Microscopic Origin of Low-Frequency Flux Noise in Josephson Circuits

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We analyze the data and discuss their implications for the microscopic origin of the low-frequency flux noise in superconducting circuits. We argue that this noise is produced by spins at superconductor insulator boundary whose dynamics is due to RKKY interaction. We show that this mechanism explains size independence of the noise, different frequency dependences of the spectra reported in large and small SQUIDs, and gives the correct intensity for realistic parameters.

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Studies of the flux noise in superconducting structures have a long history that began in the 1980s with the demonstration that it is the flux and not the critical current noise that limits the sensitivity of dc SQUIDs (superconducting quantum interference devices) [1,2]. The noise phenomenological characterization performed at this time revealed many puzzling features that defied a simple model of its microscopic origin, so this problem was put aside and largely forgotten. Recently, interest in it was renewed when it was realized that the flux noise limits the coherence of qubits based on superconducting devices [3] and that the dephasing of ''flux'' qubits is due by low-frequency flux noise with intensity comparable to the one measured in dc SQUIDs [4,5]. It is likely that the same flux noise will limit the quantum coherence in ''phase'' qubits [6].

Recently two models for the excess $1/f$ flux noise were proposed. The first one [7] proposes that the flux noise is due to the electrons that hop between traps in which their spins have fixed, random orientations. The second model [8] attributes the noise to the electrons that experience spin flips induced by the interaction with tunneling two level systems (TLSs) and phonons. Both models rely on some assumptions that are difficult to justify: for example, in order to match the intensity of the noise spectral density reported in the experiments, the number of *thermally* activated TLSs present in the oxide layer has to be much larger than in a typical glass at the same temperature. For instance, in a typical loop of radius $R = 1\ \mu$m and volume 10^7 nm^3 the observed noise value implies activation of 10^5–10^6 spin fluctuators with magnetic moment μ_B while a typical glass of the volume has about 10 thermally excited TLSs at $T = 0.1$ K. This Letter has two goals: (i) to present a critical analysis of the flux noise phenomenology and its implication for the possible models of its microscopic origin, (ii) to propose a novel mechanism in which the low-frequency noise is due to spin diffusion on the superconductor surface generated by the exchange mediated by the conduction electrons. We demonstrate that this spin dynamics together with the spatial dependence of the surface current density on the thin superconducting SQUID loop leads to low-frequency $1/f^\alpha$ flux noise spectral density with $\alpha \in \{0, 1\}$ and that the intensity of the noise does not depend on the area of the SQUID, as long the ratio R/W remains constant (W denotes the width of the SQUID line); however, the details of diffusion in large SQUIDs, with $W \sim 100\ \mu$m and small SQUIDs having $W \sim 1\ \mu$m might be different. In particular, the frequency dependence of the noise spectrum might vary depending on the size of the SQUIDs and the measured frequency range.

