Delayed-Choice Test of Quantum Complementarity with Interfering Single Photons

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We report an experimental test of quantum complementarity with single-photon pulses sent into a Mach-Zehnder interferometer with an output beam splitter of adjustable reflection coefficient R. In addition, the experiment is realized in Wheeler's delayed-choice regime. Each randomly set value of R allows us to observe interference with visibility V and to obtain incomplete which-path information characterized by the distinguishability parameter D. Measured values of V and D are found to fulfill the complementarity relation $V^2 + D^2 \leq 1$.

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As emphasized by Bohr [1], complementarity lies at the heart of quantum mechanics. A celebrated example is the illustration of wave-particle duality by considering single particles in a two-path interferometer [2], where one chooses either to observe interference fringes, associated to a wavelike behavior, or to know which path of the interferometer has been followed, according to a particlelike behavior [3]. Although interference has been observed at the individual particle level with electrons [4], neutrons [5], atoms [6,7], molecules [8], only a few experiments with massive particles have explicitly checked the mutual exclusiveness of which-path information (WPI) and interference [9–13].

In the case of photons, it has been pinpointed that meaningful two-path interference experiments demand a single-photon source [14] for which full and unambiguous WPI can be obtained, complementary to the observation of interference [14–16]. In order to rule out a too naive view of complementarity, which would assume that the particle could "know" when entering the apparatus which experimental configuration has been set (record of interference or determination of WPI) and would then adjust its behavior accordingly [17], Wheeler proposed the "delayed-choice" scheme where the choice between the two complementary measurements is made long after the particle entered the interferometer [18]. Realizations of that gedanken experiment [19–22] have confirmed that the chosen observable can be determined with perfect accuracy even if the choice, made by a quantum random number generator, is spacelike separated from the entering of the particle into the interferometer [22].

In 1978, Wooters and Zurek [23] addressed an intermediate situation in which interaction with the interferometer considered as a quantum device allows one to gain an imperfect—but significant—knowledge of WPI, without destroying the interference pattern, which remains observable with a good—although reduced—visibility. In 1988, Greenberger and Yasin noticed that in an unbalanced interferometer as used in some neutron interferometry experiments, one has partial WPI while keeping interference with limited visibility [24]. The complementary quantities—WPI and interference visibility—could then be partially determined simultaneously.

Consistent theoretical analysis of both schemes, independently published by Jaeger *et al.* [25] and by Englert [26] leads to the inequality [27]

$$V^2 + D^2 \leq 1 \tag{1}$$

which puts an upper bound to the maximum values of simultaneously determined interference visibility V and path distinguishability D, a parameter that quantifies the available WPI on the quantum system.

The all-or-nothing cases ($V = 1$, $D = 0$) or ($V = 0$, $D = 1$) [4–8,14–16] obviously fulfill inequality (1). Intermediate situations, corresponding to partial WPI and reduced visibility, have been investigated using atoms [28], nuclear spins [29] and faint laser light [30]. However, none of them has been realized in the delayed-choice scheme. Elaborating on a delayed-choice setup with true single-photon pulses in a Mach-Zehnder interferometer [22], we report here an experimental test of the complementarity inequality (1) through the realization of intermediate situations between the two all-or-nothing cases.

Following Englert [26], we point out that the distinguishability D constrained by inequality (1) corresponds to two different notions. The *a priori* distinguishability, also called "predictability," refers to a WPI obtained by using an unbalanced interferometer with different particle flux along the two paths. Only the case where path distinguishability is introduced *a posteriori*—i.e., after the entering of the particles into the interferometer—offers the opportunity of a delayed-choice test of complementarity. This *a posteriori* distinguishability can be introduced either by creating entanglement between the particle and a which-path marker [13,31] or by using an interferometer with an unbalanced output beam splitter [28]. We have chosen the latter case by implementing the scheme depicted on Fig. 1, corresponding to a Mach-Zehnder interferometer with a variable output beam splitter (VBS) of adjustable reflection coefficient R.

The experiment starts from a clock-triggered single-photon source, based on the photoluminescence of a single N-V color center in a diamond nanocrystal [32]. The linearly polarized single-photon pulses are then directed to a polarization Mach-Zehnder interferometer described in Ref. [22]. The input polarization beam splitter BS splits the light pulse into two spatially separated components of equal amplitudes, associated with orthogonal S and P polarizations. The two beams then propagate in free space for 48 m.

