Theory of Smeared Quantum Phase Transitions

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We present an analytical strong-disorder renormalization group theory of the quantum phase transition in the dissipative random transverse-field Ising chain. For Ohmic dissipation, we solve the renormalization flow equations analytically, yielding asymptotically exact results for the low-temperature properties of the system. We find that the interplay between quantum fluctuations and Ohmic dissipation destroys the quantum critical point by smearing. We also determine the phase diagram and the behavior of observables in the vicinity of the smeared quantum phase transition.

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One of the most basic questions concerning phase transitions in random systems is whether or not a sharp transition survives in the presence of quenched disorder. Initially, it was suspected that disorder destroys any critical point because different spatial regions order at different temperatures. However, it was soon realized that classical continuous phase transitions generically remain sharp in the presence of weak disorder because finite spatial regions cannot undergo a true phase transition (see Ref. [1] and references therein).

Nonetheless, rare strongly coupled spatial regions play an important role. They can be locally in the ordered phase even if the bulk system is in the disordered phase. The slow fluctuations of these regions give rise to a singular free energy in a whole temperature region around the transition (called the Griffiths phase) [2,3]. In generic classical systems, this is a weak effect, because the Griffiths singularity is only an essential one. In contrast, rare regions can play a more important role at zero-temperature quantum phase transitions where order-parameter fluctuations in space and (imaginary) time need to be considered. Quenched disorder is perfectly correlated in time direction, and this enhances the Griffiths singularities. In the prototypical random transverse-field Ising systems, the singularities take power-law forms, implying, e.g., a divergent susceptibility in the Griffiths phase [4–6]. The transition itself is governed by an exotic infinite-randomness critical point [7,8], but remains sharp.

Recently, it was noted that dissipation can further enhance rare region effects at quantum phase transitions with Ising order-parameter symmetry. Each locally ordered region acts as two-level system. When coupled to an Ohmic dissipative bath, it can undergo the localization transition of the spin-boson problem [9]. Thus, each region can order independently of the bulk system, destroying the sharp phase transition by smearing [10]. In view of this observation, it would be highly desirable to treat the nonperturbative physics of these dissipative rare regions within the framework of the renormalization group (RG) commonly used to describe phase transitions. Such a theory would not only unveil the ultimate fate of the critical point, it would also predict *quantitatively* the behavior of many observables near the transition.

An important step towards this goal was taken by Schehr and Rieger [11,12] who studied the dissipative random transverse-field Ising chain by a numerical strong-disorder RG. They confirmed the smeared transition scenario and focused on the infinite-randomness "pseudo"-critical point arising at intermediate energy scales where dissipative effects are less important.

In this Letter, we develop a comprehensive strong-disorder RG for the dissipative random transverse-field Ising chain. We derive RG flow equations for the distributions of the fields, bonds and magnetic moments and solve them analytically, providing asymptotically exact low-energy results. We prove that the quantum critical point (QCP) is destroyed by Ohmic dissipation. Instead, a smeared quantum phase transition separates a conventional paramagnet from an inhomogeneously ordered ferromagnet (Fig. 1). In the remainder of the Letter, we sketch the derivation of our theory, compute important observables, and discuss the relevance of our results. Extensive details will be given in a longer paper.

Our starting point is the dissipative random transverse-field Ising chain defined by the Hamiltonian

$$H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + \sum_{i,n} \sigma_i^z \lambda_{i,n}(a_{i,n}^\dagger + a_{i,n}) + \sum_{i,n} \nu_{i,n} a_{i,n}^\dagger a_{i,n}, \tag{1}$$

where $\sigma_i^{x,z}$ are Pauli matrices. The bonds J_i and fields h_i are independent random variables; $a_{i,n}^\dagger$ ($a_{i,n}$) are the creation (annihilation) operators of the nth oscillator coupled to spin σ_i via $\lambda_{i,n}$, and $\nu_{i,n}$ is its frequency. Initially, all baths have the same Ohmic spectral function $\mathcal{E}(\omega) = \pi \sum_n \lambda_{i,n}^2 \delta(\omega - \nu_{i,n}) = 2\pi\alpha\omega e^{-\omega/\omega_c}$, with α the dimensionless dissipation strength and ω_c the (bare) cutoff energy. (The cutoff will change under the RG and the dissipation strength will become site-dependent.)