All experiments agree on the magnitude of the noise at frequency $f \sim 1$ Hz and its area independence. Specifically, Wellstood [2] observed noise spectra $S_\Phi^{1/2}(1\ \text{Hz}) \approx 4$–$10\ \mu\Phi_0\,\text{Hz}^{-1/2}$ at temperature below 0.1 K in Nb and Pb dc SQUIDs with sizes in the range R, $W \sim 30$–$300\ \mu$m on Si/SiO substrate with Nb/NbO/PnIn Josephson junctions. Cromar *et al.* [9] reported the value $S_\Phi^{1/2}(1\ \text{Hz}) = 2.3\ \mu\Phi_0\,\text{Hz}^{-1/2}$ for Nb SQUIDs device with very high quality Nb/AlO/Nb junctions at 4 K. Finally, Bialczak *et al.* [10] measured $S_\Phi^{1/2}(1\ \text{Hz}) = 2\ \mu\Phi_0\,\text{Hz}^{-1/2}$ at 20 mK in a Al SQUID loop of size $W \sim 1\ \mu$m on sapphire substrate with Al/AlO/Al Josephson junctions. Although the details of temperature dependence were studied only in [2], all experiments agree that the noise does not decrease at very low temperatures. Similarly, all data show homogeneous noise spectra where single strong fluctuators cannot be resolved. The frequency dependence of the noise is more controversial. Namely, Wellstood's flux noise power spectra in the frequency range 1–10^3 Hz displayed $1/f^\alpha$ dependence with exponent $\alpha = 0.66$ at low temperatures $0.022\ \text{K} < T < 1$ K and with exponent $\alpha = 1$ at $1\ \text{K} < T < 4.2$ K. The data [9] show the dependence $f^{-0.7}$ in the interval 400–10^3 Hz at 4 K, below 400 Hz the frequency dependence decreases to approximately $f^{-0.1}$ and completely ceases in the 0.1–40 Hz interval. Bialczak *et al.* [10] reported $1/f^\alpha$ spectrum with $\alpha = 0.95$ in the frequency range 10^{-5}–1 Hz at 20 mK. The temperature dependence of the noise [2] shows two different temperature regimes: at <0.5 K the noise is T independent, while at $1\ \text{K} < T < 4.2$ K it displays T^2 dependence with the crossover regime ($0.5\ \text{K} < T < 1$ K) that is nonmonotonic in some samples. Two distinct regimes suggest two different microscopic mechanisms for the noise at low and high temperature; in the following we shall focus on the low temperature regime. As it will be clear below, a very important piece of information is the high frequency cutoff of the $1/f^\alpha$ dependence. Unfortunately, no direct measurements are available but the observed dephasing of the flux qubits indicates that this cutoff is at least 10 MHz [11].

We now discuss the implications of the data for the noise origin. The noise persistence at low temperatures indicates that it is due to a subsystem characterized by very low energy scales, smaller than the minimal temperature available experimentally (∼20 mK). This rules out the thermally excited TLSs [7] or vortices and points towards weakly interacting nuclear or electron spins. The spin mechanisms agree also with the observation of homogeneous low-frequency noise power spectra (which is incompatible with the vortex origin). Nuclear spins can be excluded for three reasons: first, the flux produced by each spin scales as $1/L$ (where $L \sim R$, W is the linear size of the device) thereby leading to $S_\Phi^{1/2}(\omega) \propto L^{1/2}$ while data are roughly size independent; second, *all* frequency scales associated with nuclear spins in low magnetic fields ($H \lesssim 1$ G) are very low ($f < 1$ kHz) in contrast with the results of the dephasing analysis which shows that $1/f$ persists up to 10 MHz [11]; third, one expects that nuclear spin noise would be substrate dependent [7]. Paramagnetic electron spins located on the superconductor or insulator interfaces seem to be more promising candidates since their contribution to the flux noise is roughly size independent. Properties of these spins were extensively studied for Si/SiO_2 interfaces. ESR experiments have shown that (i) the surface density of spins varies between $\nu_{2D} \approx 10^{10}$–10^{12} cm^{-2} [12], (ii) the g factor of these spins is isotropic and it has value $g = 2.001\,36 \pm 0.000\,03$ [13]. As we show below, such surface density is barely sufficient to explain the level of flux noise at 1 Hz *if* one assumes that *all* these spins remain active at low temperatures but is difficult to reconcile them with schemes in which only a small percentage of the spins remain active at low T [7]. The value $g \cong 2$ shows that the spin orbit coupling is very weak indicating that the interaction between paramagnetic spins and TLSs is very small in contrast to assumptions of Refs. [7,8].

