tune its coupling to extended regions of 2DEG, which serve as leads. One gate in particular, labeled V_G in Fig. 1(d), tunes the quantum dot to odd occupancy and controls the energy ε_0 of the singly occupied level while minimally perturbing the dot-lead coupling Γ. The dot's area is approximately 0.04 $\mu\mathrm{m}^2$, leading to a bare charging energy $U \sim 1$ meV and mean level spacing $\Delta \sim 100\ \mu\mathrm{eV}$ [19]. We measure differential

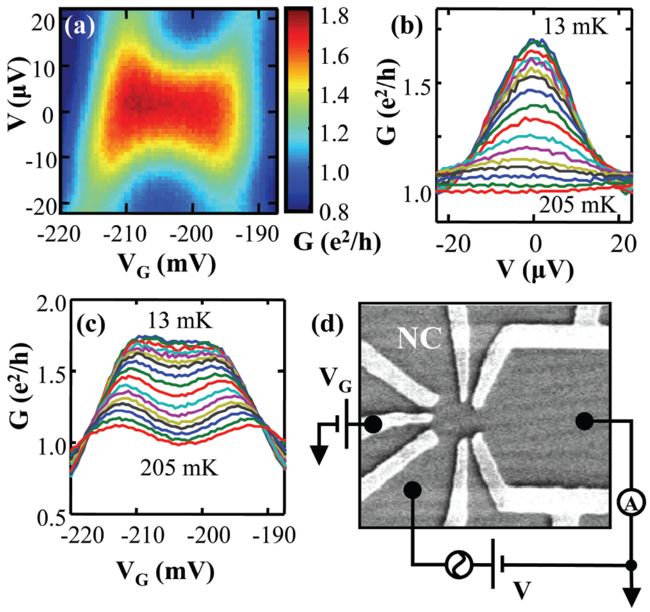

FIG. 1 (color online). (a) Differential conductance (G) measured as a function of V_G and source-drain bias at $T = 13$ mK. (b) Temperature dependence of the Kondo peak in conductance for $T = 13$–205 mK at $V_G = -203$ mV. (c) Temperature dependence of the Kondo plateau for $T = 13$–205 mK at $V = 0$ μV. (d) The SEM image shows the quantum dot device with an overlaid measurement schematic. The topmost lead (marked "NC") is pinched off from the dot and does not contribute to transport.

conductance $G = dI/dV$ using standard lock-in techniques, with an rms modulation bias of 1 μV at 337 Hz. The dot is cooled in the mixing chamber of an Oxford Instruments Kelvinox TLM dilution refrigerator with a base electron temperature of 13 mK [20].

At base temperature, the Kondo effect produces a characteristic enhancement of conductance through the quantum dot at odd occupancy ($-\varepsilon_0$, $\varepsilon_0 + U > \Gamma$), as seen in Fig. 1. The couplings of the dot to its two leads are tuned to maximize T_K while keeping the two couplings nearly equal. The saturation of conductance at a value near $1.75e^2/h$ throughout the middle of the Kondo plateau [-209 mV $< V_G < -199.5$ mV in Fig. 1(c)] confirms that $T_{\mathrm{base}} \ll T_K$ and indicates that the coupling asymmetry is around 2:1 [2]. Conductance as a function of source-drain bias in the Kondo plateau shows a narrow peak centered at zero bias, known as the Kondo peak [Fig. 1(b)].

As the temperature increases from 13 to 205 mK the overall Kondo conductance decreases [Figs. 1(b) and 1(c)]. Previous measurements found the temperature evolution of linear conductance $G(T, V = 0)$ to be well described by an empirical Kondo (EK) form derived from a fit to numerical renormalization group conductance calculations [15],

$$G_{\mathrm{EK}}(T) = G_0/(1 + (T/T_K')^2)^s. \tag{1}$$

Here, $s = 0.21$ and $T_K{}' = T_K/(2^{1/s} - 1)^{1/2}$, which defines T_K as the temperature at which the Kondo conductance is half of its extrapolated zero-temperature value: $G_{\mathrm{EK}}(T_K) = G_0/2$. The values of G_0 and T_K we extract are shown in Fig. 2(a). The Kondo conductance traces in our measurements follow the EK form very well at low temperatures ($T < T_K/4$), but deviate from it at higher temperatures, as seen in Fig. 2(b). Though the origin of the deviation at higher temperatures is not completely understood, it most likely reflects the emergence of additional transport processes at higher temperatures. We limit the temperature range for our fits to $T < 35$ mK.