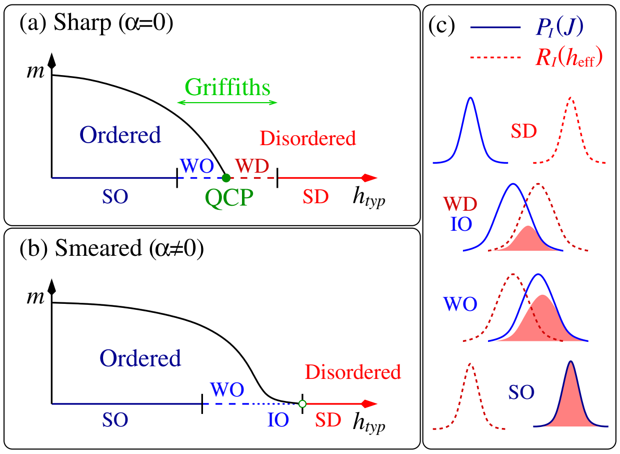

FIG. 1 (color online). Zero-temperature phase diagram and magnetization of the dissipative random transverse-field Ising chain as a function of the typical transverse field $h_{\rm typ}$. SO and SD denote the strongly ordered and disordered conventional phases; WO and WD are the weakly ordered and disordered quantum Griffiths phases. (a) No dissipation: sharp QCP. (b) Ohmic dissipation: smeared transition with the inhomogeneously ordered (IO) phase replacing the WD Griffiths phase. (c) Distributions of the bonds J and fields $h_{\rm eff}$ in the various phases. The shaded area quantifies the fraction w of J's bigger than $h_{\rm eff}$'s [see Eq. (10)].

To characterize the low-energy behavior of the system (1), we now develop a strong-disorder RG [13,14]. The idea of this method is to successively integrate out local high-energy modes. In our case, the competing energies are the transverse fields, bonds, and oscillator frequencies. Each RG step proceeds as follows: We first find the largest energy in the system $\Omega = \max(h_i, J_i, \omega_c/p)$ where $p \gg 1$ is an arbitrary constant [15]. We then lower the energy scale from Ω to $\Omega - d\Omega$ by (i) integrating out all oscillators (at all sites i) with frequencies between $p(\Omega - d\Omega)$ and $p\Omega$ and (ii) decimating all transverse fields and bonds between $(\Omega - d\Omega)$ and Ω.

For $p \gg 1$, the oscillators can be treated using adiabatic renormalization [9]. As a result, the transverse fields renormalize according to

$$\tilde{h}_i = h_i \exp\left(-\alpha_i \int_{p(\Omega - d\Omega)}^{p\Omega} \frac{d\omega}{\omega}\right) = h_i\left(1 - \alpha_i \frac{d\Omega}{\Omega}\right) \quad (2)$$

while the bonds remain unchanged. Here α_i is the renormalized dissipation strength at site i.

To decimate a strong bond $J_i = \Omega$, we assume the spins σ_i and σ_{i+1} to be locked together as an effective spin cluster $\tilde{\sigma}$ with moment $\tilde{\mu}$ and renormalized transverse field $\tilde{h}$ obtained in second order perturbation theory,

$$\tilde{\mu} = \mu_i + \mu_{i+1}, \quad (3)$$

$$\tilde{h} = h_i h_{i+1}/J_i. \quad (4)$$

$\tilde{\sigma}$ couples to a renormalized bath of dissipation strength

$$\tilde{\alpha} = \alpha_i + \alpha_{i+1} = \alpha(\mu_i + \mu_{i+1}) = \alpha\tilde{\mu}. \quad (5)$$

For a strong field, $h_i = \Omega$, the corresponding spin σ_i is delocalized in σ^z basis and thus eliminated, creating a new bond between sites $i-1$ and $i+1$,

$$\tilde{J} = J_{i-1}J_i/h_i. \quad (6)$$

Note that for spins about to be decimated, $h_i = \Omega$ is the fully renormalized tunnel splitting $h_i = h_{i0}(ph_{i0}/\omega_{c0})^{\alpha\mu_i/(1-\alpha\mu_i)}$ where h_{i0} and ω_{c0} are the field and bath cutoff of the ith cluster when it was formed at the higher energy ω_{c0}/p.