The dynamics of the electron spins in the insulator substrate is due to the interaction with other electron spins or with surrounding nuclei. For a dilute spin system such as Si/SiO_2 interfaces all energy scales associated with these interactions are too small to account for the wide frequency range observed experimentally: both dipole-dipole interaction between electron spins with density $\nu_{2D} \approx 10^{12}$ cm^{-2} and their interaction with nuclear moments of Si that have a natural concentration of 5% correspond to $f \approx 10$ kHz. Estimating the total flux noise produced by these spins, i.e., $\int S(\omega)d\omega$, we get $\sim(\mu_0\mu_B)^2 \times (R/W)\nu_{2D}\Phi_0^2$ with $(\mu_0\mu_B)^2 \sim 10^{-26}$ cm^2 which is of the right order of magnitude but somewhat smaller than the observed noise. We conclude that these spins in the insulator are unlikely to provide the dominant source of noise.

The energy scales are much larger for the electron spins in the proximity of the superconductor which allows RKKY interaction between them. Physically it is due to the conduction electron polarization by the impurity spin: although in a superconductor this total polarization of the Cooper pair is zero, the local polarization at scales $r \lesssim \xi$ is the same as in the normal metal (here ξ denotes the superconductor coherence length). At very low temperatures, this interaction might freeze the spins in a glassy state [14]. We shall show below that it is unlikely to occur in typical experimental conditions [15]. Formally, the RKKY interaction is due to the Hamiltonian: $H_K = \mathcal{J}\hat{S}\cdot\hat{\sigma}$ that couples localized spins and conduction electrons. Here $\hat{S}$ is the spin operator for the impurity, $\hat{\sigma}$ is the spin operator of a conduction electron, and $\mathcal{J}$ is the exchange constant. Integrating out the conduction electrons one gets the interaction between two spins i, j located at distance r_{ij}:

$$H_{\rm RKKY} = \sum_{i,j} V(r_{ij})\hat{S}_i\hat{S}_j, \tag{1}$$

where $V(r) = V_0(r)e^{-2r/\xi}r^{-3}\cos\varphi$ and φ changes quickly on the length scale of the Fermi wavelength λ_F [17]. The interaction "constant" $V_0(r)$ is a weak function of the distance; it is controlled by the electron density of states ν, the Fermi velocity v_F, and the Kondo temperature: $V_0(r) = (2\pi)^{-1}\nu\mathcal{J}^2(r)$, with $\mathcal{J}(r) = 2[\nu\ln^2(v_F/(rT_K))]^{-1}$ so that the average interaction at $r \ll \xi$ reads

$$\langle V^2(r)\rangle^{1/2} = \frac{1}{2\sqrt{2}\pi\nu r^3}\left(\frac{2}{\ln[v_F/(rT_K)]}\right)^2. \tag{2}$$

Because of this interaction, the magnetization $M(t, r)$ of spins averaged over the volume that contains $N \gg 1$ spins obeys the diffusion equation

$$\left[\frac{d}{dt} - \mathcal{D}\nabla^2\right]M(t, r) = 0, \tag{3}$$

with diffusion coefficient $\mathcal{D}$ which depends on the typical distance between the spins with surface density σ_s, i.e., $r = 1/\sqrt{\sigma_s} \approx 10$–$10^2$ nm and the average interaction $\langle V^2(r)\rangle^{1/2}$ (2). Typical electron density of states for Al, Pb, and Nb are, respectively, $\nu_{\rm Al} = 35/{\rm eV\,nm}^3$, $\nu_{\rm Pb} = 44/{\rm eV\,nm}^3$, and $\nu_{\rm Nb} = 160/{\rm eV\,nm}^3$. Assuming Kondo temperatures $T_K \approx 0.01$–1 K, we estimate

$$\mathcal{D} = r^2\langle V^2(r)\rangle^{1/2} \approx 10^8\text{–}10^9 \ {\rm nm}^2\,{\rm s}^{-1}. \tag{4}$$

This model neglects a few important physical effects. First, it neglects the spin orbit scattering and assumes that the diffusion process involves only electron spins located on the SI interface. As a result, the total magnetization M of the spins in contact with the superconductor is conserved.

