$\sigma \to \gamma\gamma$ Width from Nucleon Electromagnetic Polarizabilities

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(Received 13 February 2008; published 20 June 2008)

The lightest QCD resonance, the σ, has been recently fixed in the $\pi\pi$ scattering amplitude. The nature of this state remains nowadays one of the most intriguing and difficult issues in particle physics. Its coupling to photons is crucial for discriminating its structure. We propose a new method that fixes this coupling using only available precise experimental data on the proton electromagnetic polarizabilities together with analyticity and unitarity. By taking into account the uncertainties in the analysis and in the parameter values, our result is $\Gamma_{\rm pole}(\sigma \to \gamma\gamma) = 1.2 \pm 0.4$ keV.

DOI: 10.1103/PhysRevLett.100.241804 PACS numbers: 11.80.Et, 13.20.Jf, 13.40.Hq, 13.75.Lb

The lowest resonance in the QCD spectrum has the quantum numbers of the vacuum and is usually called the σ. The mass and width of this state has been recently fixed with a precision of just tens of MeV in Ref. [1] using an analytic continuation into the complex energy plane of the isopin $I = 0$ and angular momentum $J = 0$ $\pi\pi$ partial wave scattering amplitude. On the first Riemann sheet of the energy plane, the S matrix has a zero at $E = [(441^{+16}_{-8}) - i(272^{+9}_{-12})]$ MeV, which reflects the σ pole on the second sheet at the same position. This is also a zero at E^* in the inverse of the $\pi\pi$ partial wave S matrix $S = 1 + 2i\beta(t)T(t)$ on the first Riemann sheet. Here

$$T(t) = \frac{1}{\beta(t)\cot[\delta(t)] + i\beta(t)}, \quad (1)$$

where $\delta(t)$ is the scalar-isoscalar $\pi\pi$ phase shift, $\beta(t) = \sqrt{1 - 4m_\pi^2/t}$, and $t = E^2$. This result has been confirmed in Ref. [2] with the position of the σ pole at $E = [(484 \pm 17) - i(255 \pm 10)]$ MeV. The relevance of these results has to be emphasized in view of the special role played by the σ in the QCD dynamics and in the QCD nonperturbative vacuum structure.

Although the pole dominance of the σ in the scalar-isoscalar $\pi\pi$ amplitude is apparent in a wide energy region around its position, its existence is somewhat masked by the effects of its large width. For a narrow resonance, there is an observable connection between the phase dependence of the physical amplitude on the real axis and the one in the complex plane, as one crosses the pole position. This connection is, however, lost in the case of the σ with such a large width: One does not observe either a rapid variation of the amplitude phase [3] or a Breit-Wigner type behavior around the resonance position. This enormous difference in the behavior of the amplitude as one moves away from the real axis is what has made the σ existence and location so uncertain for so long.

Yet the important question about what is the nature of the σ remains unanswered [4–12]. What is its role in the chiral dynamics of QCD? Is it a $\bar{q}$-q state? Is it a π-π molecule? Is it a $(\overline{qq})$-(qq) tetraquark? Is it a glueball state? How is it possible to distinguish these different substructures? Two photon interactions can shed some light on this question from the size of the $\sigma \to \gamma\gamma$ width [13]. This is because this width is proportional to the square of the average electromagnetic charge of their constituents, while its absolute scale depends on how these constituents form the σ. Recently, the authors of Refs. [14,15] have calculated the $\gamma\gamma \to (\pi\pi)_{I=0,2}$ amplitudes using twice-subtracted dispersion relations, in order to weigh the low-energy region in the dispersive integrand. Their results take into account the now well known $\pi\pi$ final state interactions which contain the σ pole in the scalar-isoscalar contribution. For the width of the σ into two photons, they obtain 4.09 ± 0.29 keV in Ref. [14] and 1.68 ± 0.15 keV in the improved approach of Ref. [15]. Although the approach and methodology [16] are very similar in these two calculations, there is an apparent discrepancy. Its origin is discussed in Ref. [15]. The different input used for the dispersive calculation of the production amplitudes of $\gamma\gamma \to \pi\pi$ and the use of different values for the position of the σ pole on the second Riemann sheet t_σ and its coupling to two pions $g_{\sigma\pi\pi}$ are equally responsible. Notice that, although these last two inputs are not required in the dispersive calculation, the $\sigma \to \gamma\gamma$ width obtained in Refs. [14,15] depends critically on them [15].