The recursion relations (3), (4), and (6) are identical to the dissipationless case [7], the baths enter only via (2) together with the renormalization of the dissipation strengths (5). Our RG procedure is related to the one implemented numerically by Schehr and Rieger [11]. However, treating the oscillator modes on equal footing with the other degrees of freedom (by reducing the bath cutoff globally in each step) allows us to solve the problem analytically.

The complete RG step consisting of recursion relations (2)–(6) is now iterated with the energy scale Ω being decreased. At each stage, the remaining bonds J and fields h are independent, but the fields and magnetic moments are correlated. Using logarithmic variables $\Gamma = \ln(\Omega_I/\Omega)$ [where Ω_I is the initial (bare) value of Ω], $\zeta = \ln(\Omega/J)$ and $\beta = \ln(\Omega/h)$, we can thus derive RG flow equations for the bond distribution $\mathcal{P}(\zeta)$ and the joint distribution of fields and moments $\mathcal{R}(\beta, \mu)$. They read

$$\frac{\partial \mathcal{P}}{\partial \Gamma} = \frac{\partial \mathcal{P}}{\partial \zeta} + (1 - \alpha\bar{\mu}_0)\mathcal{R}_\beta(0)(\mathcal{P} \overset{\zeta}{\otimes} \mathcal{P}) + [\mathcal{P}(0) - (1 - \alpha\bar{\mu}_0)\mathcal{R}_\beta(0)]\mathcal{P}, \quad (7)$$

$$\frac{\partial \mathcal{R}}{\partial \Gamma} = (1 - \alpha\mu)\frac{\partial \mathcal{R}}{\partial \beta} + \mathcal{P}(0)(\mathcal{R} \overset{\beta,\mu}{\otimes} \mathcal{R}) - [\mathcal{P}(0) - (1 - \alpha\bar{\mu}_0)\mathcal{R}_\beta(0)]\mathcal{R}, \quad (8)$$

where $\mathcal{R}_\beta(\beta) = \int_0^\infty \mathcal{R}(\beta, \mu)d\mu$ is the distribution of the fields and $\bar{\mu}_0$ is the average moment of clusters about to be decimated (defined by $\bar{\mu}_0\mathcal{R}_\beta(0) = \int_0^\infty \mu\mathcal{R}(0, \mu)d\mu$). The symbol $\mathcal{P} \overset{\zeta}{\otimes} \mathcal{P} = \int_0^\zeta \mathcal{P}(\zeta')\mathcal{P}(\zeta - \zeta')d\zeta'$ denotes the convolution. The first term on the r.h.s. of (7) and (8) is due to the rescaling of ζ and β with Γ and the renormalization (2) of h by the baths. The second term implements the recursion relations (3), (4), and (6) for the moments, fields and bonds. The last term ensures the normalization of $\mathcal{P}$ and $\mathcal{R}$. As expected, for $\alpha = 0$, (7) and (8) become identical to the dissipationless case [7,8].

Important insight can already be obtained from the structure of the flow equations. The probability of decimating a field, $(1 - \alpha\bar{\mu}_0)\mathcal{R}_\beta(0)$, decreases with increasing dissipation strength and cluster size. Clusters with moment $\mu > 1/\alpha$ are not decimated. Thus, in the presence of dissipation, the flow equations always contain a *finite* length scale above which the cluster dynamics freezes.

We now search for stationary solutions of the flow equations (7) and (8) that describe stable phases or critical points. There are two trivial cases: If all bonds are larger than all fields, only bonds are decimated, building larger and larger clusters. This is the conventional strongly ordered (SO) ferromagnetic phase. If only fields are decimated, we are in the conventional strongly disordered (SD) paramagnetic phase.

In the more interesting case of overlapping field and bond distributions, we look for solutions invariant under a general rescaling $\eta = \zeta/f_\zeta(\Gamma)$, $\theta = \beta/f_\beta(\Gamma)$ and $\nu = \mu/f_\mu(\Gamma)$.