Second, the estimate for $\mathcal{D}$ in Eq. (4) assumes that the spins are in direct contact with the metal. However, paramagnetic spins responsible for the flux noise are likely to be located in the surface oxide having thickness $d =$ 2–3 nm, with some of them farther away from the superconducting wire. For impurity located at depth y from the superconductor, the strength of RKKY interaction decreases as $V(r, y) \sim e^{-2y/a_0} V(r)$, where a_0 is the atomic distance. A more realistic model should include "fast" spins at the surface with the diffusion constant given in Eq. (4) and with effective density $\sigma_s^{\text{fast}} = (a_0/d)\, \sigma_s^{\text{tot}}$ and slower spins coupled to the fast subsystem by a weakened RKKY interaction. Third, the diffusion approximation for the spin dynamics neglects the effect of the rare pairs of spins located at distances much smaller than the average distance between the spins. Such spins are strongly coupled with each other, the difference in the energy of their triplet and singlet state is much larger than their coupling to their neighbors, so they change their state rarely. This mechanism generates an additional noise at low frequencies.

To find the effective flux Φ_{eff} produced by the spin magnetization, we determine the spin energy E in the field of the test current I in the loop. We find that

$$\Phi_{\text{eff}} = \frac{dE}{dI} = g\mu_B \int \frac{\hat{S}(r)B(r)}{I} d^2 r. \qquad (5)$$

Here μ_B is the Bohr magneton, $\hat{S}(r)$ is the spin density operator, and $B(r)$ denotes the probing magnetic field. Conservation of the total magnetization by spin diffusion means that it would not produce any noise if the probing magnetic field were uniform. In fact, it is not: the SQUID loop is typically a strip conductor of width W greater than its thickness b with length $L \gg W$ and penetration depth $\lambda \sim$ 100–200 nm $\gtrsim b$ so that $bW \gg \lambda^2$. In these conditions, the dependence of the current density on x near the center of the strip is $J_s(x) = 2I/(\pi W)[1 - (2x/W)^2]^{-1/2}$ for $-W/2 + \lambda < x < W/2 - \lambda$, while the current density falls away exponentially to zero at the edges $\pm W/2$ [18]. This current density results in a probing magnetic field $B(x) = \frac{\mu_0}{2} J_s(x)$. The spin diffusion together with the divergency of the surface current density close to the edges of the loop generates $1/f$ flux spectral density:

$$\langle \Phi_\tau \Phi_0 \rangle = (g\mu_B)^2 L \int_{-W/2}^{W/2} dx\, dx' \, \frac{\hat{S}_\tau(x)B(x)\hat{S}_0(x')B(x')}{I^2}, \qquad (6)$$

where x is the coordinate across the wire strip. To compute the integral in Eq. (6) we expand the spin density operator $\hat{S}_t(x)$ as a series of orthonormal eigenfunctions of the diffusion equation (3) with boundary conditions ensuring zero magnetization current at the wire boundary, i.e., $\frac{d}{dt} M_t(x = \pm W/2) = 0$:

$$\hat{S}_t(x) = \sqrt{\frac{2}{W}} \sum_{q=\pi n/W} \hat{S}_q(t) \cos\left[\left(x + \frac{W}{2}\right) q\right], \qquad (7)$$

where n is positive integer. By substituting the expansion given in Eq. (7) into Eq. (6) we find

$$\langle \Phi_\tau \Phi_0 \rangle = (g\mu_B)^2 L \sum_q B_q^2 \langle \hat{S}_q(\tau) \hat{S}_q(0) \rangle, \qquad (8)$$

where

$$B_q = \sqrt{\frac{2}{W}} \int_{-W/2}^{W/2} dx \frac{B(x)}{I} \cos\left[\left(x + \frac{W}{2}\right) q\right] = \frac{\mu_0}{\sqrt{2W}} \mathcal{J}_0\left(\frac{qW}{2}\right) \cos\left(\frac{qW}{2}\right), \qquad (9)$$

where $\mathcal{J}_0(x)$ is the Bessel function. The Fourier transform of the spin density correlator (8) is found from the solution of the diffusion equation (3):

$$\langle \hat{S}_q^2(\omega) \rangle = \frac{\sigma_s}{2} \frac{\mathcal{D} q^2}{\omega^2 + (\mathcal{D} q^2)^2}. \qquad (10)$$

