Violation of the Cauchy-Schwarz Inequality in the Macroscopic Regime

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(Received 20 February 2008; published 10 June 2008)

We have observed a violation of the Cauchy-Schwarz inequality in the macroscopic regime by more than 8 standard deviations. The violation has been obtained while filtering out only the low-frequency noise of the quantum-correlated beams that results from the technical noise of the laser used to generate them. We use bright intensity-difference squeezed beams produced by four-wave mixing as the source of the correlated fields. We also demonstrate that squeezing does not necessarily imply a violation of the Cauchy-Schwarz inequality.

DOI: 10.1103/PhysRevLett.100.233601 PACS numbers: 42.50.Xa, 42.50.Dv, 42.65.Yj

The comparison between the predictions of quantum and classical theories has been a subject of study since the development of quantum mechanics. To that end, a number of different classical inequalities have been developed that provide an experimental discrimination between these theories [1,2]. Experiments showing a violation of these classical inequalities have verified quantum theory. However, to date, most of these experiments have been carried out in the regime in which particles are detected one at a time. It is thus interesting to study whether or not the quantum signature given by these tests is still present in the limit in which the system becomes macroscopic.

Among the inequalities that offer a test between quantum theory and classical electromagnetic theory is the Cauchy-Schwarz inequality (CSI) [1,2]. The first observation of a violation of this inequality was obtained by Clauser by using an atomic two-photon cascade system [3]. More recently, large violations by using four-wave mixing have been obtained [4,5], still in the photon-counting regime. For bright fields, the natural approach for analyzing their quantum nature is through noise measurements. In this case, the boundary between quantum and classical is taken to be the noise of a coherent state, or standard quantum limit (SQL), such that having a field with less noise than the SQL (squeezed light) is considered nonclassical. However, the presence of squeezing does not provide a direct discrimination between quantum theory and classical electromagnetic theory since the SQL is a result of quantum theory [2]. In addition, the CSI has some interesting implications about the nature of the field, which can be obtained from its relation to the quasiprobability functions, as described in Ref. [2].

The possibility of using a macroscopic quantum state to violate the CSI has been previously analyzed [6–9]. To date, however, only a few experiments have probed this macroscopic regime. Recently, antibunching of a small number of photons was observed in the continuous-variable regime [10]. In addition, a frequency analysis has been used to infer a violation of the CSI over limited frequency ranges [11].

In this Letter, we present a direct violation of the two-beam Cauchy-Schwarz inequality in the limit of a macroscopic quantum state. We show that the quantum-correlated fluctuations between two different modes of the electromagnetic field are responsible for the violation of the CSI. In addition to having a bright coherent carrier, we work in the high gain regime in which the mean occupation number of spontaneous correlated photons is much larger than 1 (of the order of the gain minus 1). Thus, photon counting is not an option, and continuous-variable detection schemes need to be used.

The CSI for the degree of second-order coherence $g^{(2)}$ for two distinct fields a and b is of the form [12]

$$[g_{ab}^{(2)}(\tau)]^2 \leq g_{aa}^{(2)}(0)g_{bb}^{(2)}(0), \tag{1}$$

where $g^{(2)}$ is the normalized intensity correlation function. This inequality indicates that for a classical system the cross correlation between two fields $g_{ab}^{(2)}$ cannot be larger than the geometric mean of the zero-time autocorrelations $g_{aa}^{(2)}$ and $g_{bb}^{(2)}$. According to quantum theory, however, it is possible to violate this inequality. In this case, the correlation function is defined in terms of normally ordered operators

$$g_{ab}^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t)\hat{b}^\dagger(t+\tau)\hat{b}(t+\tau)\hat{a}(t)\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle}, \tag{2}$$

where $\hat{a}$ and $\hat{b}$ are the photon annihilation operators for the two fields. A violation of the CSI indicates the presence of nonclassical correlations between the fields.

