Next-to-Leading-Order QCD Corrections to J/ψ Polarization at Tevatron and Large-Hadron-Collider Energies

Bin Gong and Jian-Xiong Wang

Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(4), Beijing, 100049, China
Theoretical Physics Center for Science Facilities, Beijing, 100049, China

(Received 27 February 2008; published 11 June 2008)

We calculate the next-to-leading-order QCD corrections to J/ψ polarization at the Fermilab Tevatron and CERN Large Hadron Collider. Our results show that the J/ψ polarization status drastically changes from transverse-polarization dominant at leading order to longitudinal-polarization dominant at next-to-leading order. Although the theoretical evaluation predicts a larger longitudinal polarization than the value measured at the Tevatron, it may provide a solution towards the previous large discrepancy for J/ψ polarization between theoretical predication and experimental measurement.

DOI: 10.1103/PhysRevLett.100.232001 PACS numbers: 12.38.Bx, 13.25.Gv, 13.60.Le

Since its discovery in 1974, study on J/ψ production never ends, because there are still many problems unsolved yet, and all of them stimulate people to investigate the essence of the mechanisms which govern the reactions. It indeed provides an opportunity to probe both perturbative and nonperturbative aspects of QCD dynamics. Especially, there exists a huge discrepancy of the experimental data and theoretical predictions about the production rate of high-p_t J/ψ; to remedy it, a color-octet mechanism [1] was proposed based on the nonrelativistic QCD (NRQCD) [2]. The factorization formalism of NRQCD provides a theoretical framework for studying the heavy-quarkonia production. It allows a consistent theoretical prediction to be made and to be improved systematically in the QCD coupling constant α_s and the heavy-quark relative velocity v. Although it seems that the picture qualitatively agrees with experimental data, a simple application of NRQCD cannot provide satisfactory quantitative estimates for J/ψ photon production at the DESY ep collider HERA [3,4], $J/\psi(\psi')$ polarization of hadron production at the Fermilab Tevatron, and J/ψ production in B factories. Indeed, for J/ψ production, the theoretical prediction at leading order (LO) fails to reach an agreement with experimental data. Kramer *et al.* find that the experimental results on inelastic J/ψ photoproduction are adequately described by the color-singlet channel alone once higher-order QCD corrections are included [4]. The authors of Ref. [5] find that the data taken by the DELPHI Collaboration at CERN LEP2 [6] evidently favor the NRQCD formalism for J/ψ production in $\gamma\gamma \to J/\psi X$ rather than the color-singlet mechanism. Moreover, the LO NRQCD calculation predicts a sizable transverse-polarization rate for high-p_t J/ψ [7], while the measurement at Fermilab Tevatron [8] demands that there exists a slight longitudinal polarization.

To solve such puzzles, one natural way is to check if the next-to-leading-order (NLO) effects can change the situation. In fact, there are a few examples showing that NLO corrections may be quite large and change the whole characteristics of the theoretical predictions on p_t distribution of J/ψ. It is found in Ref. [9] that the contribution of higher-order QCD process $\gamma\gamma \to J/\psi c\bar{c}$ is of the same order as and/or even larger than that of the LO process for a color singlet in the high-p_t region. The authors of Ref. [10] indicate that at the Tevatron the NLO process $cg \to J/\psi c$, where the c quark in the initial state is the intrinsic charm quark of the beam proton, presents a large contribution for high-p_t J/ψ. Serious discrepancies between LO theoretical predictions [11,12] and experimental results [13,14] for the single and double charmonium productions in e^+e^- annihilation at B factories have been discussed by many authors. It seems that they may be resolved by including higher-order corrections: both NLO QCD and relativistic corrections [11,15,16].

Recently, the NLO QCD corrections to J/ψ hadron production have been calculated by Campbell, Maltoni, and Tramontano [17]. Their results show that the total cross section is boosted by a factor of about 2 and the production rate of J/ψ is much increased for larger transverse momentum p_t. The NLO process $gg \to J/\psi c\bar{c}$ has been calculated by Artoisenet, Lansberg, and Maltoni [18]. It causes a sizable contribution to the p_t distribution of J/ψ, especially at the high-p_t region. An s-channel treatment to J/ψ hadron production gives longitudinal polarization in the work of Haberzettl and Lansberg [19].