The experimental results on the $\gamma\gamma \to \pi\pi$ process are scarce and, in order to extract information on the σ, unfortunately theoretically contaminated by the Born term in the charged pion channel and by the isospin $I = 2$ amplitude in all cases, interfering with the $I = 0$ amplitude in the cross section [3]. The purpose of this Letter is to point out that the coupling $g_{\sigma\gamma\gamma}$ of the σ meson found in the $\pi\pi$ scattering amplitude [1,2] is a measurable quantity, directly obtainable from the nucleon electromagnetic polarizabilities, and that it can be extracted with good precision from existing experimental values. This differs from the analysis in Ref. [17], where the properties of the σ meson of a Nambu–Jona-Lasinio model are used. The argument proceeds as follows. Besides the mass, electromagnetic charge, and magnetic moment, the electric α and magnetic β polarizabilities structure constants determine the Compton scattering amplitude [18,19] and the differential cross section up to second and third order in the energy of the photon, respectively. The available experiments of Compton scattering on protons and neutrons at low energies can be analyzed [20,21] in terms of α and β, with the sum $\alpha + \beta$ constrained by the sum rule obtained from the forward dispersion relation [22]. The results are $\alpha^{\rm expt} = 12.0 \pm 0.6$ and $\beta^{\rm expt} = 1.9 \mp 0.5$ for protons and $\alpha^{\rm expt} = 11.6 \pm 1.5$ and $\beta^{\rm expt} = 3.7 \mp 2.0$ for neutrons. Here and in the rest of the Letter, polarizabilities are given in 10^{-4} fm^3 units.

A separate theoretical determination of α and β needs more ingredients than the ones present in the forward sum rule. The authors of Ref. [23] investigated this problem by using a backward dispersion relation for the physical spin-averaged amplitude. The corresponding sum rule for $\alpha - \beta$ contains contributions from an s-channel part and a t-channel part. The first is related to the multipole content of the total photoabsorption cross section, whereas the t-channel part is related with the imaginary part of the amplitude through a dispersion relation for t, as shown in Ref. [24]. This imaginary part of the amplitude is given by the processes $\gamma\gamma \to \pi\pi$ and $\pi\pi \to N\bar{N}$ via a unitarity relation. The result is the Bernabéu-Ericson-Ferro Fontan-Tarrach (BEFT) sum rule

$$\alpha - \beta = \frac{1}{2\pi^2}\int_{\nu_{\rm th}}^{\infty}\frac{d\nu}{\nu^2}\sqrt{1+2\frac{\nu}{M_p}} \times [\sigma(\Delta\pi = {\rm yes}) - \sigma(\Delta\pi = {\rm no})] + \frac{1}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{dt}{4M_p^2 - t}\frac{\beta(t)}{t^2}\Big\{|f_+^0(t)||F_0^0(t)| - \frac{(4M_p^2 - t)(t - 4m_\pi^2)}{16}|f_+^2(t)||F_0^2(t)|\Big\}, \quad (2)$$