In the photon-counting regime, in which the occupation number is much less than 1, it is possible to obtain a large cross correlation while the zero-time autocorrelation functions are in principle equal to 2, giving a large violation of the CSI [12]. In contrast, in the large gain regime, all of the correlation functions tend to the same value (≥ 1), making it harder to observe a violation of the CSI.

We use a seeded four-wave mixing (4WM) process in a double-Λ system in Rb vapor, as described in

Refs. [13,14], as our source of bright correlated beams. Four-wave mixing is a parametric process, such that the initial and final states of the atomic system are the same. This leads to the emission of probe and conjugate photons in pairs and thus to intensity correlations between the two fields which are stronger than any correlations possible between classical optical fields.

The configuration and experimental parameters for the 4WM are the same as the ones described in Ref. [14]. A single Ti:sapphire laser and an acousto-optic modulator (AOM) are used to generate a bright pump and a weak probe which are resonant with a two-photon Raman transition between the $F = 2$ and $F = 3$ electronic ground states of ^{85}Rb. The pump is tuned 800 MHz to the blue of the D1 line at 795 nm, while the probe is down-shifted by 3 GHz with the AOM. The two beams are mixed at a small angle in a ^{85}Rb vapor cell, as shown in Fig. 1, at temperatures around 110 °C. In our double-Λ configuration, the 4WM converts two photons from the pump into one probe photon and one conjugate photon (up-shifted by 3 GHz with respect to the pump). We have measured up to 8 dB of intensity-difference squeezing at 1 MHz [14].

After the vapor cell, we separate the probe and conjugate from the pump beam with a polarizer with a $\approx 10^5$:1 extinction ratio for the pump. We then split the probe and conjugate, each into two beams of equal power ($\approx$0.5 mW), and detect the resulting four beams with separate photodiodes, as shown in Fig. 1. This setup directly measures the normally ordered correlation function defined in Eq. (2), as described in Ref. [2]. After each photodiode, a bias-T is used to separate the dc part of the photocurrent, which is recorded and then used to normalize the correlation functions. The rest of the signal is amplified and digitized with a resolution of 9 bits. The amplified time traces are sampled at a rate of 1 GS/s and 500 sets of traces, each with 10 000 points, are recorded. This setup allows us to simultaneously obtain all of the information needed to calculate the correlation functions and the noise power spectra of the different beams.

The bright correlated beams that are obtained from the seeded 4WM process consist of a large coherent part plus quantum-correlated fluctuations. As the intensity of the coherent part increases, the $g^{(2)}$ functions tend to 1, the value for a coherent state. It is thus useful to separate the

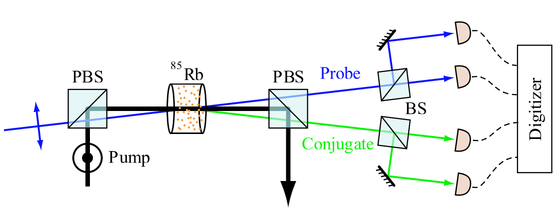

FIG. 1 (color online). Experimental setup. A 4WM process is used to generate quantum-correlated bright beams. PBS = polarizing beam splitter; BS = 50/50 beam splitter.

correlation functions into contributions for the coherent part of the field and the fluctuations, that is, $g^{(2)}_{ab} = 1 + \epsilon_{ab}$. Since the quantum correlations between the fields are in the fluctuations, we can rewrite the CSI in terms of the fluctuations such that

$$\epsilon_{ab} \leq \frac{\epsilon_{aa} + \epsilon_{bb}}{2}, \tag{3}$$

where we have kept only terms to first order in ϵ. For our experimental parameters, ϵ is of the order of 10^{-6}, as can be seen in Fig. 2. We define a violation factor $V \equiv (\epsilon_{aa} + \epsilon_{bb})/2\epsilon_{ab}$ such that $V < 1$ indicates a violation of the CSI.