As the NLO correction is so important for the p_t distribution and the total cross section, it would be interesting to investigate its effects on the polarization status. In this Letter, we calculate the NLO QCD corrections to the J/ψ polarization at the Tevatron and LHC. In the calculation, we employ our Feynman diagram calculation package (FDC) [20] with additional parts which provide a complete method for calculating tensor and scalar integrals in the dimensional regularization scheme. This package has been used in our previous work [16].

At the LO, the partonic differential cross section for process $g(p_1) + g(p_2) \to J/\psi(p_3) + g(p_4)$ is evaluated in the framework of the NRQCD factorization as

$$\frac{d\hat{\sigma}^B}{dt} = \frac{5\pi\alpha_s^3|R_s(0)|^2[s^2(s-1)^2 + t^2(t-1)^2 + u^2(u-1)^2]}{144m_c^5 s^2(s-1)^2(t-1)^2(u-1)^2}, \tag{1}$$

with

$$s = \frac{(p_1+p_2)^2}{4m_c^2}, \qquad t = \frac{(p_1-p_3)^2}{4m_c^2}, \qquad u = \frac{(p_1-p_4)^2}{4m_c^2},$$

where $R_s(0)$ is the radial wave function of J/ψ at origin and the reasonable approximation $M_{J/\psi} \approx 2m_c$ is taken. The LO total cross section is obtained by convoluting the partonic cross section with the parton distribution function (PDF) $G_g(x, \mu_f)$ in the proton:

$$\sigma^B = \int dx_1 dx_2 G_g(x_1, \mu_f) G_g(x_2, \mu_f) \hat{\sigma}^B, \tag{2}$$

where μ_f is the factorization scale. In the following, $\hat{\sigma}$ represents the corresponding partonic cross section.

The NLO corrections include virtual corrections and real corrections. There are UV, IR, and Coulomb singularities in the calculation of the virtual corrections. The UV divergences existing in the self-energy and triangle diagrams are removed by the renormalization of the QCD gauge coupling constant, charm quark mass, and charm quark and gluon fields. Here we adopt the renormalization scheme used in Ref. [21]. For the charm quark mass and charm quark and gluon fields, the renormalization constants Z_m, Z_2, and Z_3 are determined by the on-mass-shell (OS) scheme, while for the QCD gauge coupling Z_g is fixed in the modified-minimal-subtraction ($\overline{\mathrm{MS}}$) scheme:

$$\begin{aligned}
\delta Z_m^{\mathrm{OS}} &= -3C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\mathrm{UV}}} - \gamma_E + \ln\frac{4\pi\mu_r^2}{m_c^2} + \frac{4}{3}\right],\\
\delta Z_2^{\mathrm{OS}} &= -C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\mathrm{UV}}} + \frac{2}{\epsilon_{\mathrm{IR}}} - 3\gamma_E + 3\ln\frac{4\pi\mu_r^2}{m_c^2} + 4\right],\\
\delta Z_3^{\mathrm{OS}} &= \frac{\alpha_s}{4\pi}\left[(\beta_0 - 2C_A)\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)\right],\\
\delta Z_g^{\overline{\mathrm{MS}}} &= -\frac{\beta_0}{2}\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\mathrm{UV}}} - \gamma_E + \ln(4\pi)\right],
\end{aligned} \tag{3}$$

where γ_E is the Euler's constant, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient of the QCD beta function, and n_f is the number of active quark flavors. There are three light quarks u, d, and s, so $n_f = 3$. In $\mathrm{SU}(3)_c$, the color factors are given by $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. μ_r is the renormalization scale.