We can define two frequency regimes: small frequencies with $f \ll f_W$ and large frequencies with $f \gg f_W$. f_W is the characteristic equilibrium frequency for spins that diffuse across SQUID of width W:

$$f_W = \frac{\mathcal{D}}{W^2} = \begin{cases} 10^{-2}\text{–}10^{-1}\ \text{Hz} & \text{if } W \sim 100\ \mu\text{m}; \\ 10^{2}\text{–}10^{3}\ \text{Hz} & \text{if } W \sim 1\ \mu\text{m}. \end{cases}$$

At small frequencies, the flux noise spectrum given in Eq. (8) is white, with noise amplitude given by

$$\langle \Phi^2 \rangle_{\omega \to 0} = \left(\frac{\mu_0 \mu_B}{2\pi}\right)^2 \sigma_s \frac{L}{W} \frac{\mathcal{J}_0(\pi)^2}{f_W}; \qquad (11)$$

where $\mathcal{J}_0(\pi) = -0.3042$. At large frequencies, Eq. (9) reduces to $B_q = \sqrt{\frac{2}{\pi}} \frac{\mu_0}{W} \frac{1}{\sqrt{q}}$ and the flux noise spectrum given in Eq. (8) becomes

$$\langle \Phi^2 \rangle = \frac{2}{\pi^2} (\mu_0 \mu_B)^2 \sigma_s \frac{L}{W} \int_0^\infty \frac{dq}{q} \frac{\mathcal{D} q^2}{\omega^2 + (\mathcal{D} q^2)^2} = \frac{4}{\pi} (\mu_0 \mu_B)^2 \sigma_s \frac{R}{W} \frac{1}{f}, \qquad (12)$$

where we have written explicitly the length of the SQUID loop $L = 2\pi R$. At intermediate frequencies, we expect a crossover between $1/f$ and white noise behavior. It is quite straightforward to estimate the intensity of the $1/f$ noise. Assuming that $\sigma_s \approx 10^{16}\ \text{m}^{-2}$ (similar density was reported for Si/SiO_2 interfaces) and that $R/W \sim 10$ we find flux noise spectral density $S_\Phi(1\ \text{Hz}) \approx 3\ (\mu\Phi_0)^2\ \text{Hz}^{-1}$, in agreement with the observed "universal" value. Because the noise is due to the spins on the surface, its level has the same R/W dependence as in Ref. [10].

Thus, the spin diffusion model explains the excess flux noise measured in large SQUIDs [2,9], which spectra correspond to the intermediate-to-high frequencies regime, but not the $1/f$ noise observed in much smaller devices [10] since the latter was measured in the range corresponding to the low-frequency regime where purely spin diffusion model predicts a constant spectral density. However, two physical effects missing in the model that were mentioned above are very likely to produce a significant low-frequency noise in the smaller SQUIDs: the presence of weakly coupled spins further away from superconductor and the presence of strongly coupled spin pairs. Indeed, assuming flat distribution of the spin depth inside the insulating layer one gets an exponential distribution of the coupling to the spins on SI interface $P(\mathcal{J}) \propto 1/\mathcal{J}$ that directly translates into the $1/f$ spectrum of the noise generated by these spins. The intensity of this noise is determined by the areal density, σ_s^{slow} of the spins in the layer of approximately atomic depth $\sim 2a_0$. Generally, one expects $\sigma_s^{\text{fast}} \sim \sigma_s^{\text{slow}}$ which results in a smooth crossover from $1/f$ dependence due to fast spin diffusion to the one due to slow spin dynamics. A significant difference of these values might lead to a more complicated frequency dependence of the noise. Our preliminary analysis shows that close pairs of spins strongly coupled by RKKY interaction lead to $1/f$ contribution to the low-frequency noise as well.