where M_p is the proton mass, the partial wave helicity amplitudes $f_+^0(t)$ and $f_+^2(t)$ for $N\bar{N} \to \pi\pi$ are Frazer and Fulco's [25], and the partial wave helicity amplitudes $F_0^0(t)$ and $F_0^2(t)$ for $\gamma\gamma \to \pi\pi$ are defined as in Ref. [26]. The absorptive part in the s-channel contribution is obtained from that of the forward physical amplitude by changing the sign of the nonparity flip multipoles ($\Delta\pi = $ no). A reliable evaluation of this s-channel integrand [21] gives $(\alpha - \beta)^s = -(5.0 \pm 1.0)$ for protons and neutrons. The importance of the t-channel contribution was already emphasized in Ref. [24] and the connection of $(\alpha - \beta)^t$ to the isoscalar s-wave $\gamma\gamma \to \pi\pi$ amplitude $F_0^0(t)$ pointed out. The "experimental" $(\alpha - \beta)^t$ is thus 15.1 ± 1.3 for protons and 12.9 ± 2.7 for neutrons, compatible with the isoscalar selection imposed by the t-channel sum rule. It is remarkable that the products of helicity amplitudes appearing in Eq. (2) are the products of their moduli, which might take negative values if the phases of these amplitudes differ from the $\pi\pi$ phase shift in an odd number of π's. The d-wave contribution is much smaller than the s-wave one; hence, it is a good approximation to take only the Born term in the crossed channel [27], and this leads to $(\alpha - \beta)_2^t = -1.7$. Therefore, the experimental quantity to be compared with the result of the integral term containing $F_0^0(t)$ in Eq. (2) is $(\alpha - \beta)_0^t = (16.8 \pm 1.3)$. The input $|F_0^0(t)|$ amplitude in that integral is what we want to fix from this experimental value. The Frazer-Fulco $|f_+^0(t)|$ amplitude is known with sufficient accuracy for our purposes from Ref. [28], and we have assigned a 20% uncertainty to the theoretical $(\alpha - \beta)_0^t$ determination from the uncertainty of $|f_+^0(t)|$. Notice that the $1/t^2$ factor in the integrand of Eq. (2) makes the well known low-energy and, to a lesser extent, intermediate-energy contributions the dominant ones.

On the physical sheet, we use the twice-subtracted dispersion relation [16]

$$F_0^0(t) = L(t) - \Omega(t)\Big[ct + \frac{t^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{t'^2}\frac{L(t'){\rm Im}\Omega^{-1}(t')}{t' - t - i\varepsilon}\Big], \quad (3)$$

where c is a subtraction constant fixed by chiral perturbation theory (CHPT) [16,29], $c = \alpha/48\pi f_\pi^2$, with $\alpha \simeq 1/137$ the fine-structure constant and $f_\pi = 92.4$ MeV the pion decay constant,

$$\Omega(t) = \exp\Big[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{t'}\frac{\delta(t')}{t' - t - i\varepsilon}\Big] \quad (4)$$

is the scalar-isoscalar $\pi\pi$ Omnès function [30] which gives the correct right-hand cut contribution, and $L(t)$ is the left-hand cut contribution. In this way we ensure unitarity, the correct analytic structure of $F_0^0(t)$, and that the σ pole properties enter through the scalar-isoscalar phase shift $\delta(t)$ from $T(t)$ in (1). Here we shall use a simple analytic expression for $T(t)$, compatible with Roy's equations, which takes a three-parameter fit from Ref. [2] including both low-energy kaon data and high-energy data. This fit is valid up to values of t of the order of 1 GeV^2, which is enough in the integrand of the polarizability sum rule in Eq. (2).

At the σ pole position on the first Riemann sheet [3,14,15]

$$F_0^0(t_\sigma) = e^2\sqrt{6}\frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}}\frac{1}{2i\beta(t_\sigma)}, \quad (5)$$

where e is the electron charge, $g_{\sigma\pi\pi}^2$ is the residue of the $\pi\pi$ scattering amplitude at the σ pole on the second Riemann sheet, and $g_{\sigma\gamma\gamma}g_{\sigma\pi\pi}$ is proportional to the residue of the $\gamma\gamma \to \pi\pi$ scalar-isoscalar scattering amplitude on the second Riemann sheet. The proportionality factors are such that $g_{\sigma\pi\pi}$ and $g_{\sigma\gamma\gamma}$ agree with those used in Refs. [3,14]. The pole width is given by [3,14]