After having fixed the renormalization scheme, there are 129 NLO diagrams in total, including counterterm diagrams. They are shown in Fig. 1 and divided into 8 groups. Diagrams of group (e) that have a virtual gluon line connected with the quark pair lead to a Coulomb singularity, which can be isolated and absorbed into the $c\bar{c}$ wave function. By adding all diagrams together, the virtual corrections to the differential cross section can be expressed as

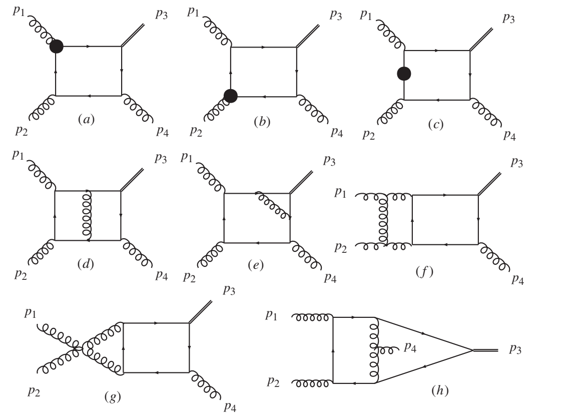

FIG. 1. One-loop diagrams for $gg \to J/\psi g$. Groups (a) and (b) are counterterm diagrams of the quark-gluon vertex and corresponding loop diagrams. Group (c) is the quark self-energy diagrams and corresponding counterterm ones. More diagrams can be obtained by permutation of external gluons.

$$\frac{d\hat{\sigma}^V}{dt} \propto 2\mathrm{Re}(M^B M^{V*}), \tag{4}$$

where M^B is the amplitude at LO and M^V is the renormalized amplitude at NLO. M^V is UV and Coulomb finite, but it still contains IR divergences:

$$\begin{aligned}
M^V|_{\mathrm{IR}} = &\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\\
&\times\left[-\frac{9}{2\epsilon_{\mathrm{IR}}^2} - \frac{3}{2\epsilon_{\mathrm{IR}}}\left(\ln\frac{s}{-t} + \ln\frac{s}{-u} - \frac{1}{3}n_f + \frac{11}{2}\right)\right]M^B.
\end{aligned} \tag{5}$$

The real corrections arise at the parton level subprocesses $gg \to J/\psi gg$, $gg \to J/\psi q\bar{q}$, and $gq(\bar{q}) \to J/\psi g q(\bar{q})$. The phase space integration of the above processes generates IR singularities, which are either soft or collinear and can be conveniently isolated by slicing the phase space into different regions. We use the two-cutoff phase space slicing method [22], which introduces two small cutoffs to decompose the phase space into three parts. Then the cross section can be written as

$$\sigma^R = \sigma^{\mathrm{H}\bar{\mathrm{C}}} + \sigma^S + \sigma^{\mathrm{HC}} + \sigma^{\mathrm{HC}}_{\mathrm{add}}. \tag{6}$$

The hard noncollinear part $\sigma^{\mathrm{H}\bar{\mathrm{C}}}$ is IR finite; one can numerically compute it by using standard Monte Carlo integration techniques. $\hat{\sigma}^S$ comes from the soft regions containing soft singularities; thus, it is calculated analytically under soft approximation. It is easy to find that the soft singularities caused by emitting soft gluons from the quark pair in the S-wave color singlet J/ψ are canceled by each other. σ^{HC} from the hard collinear regions contains collinear singularities which are factorized out, and the singularities are partly absorbed into the redefinition of the PDF (usually it is called the mass factorization [23]). Here we adopt the scale-dependent PDF using the $\overline{\mathrm{MS}}$

convention given by Ref. [22]. After redefinition of the PDF, an additional term $\sigma_{\rm add}^{\rm HC}$ is separated out. Finally, all of the IR singularities are canceled analytically for $\hat{\sigma}^S + \hat{\sigma}^{\rm HC} + \hat{\sigma}^V$.

To obtain the transverse momentum distribution of J/ψ, a transformation of integration variables ($dx_2dt \to Jdp_tdy$) is introduced. Thus we have

$$\frac{d\sigma}{dp_t} = \int Jdx_1dyG_g(x_1, \mu_f)G_g(x_2, \mu_f)\frac{d\hat{\sigma}}{dt}, \qquad (7)$$

where y and p_t are the rapidity and transverse momentum of J/ψ in the laboratory frame, respectively. The polarization factor α is defined as

$$\alpha(p_t) = \frac{d\sigma_T/dp_t - 2d\sigma_L/dp_t}{d\sigma_T/dp_t + 2d\sigma_L/dp_t}. \qquad (8)$$