To determine whether bias and temperature obey a scaling relationship at low temperatures we fit the low-energy conductance to the form

$$G(T, V) \approx G_0 - \tilde{c}_T (kT)^{P_T} - \tilde{c}_V (eV)^{P_V}. \tag{2}$$

Here P_T and P_V are exponents that characterize the temperature and bias dependence, respectively, and $\tilde{c}_T$ and $\tilde{c}_V$ are expansion coefficients. Unlike the EK form [Eq. (1)], Eq. (2) does not assume quadratic behavior at low temperature. We first extract P_V by fitting $G(T, 0) - G(T, V)$ as a power law in voltage for $|V| < 7$ μV at each temperature point below 20 mK. We find that P_V is nearly constant across the Kondo plateau with an average value of 1.9 ± 0.15 [Fig. 3(a)]. Extracting P_T is more difficult since only a few temperature points unambiguously reside in the power-law regime at each gate voltage point. Fits for T/T_K yield a mean value of $P_T = 2.0 \pm 0.6$ across the Kondo plateau [21]. These fits are consistent with temperature and bias sharing a characteristic exponent of 2, as theoretically expected for the single-channel Kondo effect, and we assume this universality for the remainder of our analysis.

Having determined the characteristic scaling exponent, we now examine to what extent the low-energy nonequilibrium conductance $G(T, V)/G_0$ is described by a universal scaling function, $F(T/T_K, eV/kT_K)$. We assume as a starting point that $G(T, 0)$ follows the universal curve given by Eq. (1) and examine the evolution of the differential

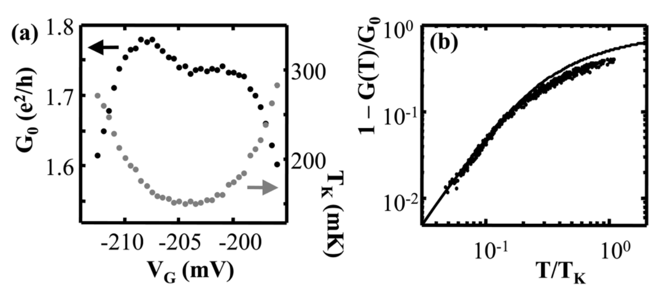

FIG. 2. (a) Values of G_0 and T_K across the Kondo plateau, extracted using the empirical Kondo (EK) form $G_{\mathrm{EK}}(0, T) = G_0/(1 - (T/T_K{}')^2)^s$ with $T_K{}' = T_K/(2^{1/s} - 1)^{1/2}$. The fit was performed using data points for temperatures between 13–35 mK. (b) A plot of the scaled conductance $1 - G(T, 0)/G_0$ versus T/T_K for all measured temperatures and gate voltage points across the Kondo plateau [18]. The solid line shows the empirical Kondo form.