$$\Gamma_{\rm pole}(\sigma \to \gamma\gamma) = \frac{\alpha^2|\beta(t_\sigma)g_{\sigma\gamma\gamma}^2|}{4M_\sigma} \quad (6)$$

that agrees, modulo normalizations, with that of Ref. [15]. This is not the observable radiative width that would be associated with a possible Breit-Wigner resonance in the physical $\gamma\gamma \to (\pi\pi)_{I=0}$ amplitude. However, in order to discuss the structure of the σ, one has to move around the pole, and $\Gamma_{\text{pole}}(\sigma \to \gamma\gamma)$ is the appropriate one.

Because of Low's low-energy theorem [18], the amplitude $F_0^0(t)$ is given by the Born term at low energies. Then, as a first approximation, we consider the left-hand cut contribution $L(t)$ in (3) to be the Born contribution $L_B(t)$ to the crossed channel describing the pion Compton scattering $\gamma\pi \to \gamma\pi$:

$$L_B(t) = e^2 \frac{1-\beta(t)^2}{\beta(t)} \log\left(\frac{1+\beta(t)}{1-\beta(t)}\right). \tag{7}$$

Inserted into the dispersion relation in Eq. (3), this contribution leads [16,27] to a Born amplitude $F_0^0(t)|_B$ for the annihilation channel $\gamma\gamma \to \pi\pi$ dressed with $\pi\pi$ final state interactions. Thus, this $F_0^0(t)|_B$ includes the σ and is compatible with unitarity and analyticity.

The evaluation of the sum rule in Eq. (2) with this $F_0^0(t)|_B$ results in a value of $(\alpha - \beta)_0^t|_B = 6.7 \pm 1.2$, much smaller than the experiment. The quoted uncertainty stems from the uncertainties in the input data needed for the sum rule in Eq. (2). The main reason for this small value is the presence of a zero in the integrand of that sum rule at a moderate t value of $t_0 \simeq 0.30\ \text{GeV}^2$, as shown in Fig. 1. The amplitude $F_0^0(t)|_B$, when analytically continued to complex t, has the σ pole in the second Riemann sheet at $t = t_\sigma = \{[(474 \pm 6) - i(254 \pm 4)]\ \text{MeV}\}^2$ with $g_{\sigma\pi\pi} = [(452 \pm 4) + i(224 \pm 2)]$ MeV and on the first Riemann sheet leads to $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B = (0.49^{+0.03}_{-0.01}) - i(0.37 \pm 0.03)$ and $\Gamma_{\text{pole}}(\sigma \to \gamma\gamma)|_B = 2.5 \pm 0.2$ keV. These results are, however, not adequate for reproducing the experimental nucleon electromagnetic polarizabilities, as we

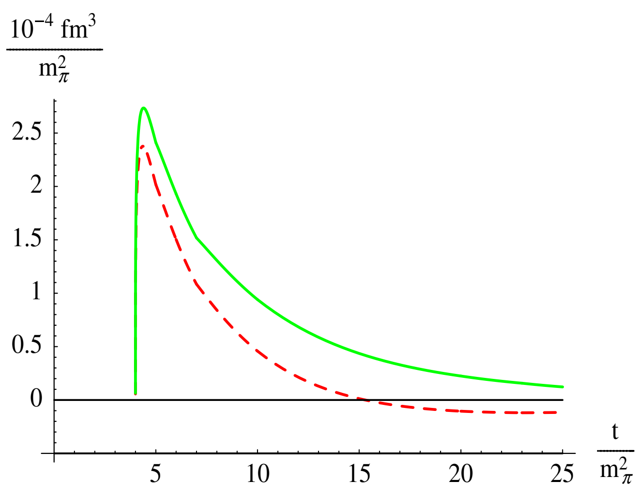

FIG. 1 (color online). The integrand of $(\alpha - \beta)_0^t$ in (2). The dashed line is when using $L(t) = L_B(t)$ in (3) and the continuous line is when using $L(t) = L_B(t) + L_A(t) + L_V(t)$ in (3) as explained in the text.

have seen, and the pion Compton scattering description has to go beyond the Born approximation $L_B(t)$, with a modification of the left-hand cut $L(t)$ contribution in Eq. (3).