To calculate $\alpha(p_t)$, the polarization of J/ψ must be explicitly retained in the calculation. The partonic differential cross section for a polarized J/ψ could be expressed as

$$\frac{d\hat{\sigma}_\lambda}{dt} = a\epsilon(\lambda)\cdot\epsilon^*(\lambda) + \sum_{i,j=1,2} a_{ij}p_i\cdot\epsilon(\lambda)p_j\cdot\epsilon^*(\lambda), \quad (9)$$

where $\lambda = T_1, T_2, L$. $\epsilon(T_1)$, $\epsilon(T_2)$, and $\epsilon(L)$ are the two transverse-polarization vectors and the longitudinal one of J/ψ, and the polarizations of all of the other particles are summed over in n dimensions. It causes a more difficult tensor reduction path than that with all of the polarizations being summed over in the calculation of virtual corrections. One can find that a and a_{ij} are finite when the virtual corrections and real corrections are summed up.

To make a cross-check, we carry out another calculation. Namely, we calculate the differential cross section $\sigma_{\rm add}^{\rm HC}$ and $\hat{\sigma}^S + \hat{\sigma}^{\rm HC} + \hat{\sigma}^V$ with the polarizations of all particles being summed up analytically. The results are numerically compared with that obtained without summing up the polarization of J/ψ. Moreover, to check the gauge invariance, in the expression we explicitly keep the gluon polarization vector and then replace it by its 4-momentum in the final numerical calculation. Definitely the result must be zero, and our results confirm it. To calculate the real correction $\sigma^{\rm H\bar{C}}$ to the process, only numerical computation is carried out, and we sum over only the physical polarizations of the gluons to avoid involving diagrams which contain external ghost lines.

In our numerical calculations, the CTEQ6L1 and CTEQ6M PDFs [24] and the corresponding fitted values $\alpha_s(M_Z) = 0.130$ and $\alpha_s(M_Z) = 0.118$ are used for LO and NLO calculations, respectively. For the charm quark mass and the wave function at the origin of J/ψ, we take $m_c = 1.5$ GeV and $|R_s(0)|^2 = 0.810$ GeV^3, respectively. The two phase space cutoffs $\delta_s = 10^{-3}$ and $\delta_c = \delta_s/50$ are chosen. To check whether the final results depend on the two cutoffs, different values of δ_s and δ_c are used, where δ_s can be as small as $\delta_s = 10^{-5}$, and the invariance is obviously observed within the error tolerance of less than 1%. It is well known that the perturbative expansion cannot be applicable to the regions with small transverse momentum and large rapidity of J/ψ. Therefore, the result is restricted to the domain $p_t > 3$ GeV and $|y_{J/\psi}|$ being less than 3 or 0.6.

The dependence of the total cross section at the renormalization scale μ_r and factorization scale μ_f is obtained, and it agrees with Ref [17]. In Fig. 2, the p_t distribution of J/ψ is presented, and the J/ψ polarization factor α as a function of p_t is shown in Fig. 3. At LO, α is always positive and becomes closer to 1 as p_t increases, and this figure means that the transverse polarization is more than the longitudinal one and even becomes dominant in the high-p_t region. But there is a dramatic change when the NLO QCD corrections are taken into account. α is always negative and becomes closer to -0.9 as p_t increases; this new figure indicates that the longitudinal polarization is always more than the transverse one and even becomes dominant in the high-p_t region. Meanwhile, the J/ψ polarization of process $gg \to J/\psi c\bar{c}$ is nearly zero. By including the contribution of this subprocess, the curve of NLO$^+$ in Fig. 3 is closer to the experimental data.

In summary, we have calculated the NLO QCD correction to the J/ψ hadron production at the Tevatron and LHC. Dimensional regularization is applied to deal with the UV and IR singularities in the calculation, and the Coulomb singularity is isolated and absorbed into the $c\bar{c}$ bound state wave function. To deal with the soft and collinear singularities in the real corrections, the two-cutoff phase space slicing method is used. By summing over all of the contributions, a result which is UV, IR, and Coulomb finite is obtained. Numerically, we obtain a K factor (the ratio of the NLO contribution to the LO) of about 2 for the total cross section with $\mu_r = \mu_f = \sqrt{(2m_c)^2 + p_t^2}$. The transverse momentum distribution of J/ψ is presented in