In the ideal case, the 4WM process can be described by the two-photon squeeze operator $\hat{S}_{ab} = \exp(s\hat{a}\hat{b} - s\hat{a}^\dagger\hat{b}^\dagger)$, where s is the squeeze parameter ($s > 0$). The bright quantum-correlated beams are obtained by applying this operator to an input coherent state $|\alpha\rangle$ for the probe and the vacuum for the conjugate. In the limit in which the number of photons in the probe seed is much larger than 1 ($|\alpha| \gg 1$), V takes the form

$$V = 1 - \frac{1}{2G}, \tag{4}$$

where $G = \cosh^2(s)$ is the gain of the process and we have made the single frequency approximation for each beam. For an ideal seeded 4WM process, V is always less than 1, so that a violation of the CSI should always be obtained. In general, however, the presence of squeezing does not guarantee a violation of the CSI. As Eq. (4) shows, the amount of violation is inversely proportional to the gain. This is in contrast to the amount of intensity-difference squeezing that is expected from a seeded 4WM process, for which the noise scales as $1/(2G - 1)$. Thus, a large amount of

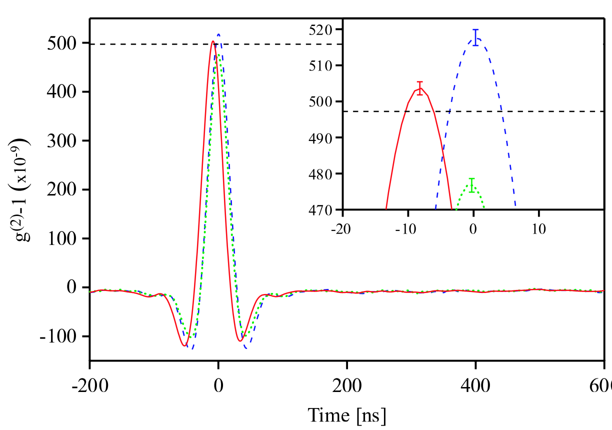

FIG. 2 (color online). Correlation functions of the fluctuations for the probe (dotted curve), conjugate (dashed curve), and cross $g^{(2)}$ (solid curve). The inset shows an expanded view of the peaks of the correlation functions. The horizontal dashed line shows the mean value of the zero-time autocorrelation functions for the probe and the conjugate. The indicated uncertainties are discussed in the text.

squeezing does not imply a large violation of the CSI, as has been pointed out in Ref. [15].

A typical set of correlation functions that exhibits a violation of the CSI is shown in Fig. 2. Here the horizontal dashed line indicates the mean value of the zero-time autocorrelation functions of the fluctuations for the probe and the conjugate, such that a cross correlation larger than this level indicates a violation of the CSI. A violation can clearly be seen in the inset in Fig. 2. In obtaining these correlation functions, we have filtered out only the technical noise below 500 kHz. The bandwidth of the detection system (>40 MHz) is larger than the bandwidth of the quantum correlations.

The uncertainties indicated in Fig. 2 are obtained by directly calculating the correlation functions and obtaining the standard deviation over the 500 sets of traces [16]. These uncertainties are not statistically independent since the probe and conjugate contain classical fluctuations that are strongly correlated as a result of slow intensity drifts of the pump and probe seed beams between data sets. This leads to a violation of the CSI that is more significant than what can be inferred from the inset. An accurate measure of the uncertainty is obtained by calculating V for each set of traces and using these results to derive its standard deviation. For the results shown in Fig. 2, the gain of the process is around 10 and $V = 0.9874(14)$, giving a violation of the CSI by more than 8 standard deviations.

The cross-correlation function shows a delay in the arrival time between probe and conjugate fluctuations; for the case shown in Fig. 2, the delay is around 8 ns. The delay results from the combination of 4WM in the double-Λ system and propagation through the vapor cell [17,18]. A property of the double-Λ system is that the relative delay between probe and conjugate for fluctuations of different frequencies (dispersion) is nearly fixed. Such a fixed delay causes only the cross correlation to be shifted in time and will not have an effect on V. In contrast, a large dispersion would make the cross-correlation peak wider and reduce its maximum value, degrading the amount of violation.