At intermediate energies, this modification is due to resonance exchanges $\gamma\pi$ with the leading ones being $R = a_1$, ρ, and ω [15,16]. The a_1 exchange contribution to $L(t)$ is

$$L_A(t) = e^2 \frac{C}{32\pi f_\pi^2}\left[t + \frac{M_{a_1}^2}{\beta(t)} \log\left(\frac{1+\beta(t)+t_A/t}{1-\beta(t)+t_A/t}\right)\right], \tag{8}$$

while the ρ and ω resonances exchange contribution to $L(t)$ in nonet symmetry ($M_\rho = M_\omega = M_V \simeq 782$ MeV) is

$$L_V(t) = e^2 \frac{4}{3} R_V^2 \left[t - \frac{M_V^2}{\beta(t)} \log\left(\frac{1+\beta(t)+t_V/t}{1-\beta(t)+t_V/t}\right)\right], \tag{9}$$

with $t_R = 2(M_R^2 - m_\pi^2)$. The low-energy limit of $L_V(t)$ goes as t^2, and we fix $R_V^2 = 1.49\ \text{GeV}^{-2}$ by using the well known $\omega \to \pi\gamma$ decay. Though the low-energy limit of $L_A(t)$ goes as t and corresponds to the pion electromagnetic polarizability $(\bar{\alpha} - \bar{\beta})_{\pi^\pm}$ or equivalently to $L_9 + L_{10} = (1.4 \pm 0.3) \times 10^{-3}$ in CHPT [31], we consider $L_A(t)$ as an effective contribution for moderate higher values of t with C a real constant to be determined phenomenologically and not connected to the pion polarizability. This is supported by the fact that the $a_1 \to \pi\gamma$ coupling is not so well known at intermediate energies. We fix C by requiring that the experimental value of $(\alpha - \beta)_0^t$ is reproduced within 1.5 standard deviations of the total uncertainty when $L(t)$ in (3) is given by $L(t) = L_B(t) + L_A(t) + L_V(t)$. This procedure leads to $C = 0.59 \pm 0.20$, and the integrand of the sum rule is given in Fig. 1 as a continuous line. Notice that C has to be positive in order to match the experimental value of $(\alpha - \beta)_0^t$ and that the zero at t_0 in the dressed Born amplitude has clearly disappeared. Moreover, in spite of the fundamental dynamics of the σ resonance in the t-channel polarizability sum rule, there is no trace of a resonant Breit-Wigner-type behavior when going to the physical real t axis; see Fig. 1.

The low-energy $\gamma\gamma \to \pi^0\pi^0$ cross sections obtained for the two cases studied above are similar [15]. The central values are compatible with the data for values of t below $(450\ \text{MeV})^2$ and are above the data but compatible within 2 standard deviations for larger values of t up to $(600\ \text{MeV})^2$ and within 1 standard deviation for t between $(600\ \text{MeV})^2$ and $(800\ \text{MeV})^2$.

When $F_0^0(t)$ is analytically continued to the complex plane, at t_σ on the first Riemann sheet one gets $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = (0.23^{+0.05}_{-0.09}) - i(0.30 \pm 0.03)$, which has a smaller absolute value when compared with $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B$ and leads to $\Gamma_{\text{pole}}(\sigma \to \gamma\gamma) = (1.0 \pm 0.3)$ keV. This is the main result of this Letter. The error quoted here is from the uncertainties in the experimental value of $(\alpha - \beta)_0^t$ and the inputs of the sum rule (2) only.