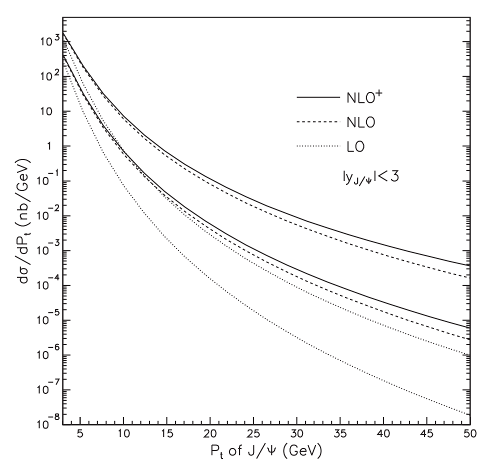

FIG. 2. Transverse momentum distribution of differential cross section with $\mu_r = \mu_f = \sqrt{(2m_c)^2 + p_t^2}$ at the LHC (upper curves) and the Tevatron (lower curves). The center mass energy is $\sqrt{s_{\rm Tevatron}} = 1.98$ TeV and $\sqrt{s_{\rm LHC}} = 14$ TeV, respectively. NLO$^+$ denotes the result including a contribution from $gg \to J/\psi c\bar{c}$ at NLO.

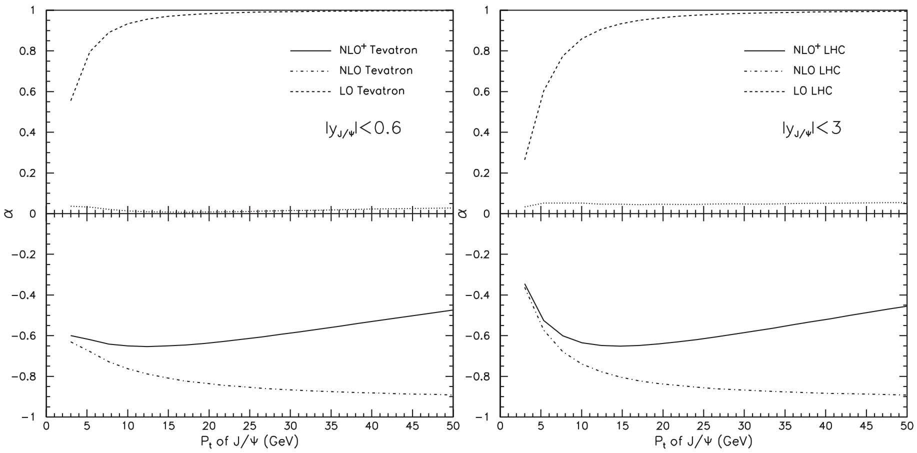

FIG. 3. Transverse momentum distribution of polarization α with $\mu_r = \mu_f = \sqrt{(2m_c)^2 + p_t^2}$ at the Tevatron (left) and the LHC (right). The unlabeled dotted line denotes the polarization of $gg \to J/\psi c\bar{c}$, and NLO$^+$ denotes the result including that of $gg \to J/\psi c\bar{c}$.

the text, and we show that the NLO corrections increase the differential cross section more as p_t becomes larger and eventually can enhance it by 2 or 3 orders in magnitude at $p_t = 50$ GeV. It confirms the calculation given in Ref. [17]. The real correction process $gg \to J/\psi c\bar{c}$ is also calculated, and the results agree with those of Ref. [18].

The NLO contributions to the J/ψ polarization are studied for the first time, and our results indicate that the J/ψ polarization is dramatically changed from more transverse polarization at LO into more longitudinal polarization at NLO. Even though our calculation results in a more longitudinal-polarization state than the recent experimental data [8] at the Tevatron, it raises a hope to solve the large discrepancy between the LO theoretical predication on J/ψ polarization and the experimental measurement and suggests that the next important step is to calculate the NLO correction to the hadron production of color-octet state $J/\psi^{(8)}$. By refixing the color-octet matrix elements, we will see what an involvement of the NLO corrections can induce for the polarization of J/ψ.

We thank Xue Qian Li for useful comments and discussions. This work is supported by the National Natural Science Foundation of China (No. 10775141) and by the Chinese Academy of Sciences under Project No. KJCX3-SYW-N2.

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