Finally, we discuss experimental tests of the proposed model. The crucial ingredient of our analysis is the rough temperature independence of the noise below 200 mK [2], it would be important to verify it for small devices. The spin origin of the noise can be tested by applying a significant external magnetic field. If this field is larger than the local field B_{loc} produced by the spin neighbors, the spin rotates around the axis determined by the external field. If it is orthogonal to the probing field, fast rotation of the spin implies that the effective spin noise is shifted to high frequencies. If these fields are parallel, the effect is much less. The effective probing field acting on the spins on the insulator boundary inside the SQUID loop is mostly perpendicular to the surface of the sample, while the probing field of the spins on SI boundary is parallel to it. Thus, applying magnetic field in different directions one can verify the spin mechanisms and determine the spin location. The local field that should be exceeded in these experiments is of the order of $B_{\text{loc}} \lesssim 100$ G for the spins 10 nm apart on SI surface and of the order of $B_{\text{loc}} \lesssim 0.1$ G for the spins in the insulator. The random position of spins in these models implies that there will be always strongly coupled spin pairs capable of producing the low-frequency noise but the number of such pairs should go down rapidly with field. The validity of the spin models discussed in this Letter can be also tested directly by fabrication of the samples with decreased density of spin defects on the surface of the insulator and by protecting the surface of superconductor of a layer of another metal, e.g., Re. In this Letter we have not discussed complicated mechanisms involving the combined effects of electron and nuclear spins such as electron spin rotation induced by nuclear spin nearby. We believe that it is unlikely that these mechanisms can produce sufficiently high upper frequency cutoff and sufficient noise level for a natural Si with a low concentration of nuclear spins but this should be also verified experimentally by measuring noise on isotopically pure Si substrates. Finally, in our model the $1/f$ dependence of the noise is due to fast diffusion and the divergent dependence of the probe magnetic field at the edge of the SQUID loop. Thicker SQUID loops have different spatial current distribution; this should affect the frequency dependence of the noise.

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[1] R. H. Koch *et al.*, J. Low Temp. Phys. **51**, 207 (1983).
[2] F. C. Wellstood, C. Urbina, and J. Clarke, IEEE Trans. Magn. **23**, 1662 (1987).
[3] J. Q. You and F. Nori, Phys. Today **58**, No. 11, 42 (2005).
[4] F. Yoshihara *et al.*, Phys. Rev. Lett. **97**, 167001 (2006).
[5] K. Kakuyanagi *et al.*, Phys. Rev. Lett. **98**, 047004 (2007).
[6] J. M. Martinis *et al.*, Phys. Rev. Lett. **89**, 117901 (2002).
[7] R. H. Koch, D. P. DiVincenzo, and J. Clarke, Phys. Rev. Lett. **98**, 267003 (2007).
[8] R. de Sousa, Phys. Rev. B **76**, 245306 (2007).
[9] M. W. Cromar *et al.*, IEEE Trans. Magn. **25**, 1005 (1989).
[10] R. C. Bialczak *et al.*, Phys. Rev. Lett. **99**, 187006 (2007).
[11] Y. Nakamura (private communication).
[12] T. Schenkel *et al.*, Appl. Phys. Lett. **88**, 112101 (2006).
[13] A. Stesmans and B. Nouwen, Phys. Rev. B **61**, 16 068 (2000).
[14] J. A. Mydosh, *Spin Glasses: An Experimental Introduction* (Taylor & Francis, London, 1993).
[15] Very recent data [16] indicate, however, that such freezing might occur at lowest temperatures.
[16] S. Sendelbach *et al.*, following Letter, Phys. Rev. Lett. **100**, 227006 (2008).
[17] V. M. Galtiski and A. I. Larkin, Phys. Rev. B **66**, 064526 (2002).
[18] T. V. Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits* (Prentice-Hall, New Jersey, NJ, 1998).