Without dissipation, $\alpha = 0$, there are three types of well-behaved solutions [8]: a line of fixed points (parameterized by $\mathcal{R}_0$) with $f_\beta = 1$, $f_\zeta = \exp(\mathcal{R}_0\Gamma)$ and the average moment increasing as Γ. It corresponds to the weakly disordered (WD) Griffiths phase. There is another line of fixed points with $f_\zeta = 1$ and $f_\beta = f_\mu = \exp(\mathcal{P}_0\Gamma)$ (parameterized by $\mathcal{P}_0$) which corresponds to the weakly ordered (WO) Griffiths phase; and, separating these two phases, an infinite-randomness QCP with $f_\zeta = f_\beta = \Gamma$ and $f_\mu = \Gamma^\phi$, with $2\phi = 1 + \sqrt{5}$.

In the presence of dissipation, $\alpha \neq 0$, the scenario changes dramatically. For overlapping bond and field distributions, we found only *one* line of well-behaved fixed points (parameterized by $\mathcal{P}_0 > 0$) corresponding to the ordered phase [16]. Here, $f_\zeta = 1$, $f_\mu = \exp(\mathcal{P}_0\Gamma)$, $f_\beta = \Gamma\exp(\mathcal{P}_0\Gamma)$. The fields become much smaller than the bonds, justifying the perturbative treatment of the RG step. The fixed-point distributions are

$$\mathcal{P}^*(\zeta) = \mathcal{P}_0 e^{-\mathcal{P}_0\zeta}, \tag{9a}$$

$$\mathcal{R}^*(\theta, \nu) = \mathcal{R}_0\exp(-\mathcal{R}_0\nu)\delta(\theta - \alpha\nu); \tag{9b}$$

i.e., fields and moments are perfectly correlated. Here, $\mathcal{R}_0$ is a nonuniversal constant. This fixed point is similar to the WO Griffiths phase for $\alpha = 0$, but $f_\beta/f_\mu \to \infty$ as $\Gamma \to \infty$. Transforming the field distribution (9b) back to the original transverse fields h gives power-law behavior $\sim h^{\mathcal{R}_0/(\alpha f_\beta)-1}$. We could not analytically solve for the nonuniversal constants $\mathcal{P}_0$ and $\mathcal{R}_0$ in terms of the bare distributions and α. Their numerical values will be given elsewhere.

We emphasize that we have shown that there is no fixed-point solution with $f_\zeta/f_\beta \to$ const as $\Gamma \to \infty$ in the presence of dissipation, implying that there is no QCP where fields and bonds compete at *all* energy scales. This important result proves that Ohmic dissipation destroys Fisher's [7,8] infinite-randomness critical point. Physically, it is due to the fact that *finite* spin clusters (of size $\sim 1/\alpha$) can develop true magnetic order.

The complete low-energy thermodynamics can be obtained from the RG fixed-point solutions. To characterize the phase diagram (Fig. 1) in terms of the bare variables we introduce the probability

$$w = \int_0^\infty dJ P_I(J) \int_0^J dh_{\rm eff} R_I(h_{\rm eff}), \tag{10}$$

of a bare bond J being greater than an effective field (a bare field, fully renormalized by the baths) $h_{\rm eff} = h(ph/\omega_c)^{\alpha/(1-\alpha)}$. $P_I(J)$ and $R_I(h_{\rm eff})$ are the bare initial distributions of these variables [see Fig. 1(c)].

For $w = 0$ and $w = 1$, these distributions do not overlap. The system is in one of the conventional phases (SD or SO) without Griffiths singularities where disorder is RG irrelevant. For $0 < w \ll 1$, arbitrarily large rare clusters can form under renormalization. Without dissipation, $\alpha = 0$, these clusters have small but nonzero effective fields. They thus slowly fluctuate, and the system is in the WD Griffiths phase [see Fig. 1(a)]. In the presence of dissipation, $\alpha \neq 0$, clusters with moment $\mu > 1/\alpha$ have zero effective field. They freeze and order independently from the bulk. The sharp transition is thus destroyed by smearing, and the WD Griffiths phase is replaced by an inhomogeneously ordered (IO) ferromagnetic phase [see Fig. 1(b)]. Finally, with w approaching 1, the system develops bulk magnetic order but rare fluctuating clusters still exist; i.e., we are in the WO Griffiths phase. In the presence of dissipation, the IO and WO phases are separated by a crossover rather than a QCP. The asymptotic low-energy properties of both phases are described by the solution (9) with $\mathcal{P}_0$ monotonically decreasing with w.

