Optimum Spin Squeezing in Bose-Einstein Condensates with Particle Losses

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The problem of spin squeezing with a bimodal condensate in the presence of particle losses is solved analytically by the Monte Carlo wave function method. We find the largest obtainable spin squeezing as a function of the one-body loss rate, the two-body and three-body rate constants, and the s-wave scattering length.

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Spin squeezed states, first introduced in [1], generalize to spin operators the idea of squeezing developed in quantum optics. In atomic systems, effective spins are collective variables that can be defined in terms of two different internal states of the atoms [2] or two orthogonal bosonic modes [3]. States with a large coherence between the two modes, that is with a large mean value of the spin component in the equatorial plane of the Bloch sphere, can still differ by their spin fluctuations. For an uncorrelated ensemble of atoms, the quantum noise is evenly distributed among the spin components orthogonal to the mean spin. However, quantum correlations can redistribute this noise and reduce the variance of one spin quadrature with respect to the uncorrelated case, achieving spin squeezing. Besides applications in quantum communication and quantum information [4], these multiparticle entangled states have practical interest in atom interferometry, and high precision spectroscopy [5] where they could be used to beat the standard quantum limit already reached in atomic clocks [6].

Different techniques to create spin squeezed states in atomic systems have been proposed and successfully realized experimentally including transfer of squeezing from light to matter [7] and quantum nondemolition measurements of the atomic state [8]. To go further, it was shown that coherent interactions between cold atoms in a bimodal Bose-Einstein condensates [3] can in principle provide a huge amount of entanglement and spin squeezing. It is thus important to determine the ultimate limitations imposed by decoherence to the maximum spin squeezing that can be obtained by this method. Several forms of decoherence may be present in the experiment. The case of a dephasing perturbation was studied in [9]. In this Letter, we deal with particle losses, an unavoidable source of decoherence in cold atom systems, due, e.g., to collisions of condensed atoms with the hot background gas, and to three-body collisions leading to molecule formation.

As shown in [3], bimodal Bose-Einstein condensates realize the one-axis twisting model proposed in [1] to create spin squeezing. This exactly solvable model predicts a perfect squeezing in the limit of a very large system: Formally $\xi^2 \to 0$ for $N \to \infty$, N being the number of particles in the system and ξ^2 the squeezing parameter defined in Eq. (8). We expect losses to degrade the squeezing [3] that is $\xi^2_{\text{no loss}} \le \xi^2_{\text{with loss}}$ for any value of N. However, as $\xi^2_{\text{no loss}} \to 0$ as $N \to \infty$, this inequality does not tell us what will be the best squeezing in the presence of losses. In particular, the limit $\lim_{N\to+\infty} \xi^2_{\text{with loss}}$ could be zero (perfect squeezing), a very small constant, or a constant close to one (one meaning no squeezing). We show that the second possibility is the correct one if the trap frequency is optimized. The best achievable squeezing is reached for $N \to +\infty$, and we derive its explicit expression, as a function of the scattering length and the loss constants K_1, K_2, K_3.

We consider two spatially separated symmetric condensates a and b prepared in an initial state with N particles and a well defined relative phase [10]

$$|\phi\rangle \equiv \frac{1}{\sqrt{N!}}\left(\frac{e^{i\phi}a^\dagger + e^{-i\phi}b^\dagger}{\sqrt{2}}\right)^N |0\rangle. \tag{1}$$

We assume that $\phi = 0$ initially. Correspondingly, the x component of the collective spin $S_x = (a^\dagger b + b^\dagger a)/2$ has a mean value $\langle S_x \rangle = N/2$. Here, we assume that no excitation is created during the preparation process, and we neglect all the other modes than the condensate modes a and b. When expanded over Fock states $|N_a, N_b\rangle$, the state (1) shows binomial coefficients which, for large N, are peaked around the average number of particles in a and b, $\bar{N}_a = \bar{N}_b = N/2$. We use this fact to approximate the Hamiltonian with its quadratic expansion around $\bar{N}_a$ and $\bar{N}_b$ [11]: $H_0 = \sum_{\epsilon=a,b} E(\bar{N}_\epsilon) + \mu_\epsilon(\hat{N}_\epsilon - \bar{N}_\epsilon) + \frac{1}{2}\mu'_\epsilon(\hat{N}_\epsilon - \bar{N}_\epsilon)^2$ where μ_ϵ is the chemical potential for the ϵ condensate and $\mu'_\epsilon \equiv (\partial_{N_\epsilon}\mu_\epsilon)_{\bar{N}_\epsilon}$. In the symmetric case, we can write

$$H_0 = f(a^\dagger a + b^\dagger b) + \frac{\hbar\chi}{4}(a^\dagger a - b^\dagger b)^2 \tag{2}$$

where $\chi = \mu'_a/\hbar$. The first term in H_0 is some function f of the total atom number: It commutes with the density operator ρ of the system and can be omitted.