The variable output beam splitter VBS is the association of a polarization beam splitter (PBS) which spatially overlaps the two beams, an electro-optical modulator (EOM) which acts as an adjustable wave plate, and a Wollaston prism (WP) with its polarization eigenstates corresponding to the S and P polarized channels of the interferometer (Fig. 2). Given the relative orientation β of the EOM, the VBS reflection coefficient R depends on the voltage V_{EOM} applied to the EOM, according to the relation

$$R = \sin^2 2\beta \, \sin^2\left(\frac{\pi}{2}\frac{V_{\mathrm{EOM}}}{V_\pi}\right) \tag{2}$$

where V_π is the half-wave voltage of the EOM. The parameters β and V_π have been independently measured for our experimental conditions and found equal to $\beta = 24 \pm 1^\circ$ and $V_\pi = 217 \pm 1$ V at the wavelength $\lambda = 670$ nm which is the emission peak of the negatively charged N-V color center [32]. This allows R to vary between 0 and 0.5 when V_{EOM} is varied between 0 and 170 V.

When $R = 0$, the VBS is equivalent to a perfectly transparent (or absent) beam splitter. Then, each "click" of one of the two photodetectors (P_1 or P_2) placed on the output ports of the interferometer is associated to a specific path. It then gives access to the full WPI ($D = 1$), and no interference effect will be observed ($V = 0$). When $R \neq 0$, paths 1 and 2 are partially recombined by the VBS. The WPI is then partially washed out, up to be totally erased when $R = 0.5$. On the other hand, interference can be observed when the phase-shift Φ between paths 1 and 2 is varied. The experiment will consist of checking the relation between D and V for a given value of R, controlled by the EOM voltage V_{EOM}.

In order to perform the experimental test of complementarity in the delayed-choice regime, the chosen configuration of the interferometer, defined by R, has to be causally isolated from the entering of the photon into the interferometer. This condition is ensured by a relativistically spacelike separated random choice [22]. For each measurement, the value of the reflection coefficient of VBS is randomly chosen between 0 and a given value of R, using a quantum random number generator (QRNG) located at the output of the interferometer (Fig. 2). The random numbers are generated from the amplified shotnoise of a white light beam which is an intrinsic quantum random process. For each single-photon trigger pulse, fast comparison of the amplified shotnoise to the zero level generates a binary random number 0 or 1 which sets the VBS reflectivity to either 0 or R by applying the corresponding voltage to the EOM [see Eq. (2)]. In the laboratory framework, the random choice of VBS reflectivity is realized simultaneously with the entering of the corresponding emitted photon into the interferometer, ensuring the required spacelike separation [22].

As meaningful illustration of complementarity requires the use of single particles, the quantum behavior of the light field is first tested using the two output detectors feeding single and coincidence counters with no voltage applied to the EOM. The output beam splitter is then absent, and each detector is therefore univocally associated

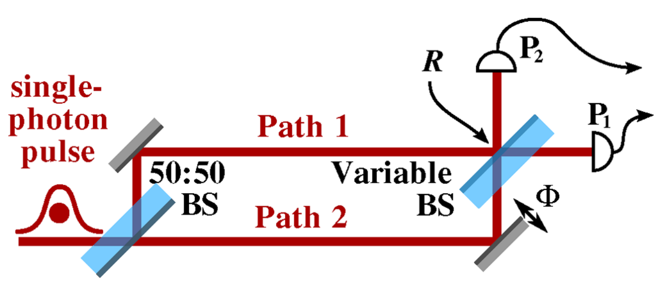

FIG. 1 (color online). Delayed-choice complementarity-test experiment. A single-photon pulse is sent into a Mach-Zehnder interferometer, composed of a 50/50 input beam splitter (BS) and a variable output beam splitter (VBS). The reflection coefficient is randomly set either to the null value or to an adjustable value R, after the photon has entered the interferometer. The single-photon photodetectors P_1 and P_2 allow to record both the interference and the WPI.