In order to obtain the rest of the uncertainty, we modify the σ properties in the pion scattering as follows. We still use the three-parameter fit formula including low-energy kaon data and high-energy data for $\cot[\delta(t)]$ in Ref. [2] as input in the amplitude $T(t)$ but with parameter values slightly modified in order to reproduce the σ pole position $t_\sigma = \{[(441 \pm 6) - i(272 \pm 4)]\ \text{MeV}\}^2$ found in Ref. [1]. In that case, we get $g_{\sigma\pi\pi} = [(480 \pm 7) + i(191 \pm 3)]$ MeV. With this $T(t)$ and the dressed Born amplitude in (3), one gets $(\alpha - \beta)_0^t|_B = 6.1 \pm 1.1$, $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B = (0.57 \pm 0.02) - i(0.41 \pm 0.03)$, and $\Gamma_{\text{pole}}(\sigma \to \gamma\gamma)|_B = 3.8 \pm 0.4$ keV. The integrand of $(\alpha - \beta)_0^t$ in (2) for this case is very similar to the dashed line of Fig. 1. The effective value of C in (8) moves to $C = 0.62 \pm 0.20$ when fixed to reproduce the experimental value of $(\alpha - \beta)_0^t$ within 1.5 standard deviations of the total uncertainty. With this new C, the analytic continuation to the new t_σ gives $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = (0.31^{+0.05}_{-0.07}) - i(0.32 \pm 0.03)$ and $\Gamma_{\text{pole}}(\sigma \to \gamma\gamma) = 1.5 \pm 0.4$ keV. Again, the integrand of $(\alpha - \beta)_0^t$ in (2) for this case is very similar to the continuous curve of Fig. 1.

As a final result for the electromagnetic pole width of the σ found in the $\pi\pi$ scattering amplitude, we quote

$$\Gamma_{\text{pole}}(\sigma \to \gamma\gamma) = 1.2 \pm 0.4\ \text{keV}, \tag{10}$$

which is the weighted average for the results of the $\sigma \to \gamma\gamma$ width using the $g_{\sigma\gamma\gamma}$ coupling in (6) obtained when the $F_0^0(t)$ amplitude in (3) is analytically continued to σ pole position t_σ on the first Riemann sheet (5) in two cases: first, when using for $\cot[\delta(t)]$ in (1) the three-parameter fit formula from Ref. [2] including both low-energy kaon data and high-energy data and, second, when varying the parameters of the fit for $\cot[\delta(t)]$ found in Ref. [2] in order to mimic the pole position found in Ref. [1]. In both cases, this $F_0^0(t)$ reproduces within 1.5 standard deviations the experimental value of $(\alpha - \beta)_0^t$ when inserted in the BEFT sum rule (2).

To conclude, we have shown that the scalar-isoscalar $\gamma\gamma \to \pi\pi$ amplitude $F_0^0(t)$ may be fixed by using analyticity, unitarity, and experimental information on the nucleon electromagnetic polarizabilities. This is possible and direct because this component is projected out in the sum rule (2). When both $F_0^0(t)$ in (3) and $T(t)$ in (1) are analytically continued to the complex plane, the σ pole position and its $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}$ and $g_{\sigma\pi\pi}$ residues become fixed.

We thank the CERN Theory Unit where this work was initiated for warm hospitality. This work has been supported in part by the European Commission (EC) RTN network FLAVIAnet Contract No. MRTN-CT-2006-035482 (J. P.), by MEC, Spain, and FEDER (EC) Grants No. FPA2005-01678 (J. B.) and No. FPA2006-05294 (J. P.), Sabbatical Grant No. PR2006-0369 (J. P.), by Junta de Andalucía Grants No. P05-FQM 101 (J. P.), No. P05-FQM 467 (J. P.), and No. P07-FQM 03048 (J. P.), and by the Spanish Consolider-Ingenio 2010 Programme CPAN Grant No. CSD2007-00042. We also thank Heiri Leutwyler, José A. Oller, José Ramón Peláez, and Mike Pennington for useful discussions and sharing unpublished results.

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