In presence of one-, two-, and three-body losses, the evolution of the density operator, in the interaction picture with respect to H_0, is ruled by the master equation

$$\frac{d\tilde{\rho}}{dt} = \sum_{m=1}^{3} \sum_{\epsilon=a,b} \gamma^{(m)} \Big[c_\epsilon^m \tilde{\rho} c_\epsilon^{\dagger m} - \frac{1}{2}\{c_\epsilon^{\dagger m} c_\epsilon^m, \tilde{\rho}\} \Big] \quad (3)$$

where $\tilde{\rho} = e^{iH_0 t/\hbar} \rho e^{-iH_0 t/\hbar}$, $c_a = e^{iH_0 t/\hbar} a e^{-iH_0 t/\hbar}$, and similarly for b, $\gamma^{(m)} = \frac{K_m}{m} \int d^3 r |\phi(r)|^{2m}$, where K_m is the m-body rate constant and $\phi(r)$ is the condensate wave function in one of the two modes. In the Monte Carlo wave function approach [12], we define an effective Hamiltonian $H_{\rm eff}$ and the jump operators $J_\epsilon^{(m)}$

$$H_{\rm eff} = -\sum_{m=1}^{3} \sum_{\epsilon=a,b} \frac{i\hbar}{2} \gamma^{(m)} c_\epsilon^{\dagger m} c_\epsilon^m; \quad (4)$$

$$J_\epsilon^{(m)} = \sqrt{\gamma^{(m)}} c_\epsilon^m. \quad (5)$$

We assume that a small fraction of particles will be lost during the evolution so that we can consider χ and $\gamma^{(m)}$ ($m = 2, 3$) as constant parameters of the model. The state evolution in a single quantum trajectory is a sequence of random quantum jumps at times t_j and nonunitary Hamiltonian evolutions of duration τ_j:

$$|\psi(t)\rangle = e^{-iH_{\rm eff}(t-t_k)/\hbar} J_{\epsilon_k}^{(m_k)}(t_k) e^{-iH_{\rm eff}\tau_k/\hbar} \times J_{\epsilon_{k-1}}^{(m_{k-1})}(t_{k-1}) \ldots J_{\epsilon_1}^{(m_1)}(t_1) e^{-iH_{\rm eff}\tau_1/\hbar} |\psi(0)\rangle. \quad (6)$$

The expectation value of any observable $\hat{\mathcal{O}}$ is obtained by averaging over all possible stochastic realizations, that is all kinds, times, and number of quantum jumps, each trajectory being weighted by its probability [12]

$$\langle \hat{\mathcal{O}} \rangle = \sum_k \int_{0<t_1<t_2<\cdots t_k<t} dt_1 dt_2 \cdots dt_k \sum_{\{\epsilon_j, m_j\}} \langle \psi(t) | \hat{\mathcal{O}} | \psi(t) \rangle. \quad (7)$$

We want to calculate spin squeezing. In the considered symmetric case with zero initial relative phase, the mean spin remains aligned to the x axis $\langle S_x \rangle = \langle b^\dagger a \rangle$, and the spin squeezing is quantified by the parameter [3,5]

$$\xi^2 = \min_\theta \frac{\langle \hat{N} \rangle \Delta S_\theta^2}{\langle S_x \rangle^2}, \quad (8)$$

where $S_\theta = (\cos\theta) S_y + (\sin\theta) S_z$, $S_y = (a^\dagger b - b^\dagger a)/(2i)$, $S_z = (a^\dagger a - b^\dagger b)/2$, and $\hat{N} = a^\dagger a + b^\dagger b$. The noncorrelated limit yields $\xi^2 = 1$, while $\xi^2 < 1$ is the mark of an entangled state [3,4]. In all our analytic treatments, it turns out that $\Delta S_z^2 = \langle \hat{N} \rangle / 4$. This allows to express ξ^2 in a simple way:

$$\xi^2 = \frac{\langle a^\dagger a \rangle}{\langle b^\dagger a \rangle^2} (\langle a^\dagger a \rangle + A - \sqrt{A^2 + B^2}), \quad (9)$$

with

$$A = \frac{1}{2} \mathrm{Re}(\langle b^\dagger a^\dagger a b - b^\dagger b^\dagger a a \rangle) \quad (10)$$

$$B = 2\,\mathrm{Im}(\langle b^\dagger b^\dagger b a \rangle). \quad (11)$$

With one-body losses only, the problem is exactly solvable. Following a similar procedure as in [11], we get

$$\xi^2(t) = \frac{1 + \frac{1}{4}(N-1) e^{-\gamma t} [\tilde{A} - \sqrt{\tilde{A}^2 + \tilde{B}^2}]}{\Big[\frac{\gamma^2 + \chi[\gamma \sin(\chi t) + \chi \cos(\chi t)] e^{-\gamma t}}{\gamma^2 + \chi^2} \Big]^{2N-2}} \quad (12)$$

with $\gamma \equiv \gamma^{(1)}$ and

$$\tilde{A} = 1 - \Big[\frac{\gamma^2 + 2\chi[\gamma \sin(2\chi t) + 2\chi \cos(2\chi t)] e^{-\gamma t}}{\gamma^2 + 4\chi^2} \Big]^{N-2}$$

$$\tilde{B} = 4 \sin\chi t \Big[\frac{\gamma^2 + \chi[\gamma \sin(\chi t) + \chi \cos(\chi t)] e^{-\gamma t}}{\gamma^2 + \chi^2} \Big]^{N-2}.$$

The key points are that (i) $H_{\rm eff}$ is proportional to $\hat{N}$, so it does not affect the state, and (ii) a phase state $|\phi\rangle$ is changed into a phase state with one particle less after a quantum jump, $c_{a,b}|\phi\rangle \propto |\phi \mp \chi t/2\rangle$ where t is the time of the jump, the relative phase between the two modes simply picking up a random shift $\mp \chi t/2$ which reduces the squeezing.

When two- and three-body losses are taken into account, an analytical result can still be obtained by using a constant loss rate approximation [11]

$$H_{\rm eff} \simeq -\sum_{m=1}^{3} \sum_{\epsilon=a,b} \frac{i\hbar}{2} \gamma^{(m)} \bar{N}_\epsilon^m \equiv -\frac{i\hbar}{2} \lambda. \quad (13)$$

We verified by simulation (see Fig. 1) that this is valid for the regime we consider, where a small fraction of particles is lost at the time at which the best squeezing is achieved. In this approximation, the mean number of particles at time t is

$$\langle \hat{N} \rangle = N[1 - \sum_m \Gamma^{(m)} t]; \qquad \Gamma^{(m)} \equiv (N/2)^{m-1} m \gamma^{(m)} \quad (14)$$

where $\Gamma^{(m)} t$ is the fraction of lost particles due to m-body losses. Spin squeezing is calculated from (9) with

$$\langle b^\dagger a \rangle = \frac{e^{-\lambda t}}{2} \cos^{N-1}(\chi t) \tilde{N} F_1 \quad (15)$$

$$A = \frac{e^{-\lambda t}}{8} \tilde{N}(\tilde{N} - 1)[F_0 - F_2 \cos^{N-2}(2\chi t)] \quad (16)$$

$$B = \frac{e^{-\lambda t}}{2} \cos^{N-2}(\chi t) \sin(\chi t) \tilde{N}(\tilde{N} - 1) F_1 \quad (17)$$

where the operator $\tilde{N} = (N - \partial_\alpha)$ acts on the functions

$$F_\beta(\alpha) = \exp\Big[\sum_{m=1}^{3} 2\gamma^{(m)} t e^{\alpha m} \frac{\sin(m\beta\chi t)}{(m\beta\chi t)\cos^m(\beta\chi t)} \Big], \quad (18)$$

and all expressions should be evaluated in $\alpha = \ln \bar{N}_a$.