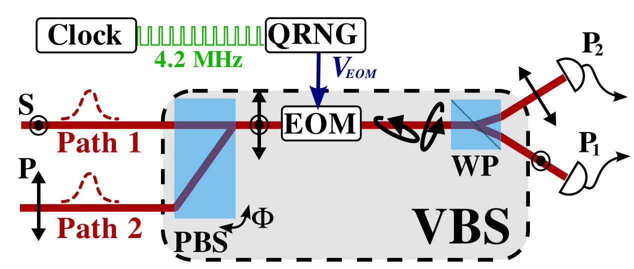

FIG. 2 (color online). Variable output beam splitter (VBS) implementation. The optical axis of the polarization beam splitter (PBS) and the polarization eigenstates of the Wollaston prism (WP) are aligned, and make an angle β with the optical axis of the EOM. The voltage V_{EOM} applied to the EOM is randomly chosen accordingly to the output of a Quantum Random Number Generator (QRNG), located at the output of the interferometer and synchronized on the 4.2-MHz clock that triggers the single-photon emission.

to a given path. In this situation, we measure the correlation parameter α [14,16] which is equivalent to the second order correlation function at zero delay $g^{(2)}(0)$. For an ideal single-photon source, quantum optics predicts a perfect anticorrelation $\alpha = 0$, in agreement with the particlelike image that the photon cannot be detected simultaneously in the two paths of the interferometer. With our source [32], we find $\alpha = 0.15 \pm 0.01$. This value, much smaller than 1, shows that we are indeed close to the pure single-photon regime [33].

The delayed-choice test of complementarity with single-photon pulses is performed with the EOM randomly switched for each photon sent in the interferometer, corresponding to a random choice between two values 0 and R of the VBS reflectivity. The phase-shift Φ between the two arms of the interferometer is varied by tilting the polarization beam splitter PBS of VBS with a piezoelectric actuator (see Fig. 2). For each photon, we record the chosen configuration of the interferometer, the detection events, and the actuator position. All raw data are saved in real time and are processed only after a run is completed. The events corresponding to each configuration of the interferometer are finally sorted. For a given value R, the wavelike information of the light field is obtained by measuring the visibility of the interference, predicted to be

$$V = 2\sqrt{R(1-R)}. \tag{3}$$

The results, depicted in Fig. 3, show a reduction of V when the randomly applied value of R decreases.

To test inequality (1), a value of the distinguishability D is then required, to qualitatively qualify the amount of WPI which can be extracted for each value of R. We introduce the quantity D_1 (resp. D_2), associated to the WPI on path 1 (resp. path 2):

$$D_1 = |p(P_1, \text{path } 1) - p(P_2, \text{path } 1)| \tag{4}$$

$$D_2 = |p(P_1, \text{path } 2) - p(P_2, \text{path } 2)| \tag{5}$$

where $p(P_i, \text{path } j)$ is the probability that the particle follows path j and is detected on detector P_i. For a single particle arriving on the output beam splitter, one obtains

$$D_1 = D_2 = \left|\frac{1}{2} - R\right|. \tag{6}$$

The distinguishability parameter D is finally defined as [26]

$$D = D_1 + D_2 = |1 - 2R|. \tag{7}$$

In order to test this relation, we estimate the values of D_1 and D_2 by blocking one path of the interferometer and measuring the number of detections N_1 and N_2 on detectors P_1 and P_2, which are statistically related to D_1 and D_2 according to [22,28]:

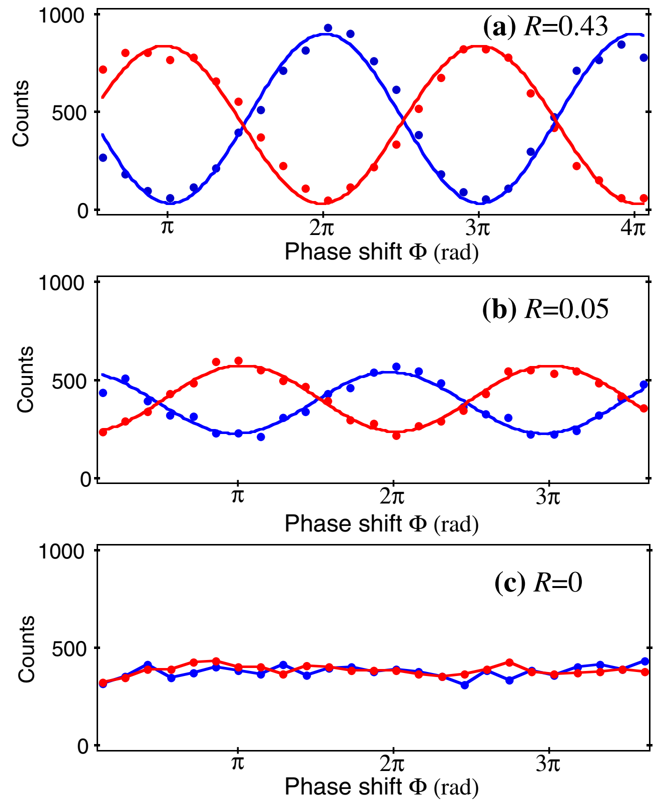

FIG. 3 (color online). Interference visibility V measured in the delayed-choice regime for different values of V_{EOM}. (a)–(c) correspond to $V_{\mathrm{EOM}} \approx 150$ V ($R = 0.43$ and $V = 93 \pm 2\%$), $V_{\mathrm{EOM}} \approx 40$ V ($R = 0.05$ and $V = 42 \pm 2\%$), and $V_{\mathrm{EOM}} = 0$ ($R = 0$ and $V = 0$). Each point is recorded with 1.9 s acquisition time. Detectors dark counts, corresponding to a rate of 60 s^{-1} for each, have been substracted to the data.

$$D_1 = \frac{1}{2}\frac{|N_1 - N_2|}{N_1 + N_2}\Big]_{\text{path 2 blocked}} \tag{8}$$

$$D_2 = \frac{1}{2}\frac{|N_1 - N_2|}{N_1 + N_2}\Big]_{\text{path 1 blocked}}. \tag{9}$$

These measurements are also performed in the delayed-choice regime, using the procedure described above. We finally obtain independent measurements of D and V for different values of the reflection coefficient R, randomly applied to the interferometer. The final results, depicted on Fig. 4, lead to $V^2 + D^2 = 0.97 \pm 0.03$, close to the maximal value permitted by inequality (1) even though each quantity varies from 0 to 1.

The effects observed in this delayed-choice experiment are in perfect agreement with quantum mechanics predictions. No change is observed between a so-called "normal-choice" experiment and the "delayed-choice" version. It demonstrates that the complementarity principle cannot be interpreted in a naive way, assuming that the photon at the input of the interferometer could adjust its nature according to the experimental setup installed. As Bohr pointed

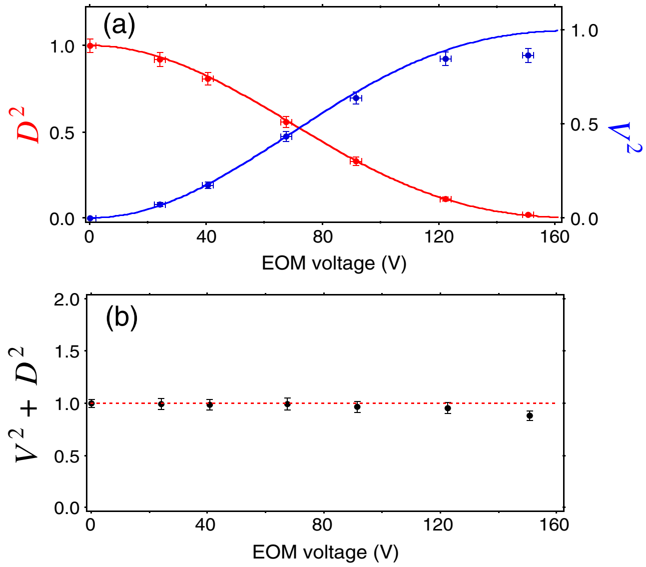

FIG. 4 (color online). Delayed choice test of complementarity with single-photon pulses. (a) Wavelike information V^2 and which-path information D^2 as a function of the EOM voltage corresponding to a given value R of the VBS reflectivity. The solid lines are the theoretical expectations, with $\beta = 24°$ and $V_\pi = 217$ V, using Eqs. (2), (3), and (7). (b) $V^2 + D^2$ as a function of the EOM voltage.

out [34], "It obviously can make no difference as regards observable effects obtainable by a definite experimental arrangement, whether our plans of constructing or handling the instrument are fixed beforehand or whether we prefer to postpone the completion of our planning until a later moment when the particle is already on its way from one instrument to another." Such an intriguing property of quantum mechanics forces one to renounce some common-sense representations of the physical reality.

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