One of the difficulties in obtaining a violation of the CSI is that any source of excess uncorrelated noise will decrease it. In order to see this, we need to rewrite the CSI in terms of the noise power spectra of the different beams. We express the correlation functions in terms of the noise power spectra for the probe (S_p), conjugate (S_c), and intensity difference ($S_{\rm diff}$), as done in Ref. [11]. The fluctuations of the correlation functions ϵ can be obtained from these expressions and substituted into Eq. (3), such that the CSI takes the form

$$\int d\Omega\left(\frac{S_{\rm diff}(\Omega)}{\langle\hat{n}_p\rangle+\langle\hat{n}_c\rangle}-1\right)\geq\frac{\langle\hat{n}_p\rangle-\langle\hat{n}_c\rangle}{\langle\hat{n}_p\rangle+\langle\hat{n}_c\rangle}\int d\Omega\left[\left(\frac{S_p(\Omega)}{\langle\hat{n}_p\rangle}-1\right)-\left(\frac{S_c(\Omega)}{\langle\hat{n}_c\rangle}-1\right)\right]. \quad (5)$$

The terms in parentheses represent the excess noise (or noise reduction) with respect to the corresponding SQL. For the ideal seeded 4WM process, the normalized noise power spectra for the probe and the conjugate are equal, so that the term in square brackets is zero, and $\langle\hat{n}_p\rangle > \langle\hat{n}_c\rangle$. The presence of squeezing in the intensity difference can make the integral on the left-hand side negative, leading to a violation of the CSI. Excess noise can have an impact on the violation in two different ways. The presence of excess uncorrelated noise on either beam can lead the intensity-difference noise to go above the SQL for some frequency ranges such that the integral on the left-hand side can become positive. In addition, excess noise on the conjugate can make the right-hand side of the inequality negative enough (given that $\langle\hat{n}_p\rangle > \langle\hat{n}_c\rangle$) so that, even if squeezing is present, a violation might not be obtained.

For the results shown in Fig. 2, the corresponding normalized noise power spectra are shown in Fig. 3(a). All of the noise power spectra are calculated by taking the fast Fourier transform of the time traces and averaging over the 500 sets. The SQL for the probe and conjugate are calculated by obtaining the difference of the corresponding photocurrents, while the one for the intensity-difference noise is given by the sum of the SQLs for the probe and conjugate. As is expected for a 4WM process, both the probe and the conjugate have excess noise with respect to the SQL, and their spectra are almost the same. The measured intensity-difference squeezing has a bandwidth of 15 MHz, consistent with the gain bandwidth of the 4WM process [18], with a maximum squeezing of 6 dB. For this case, the system acts almost as an ideal 4WM medium. The measured squeezing is limited by a total detection efficiency, including optical path transmission and photodiode efficiencies, of $(80 \pm 3)\%$ [16]. We have verified that $g^{(2)}$ is not affected by loss so that any source of loss after the vapor cell will not have an impact on the violation of the CSI.

When the gain of the process is reduced to 2, we find a situation in which the noise power spectra of the probe and

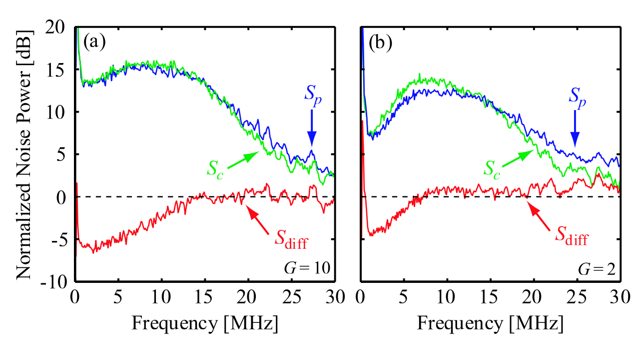

FIG. 3 (color online). Normalized noise power spectra for the probe (S_p), conjugate (S_c), and intensity difference ($S_{\rm diff}$) for a gain of (a) 10 and (b) 2. All of the spectra are normalized to their respective SQL, represented by the dashed line.