We now turn our attention to observables near the smeared phase transition, focusing on the IO ferromagnetic phase which is the novel feature of our system. The magnetization is dominated by the large frozen droplets which arise in rare regions where the bonds are greater than the local fields. Because they are static, any weak coupling mediated by the bulk is sufficient to align them. Hence, the magnetization is proportional to the volume of the rare frozen droplets, which for α and $w \ll 1$, is

$$m \sim w^{1/\alpha}. \tag{11}$$

The low-temperature magnetic susceptibility can be computed by running the RG to energy scale $\Omega = T$ and assuming the remaining spin clusters to be free. For asymptotically low energies, the RG flow is dictated by the fixed-point solution (9), leading to

$$\chi \sim T^{-1-1/z}, \tag{12}$$

with $z = 1/\mathcal{P}_0$. Note, however, that at higher energies, the flow is dominated by strong fields and the susceptibility therefore behaves as in the weakly disordered undamped Griffiths phase [8]:

$$\chi \sim \delta^{4-2\phi}[\ln(1/T)]^2 T^{-1+1/z'}, \tag{13}$$

with $z' \approx 1/(2\delta)$, and $\delta \approx \langle \ln h_{\rm eff}\rangle - \langle \ln J\rangle$. The crossover energy Ω_c separating the two regimes can be estimated as the energy in which the high-energy mean moment cluster,

$\bar{\mu} \sim \Gamma\delta^{1-\phi}$, reaches the critical size $1/\alpha$. Hence, $\alpha \ln(\Omega_I/\Omega_c) \sim \delta^{\phi-1}$. Below Ω_c, the mean magnetic moment increases much more rapidly, $\bar{\mu} \sim \exp(\mathcal{P}_0\Gamma)$.

In summary, we have developed an asymptotically exact strong-disorder RG theory for the dissipative random transverse-field Ising chain. We have solved the resulting flow equations analytically and proven that the QCP is destroyed by smearing. The smearing is the result of the *interplay* between disorder and dissipation. Dissipation alone leads to a conventional critical point [17], while disorder alone leads to an exotic infinite-randomness critical point [7,8], but the transition remains sharp. In the remaining paragraphs, we put our results in broader perspective, and we discuss further implications.

We first consider the dissipative random transverse-field Ising model in higher dimensions. The recursion relations (2)–(5) are the same as in one dimension while decimating a field now generates couplings between all nearest neighbor sites, changing the topology of the lattice. An analytical solution of the RG flow equations thus appears impossible. However, the dissipation terms are local and take the same form as in one dimension. In particular, the probability of decimating a field, $(1 - \alpha\bar{\mu}_0)\mathcal{R}_\beta(0)$, is reduced with increasing dissipation and vanishes for clusters with finite moment $\mu > 1/\alpha$. Thus, a critical fixed-point solution is impossible, and the infinite-randomness critical point found in the dissipationless case [18,19] is destroyed by smearing. Moreover, the weakly disordered Griffiths phase is replaced by the inhomogeneously ordered ferromagnet. Note that Ohmic dissipation also suppresses the quantum Griffiths singularities at the percolation quantum phase transition [20] in a *diluted* transverse-field Ising model [21]. However, the percolation transition remains sharp because it is driven by the critical geometry of the lattice.

Our results for a dissipative *Ising* magnet must be contrasted with the behavior of systems with *continuous* $O(N)$ symmetry. While large Ising clusters freeze in the presence of Ohmic dissipation, $O(N)$ clusters continue to fluctuate with a rate exponentially small in their moment [22]. This leads to a sharp transition controlled by an infinite-randomness critical point in the same universality class as the dissipationless random transverse-field Ising model [23]. All these results are in agreement with a classification of weakly disordered phase transitions according to the effective dimensionality of the rare regions [24]. If their dimension is below the lower critical dimension d_c^- of the problem, the behavior is conventional; if it is right at d_c^-, the transition is of the infinite-randomness type, and if it is above d_c^-, finite clusters can order independently leading to a smeared transition.

To the best of our knowledge, this work is the first quantitative analytical theory of a smeared phase transition. The results directly apply to quantum phase transitions in disordered systems with discrete order-parameter symmetry and Ohmic damping. Our renormalization group approach should be broadly applicable to a variety of disordered dissipative quantum systems such as arrays of resistively shunted Josephson junctions [25,26].

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