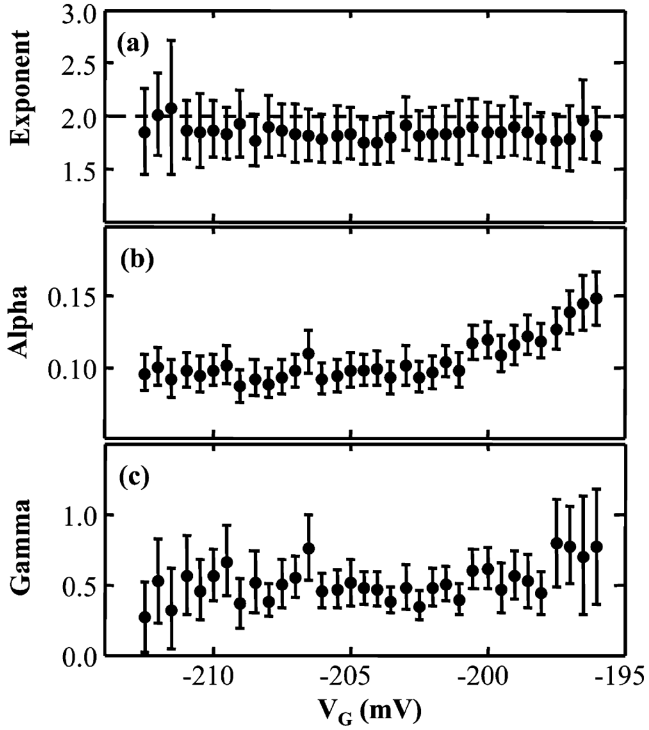

FIG. 3. (a) Values of the Kondo scaling exponent for bias (P_V) across the Kondo plateau. The horizontal dashed line shows the expected single-channel Kondo exponent $P = 2$. Values of (b) α and (c) γ extracted across the Kondo plateau. The fitting methods are described in the text and in Ref. [21].

conductance with temperature at finite bias. For each point in the Kondo plateau we fit the Kondo peak using the low bias expansion of a form that Refs. [22,23] found applicable over a wide temperature range:

$$G(T, V) = G_{\mathrm{EK}}(T, 0)\left(1 - \frac{c_T\alpha}{1 + c_T(\frac{\gamma}{\alpha} - 1)(\frac{T}{T_K})^2}\left(\frac{eV}{kT_K}\right)^2\right). \tag{3}$$

The coefficient $c_T \approx 5.49$ is fixed by the definition of T_K via Eq. (1): $G_{\mathrm{EK}}(T, 0) \approx (1 - c_T(T/T_K)^2)$. The coefficients α and γ characterize the zero-temperature curvature and temperature broadening of the Kondo peak, respectively, and are defined appropriately to be independent of how T_K is defined. The form of Eq. (3) is chosen so that at low temperature Eq. (3) reduces to a universal scaling function expansion suggested by Schiller *et al.* [4]:

$$\frac{G(T,0) - G(T,V)}{c_T G_0} = F\left(\frac{T}{T_K}, \frac{eV}{kT_K}\right) \approx \alpha\left(\frac{eV}{kT_K}\right)^2 - c_T\gamma\left(\frac{T}{T_K}\right)^2\left(\frac{eV}{kT_K}\right)^2. \tag{4}$$

Figures 3(b) and 3(c) show the extracted values of α and γ across the Kondo plateau, fitted over the regime $T/T_K < 0.2$ and $eV/kT_K < 0.4$. The coefficients are nearly constant across the plateau and have average values $\alpha = 0.10 \pm 0.015$ and $\gamma = 0.5 \pm 0.1$ along the plateau [21,24]. Both α and γ increase slightly on the right side of the Kondo plateau ($V_G > -199.5$ mV). This may reflect the expected breakdown of the universal scaling relations in the regime where charge fluctuations modify Kondo processes (mixed valence regime).

Overall, we can conclude that low-energy transport through a quantum dot in the Kondo regime is well described by the universal scaling function given by Eqs. (3) and (4). The degree of agreement can be visualized by plotting the scaled conductance $(1 - G(T, V)/G(T, 0))/\tilde{\alpha}_V$ versus $(eV/kT_K)^2$, where $\tilde{\alpha}_V = c_T\alpha/(1 + c_T(\frac{\gamma}{\alpha} - 1) \times (\frac{T}{T_K})^2)$, using the midplateau values of α and γ. As Fig. 4 shows, the conductance data across the Kondo plateau for $T/T_K < 0.6$ collapse nicely onto a single universal curve for bias values up to $(eV/kT_K)^2 \sim 0.5$, above which deviations due to higher-order terms or non-Kondo processes become apparent.

Previous experimental work on spin-$\frac{1}{2}$ Kondo dots mainly examined the full-width-at-half-maximum (FWHM) of the Kondo peak at fixed values of T_K, rather than the whole Kondo plateau, and did not investigate the low bias power-law regime [14,16,22,23,25]. For comparison purposes we extract rough values of α and γ from a few existing measurements by approximating the Kondo peak as a Lorentzian. A previous measurement of Kondo transport in the nonequilibrium regime gives $\alpha \approx 0.25$ and $\gamma \approx 0.5$ in the middle of the Kondo plateau [16]. From transport measurements through magnetic impurities coupled to highly asymmetric leads [26], which probe the equilibrium rather than nonequilibrium spectral function, we extract $\alpha \approx 0.05$ and $\gamma \approx 0.1$ in Ref. [23] and $\alpha \approx 0.15$ and $\gamma \approx 0.5$ in Ref. [27]. Though these values are comparable to our measured values, the wide variation they exhibit emphasizes the importance of focusing on low energies in order to extract meaningful coefficients.