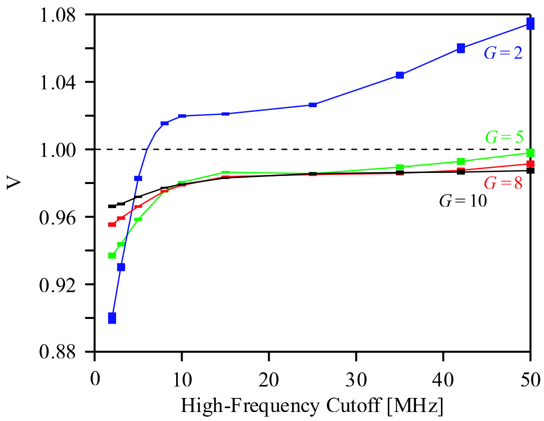

FIG. 4 (color online). Effect of frequency filtering on the violation of the CSI. Violation parameter (V) as a function of high-frequency cutoff for $G = 2$, $G = 5$, $G = 8$, and $G = 10$. $V < 1$ indicates a violation of the CSI. The height of the rectangles represents the statistical uncertainties.

the conjugate are noticeably different, as shown in Fig. 3(b). This difference in noise leads to a reduction of the intensity-difference squeezing bandwidth from 15 to 7 MHz and a small amount of excess noise at higher frequencies. For this particular case, we find that this excess noise is enough to prevent a violation of the CSI, such that $V = 1.0745(33)$, even though there is more than 4 dB of squeezing at low frequencies.

The relative delay between the probe and conjugate (8 ns for $G = 10$ and 13 ns for $G = 2$) has been compensated when calculating the intensity-difference noise power spectra shown in Fig. 3. This makes it possible to see the real squeezing bandwidth that results from the 4WM process in Fig. 3(a). While the relative delay has no effect on the violation of the CSI, it introduces a frequency-dependent phase shift such that the intensity-difference noise power spectrum oscillates between the noise power spectrum of the intensity difference and the intensity sum [19].

The effect of the excess noise can be further analyzed by filtering out the high frequencies, where most of the uncorrelated excess noise is present. The filtering is done on the digitized traces by applying a tenth-order Butterworth bandpass filter with a low-frequency cutoff of 500 kHz that filters out the technical noise of the laser and a variable high-frequency cutoff. We have done this analysis for a number of different gains, as shown in Fig. 4. The gain is changed by modifying the temperature of the cell and thus the atomic number density.

If we look at the lowest high-frequency cutoff points in Fig. 4, we see that the violation follows the trend given by Eq. (4) for an ideal 4WM process; that is, the violation gets better with smaller gains. However, once we increase the high-frequency cutoff, V starts to degrade, with lower gains degrading faster. Increasing this cutoff takes into account higher frequencies of the noise power spectrum that correspond to different regions of the gain profile. This leads to competition with other processes, such as Raman gain on the conjugate, that add excess noise. Except for the case of $G = 2$, a violation of the CSI is obtained for the different gains shown in Fig. 4 when only the low-frequency technical noise of the laser is filtered. Even for the case in which the system contains excess uncorrelated noise, a violation of the CSI can be recovered with enough filtering, as shown for the case of $G = 2$. This approaches a spectral analysis of the noise, as is regularly done with bright beams.

If we compare the case of $G = 2$ in Figs. 3 and 4, we find that the violation is lost at a high-frequency cutoff around 6 MHz while the squeezing is present over a larger frequency range than the filtering bandwidth used to calculate V, up to around 7 MHz, once the relative delay between probe and conjugate has been compensated. This gives a region in which squeezing is present but not a violation of the CSI. The excess noise on the conjugate is enough to destroy the violation but not the squeezing.

In conclusion, we have observed a violation of the Cauchy-Schwarz inequality in the macroscopic regime. The presence of excess uncorrelated noise can prevent a violation of the CSI. The ability to obtain a violation of the CSI with this system shows that the 4WM process provides a low-noise source of quantum-correlated bright beams over a large frequency range.

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