We now compare our carefully extracted values for α and γ to existing theory for nonequilibrium transport through a spin-$\frac{1}{2}$ Kondo quantum dot. Existing calculations

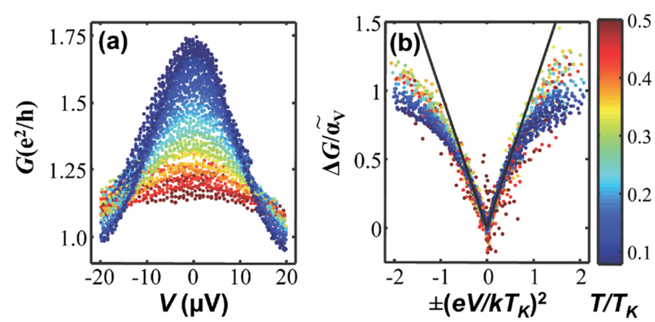

FIG. 4 (color online). (a) Conductance as a function of bias for $0 < T/T_K < 0.6$ for six points across the Kondo plateau (b) Scaled conductance $(1 - G(T, V)/G(T, 0))/\tilde{\alpha}_V$ versus $(eV/kT_K)^2$, where $\tilde{\alpha}_V = c_T\alpha/(1 + c_T(\frac{\gamma}{\alpha} - 1)(\frac{T}{T_K})^2)$. We use $\alpha = 0.10$ and $\gamma = 0.5$, which are the average values of the extracted scaling coefficients along the Kondo plateau. The solid line shows the associated universal curve described by Eq. (3).

are based on either the Anderson [6,9,11] or Kondo models [4,5,12], and focus mainly on determining α. Quantum dot measurements have been generally well described by the Anderson model. Calculations using this framework predict a value of 0.15 for α in both the strong coupling nonequilibrium [6] and equilibrium limits [12]. Other theoretical treatments show a greater level of disagreement with our results [5,8,9]. A comparison to our measured value of $\sim$0.1 indicates that the observed Kondo conductance decreases with bias slightly more slowly than predicted by the Anderson-model calculations. The origin of this discrepancy is currently not understood. One possibility is that the nonequilibrium calculations overestimate how quickly Kondo processes diminish with increasing bias. A second possibility is that additional non-Kondo transport processes, such as inelastic cotunneling [28], contribute to a concurrent bias-dependent conductance in our measurement. The contribution, if any, of additional transport processes could be resolved in future experiments by extracting scaling parameters at different dot (U, Δ) and coupling (Γ) parameters for identical T_K and G_0 values.

We have performed detailed measurements of the Kondo conductance through a quantum dot as a function of temperature and bias. Our nonequilibrium Kondo conductance measurements are consistent with universality in temperature and bias characterized by a quadratic scaling exponent. The conductance is well described by a universal nonequilibrium Kondo scaling function whose coefficients are constant along the Kondo plateau. The extracted coefficient values show general agreement with Anderson-model calculations and provide a well-defined reference point for further experimental and theoretical work.

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[18] Kondo plateau refers to the parameter region where the Kondo conductance is saturated at base temperature [-209 mV $< V_G <$ 199.5 mV, as seen in Fig. 1(c)]. The Kondo valley is the wider parameter region where Kondo processes affect conductance ($-220\,\mathrm{mV} < V_G < 188\,\mathrm{mV}$).
[19] The bare U value corresponds to weak dot-lead coupling. The actual U value is about an order of magnitude smaller due to the stronger coupling in our measurements.
[20] The experiment was performed in a 10 mT magnetic field, which had no affect on our measurements.
[21] See EPAPS Document No. E-PRLTAO-100-050825 for more information on our fitting methods, error bar determination, and the use of a conductance offset in the empirical Kondo form fit [Eq. (1)]. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html
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