We want to find simple results for the best squeezing and the best squeezing time in the large N limit. In the absence of losses [1], the best squeezing and the best squeezing time in units of $1/\chi$ scale as $N^{-2/3}$. We then set $N = \varepsilon^{-3}$

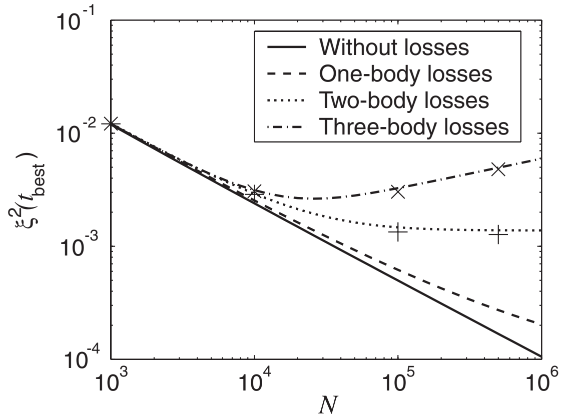

FIG. 1. Spin squeezing obtained by a minimization of ξ^2 over time, as a function of the initial number of particles, without loss of particles (solid line), with one-body losses (dashed line), with two-body losses (dotted line), with three-body losses (dash-dotted line), respectively. Parameters: $a = 5.32$ nm, $\bar{\omega} = 2\pi \times 200$ Hz, $K_1 = 0.1 \mathrm{s}^{-1}$, $K_2 = 2 \times 10^{-21}$ m^3/s [16], $K_3 = 18 \times 10^{-42}$ m^6/s. The symbols: plus (crosses) are results of a full numerical simulation with 400 Monte Carlo realizations for two-body (three-body) losses.

and rescale the time as $\chi t = \tau \varepsilon^2$. We expand the results (12) and (15)–(17) for $\varepsilon \ll 1$ keeping $\Gamma^{(m)}/\chi$ constant, and we obtain in both cases

$$\xi^2(t) \simeq \frac{1}{N^2(\chi t)^2} + \frac{1}{6} N^2 (\chi t)^4 + \frac{1}{3}\Gamma_{\rm sq} t, \qquad (19)$$

with

$$\Gamma_{\rm sq} = \sum_m \Gamma_{\rm sq}^{(m)} \quad \text{and} \quad \Gamma_{\rm sq}^{(m)} = m\Gamma^{(m)}. \qquad (20)$$

For equal loss rates $\Gamma^{(m)}$, the larger m, the more the squeezing is affected. Introducing the squeezing $\xi_0^2(t)$ in the no-loss case, the above result can be written as

$$\xi^2(t) = \xi_0^2(t)\left[1 + \frac{1}{3}\frac{\Gamma_{\rm sq} t}{\xi_0^2(t)}\right]. \qquad (21)$$

This shows that (i) the fact that only a small fraction of atoms is lost at the best squeezing time does not imply that the correction on the squeezing due to losses is small; (ii) the more squeezed the state is, the more sensitive to the losses. In presence of losses, the best squeezing time and the corresponding squeezing are

$$t_{\rm best} = \left[\frac{f(C)}{2}\right]^{1/3} \frac{N^{-2/3}}{\chi}, \qquad (22)$$

$$\xi^2(t_{\rm best}) = \left[\frac{1}{f(C)^{2/3}} + \frac{f(C)^{4/3}}{24} + \frac{C f(C)^{1/3}}{3}\right]\left(\frac{2}{N}\right)^{2/3} \qquad (23)$$

$$f(C) = \sqrt{C^2 + 12} - C; \qquad C = \frac{\Gamma_{\rm sq}}{2\chi}. \qquad (24)$$

In order to find optimal conditions to produce spin squeezing in presence of losses and set the ultimate limits of this technique, from now on, we assume that the number of particles is large enough for the condensates to be in the Thomas-Fermi regime so that

$$\mu = \frac{1}{2}\hbar\bar{\omega}\left(\frac{15}{2}\frac{Na}{a_0}\right)^{2/5}, \qquad (25)$$

where $a_0 = \sqrt{\hbar/M\bar{\omega}}$ is the harmonic oscillator length, M is the mass of a particle, and $\bar{\omega}$ is the geometric mean of the trap frequencies,

$$\chi = \frac{2^{3/5} 3^{2/5}}{5^{3/5}}\left(\frac{\hbar}{M}\right)^{-1/5} a^{2/5} \bar{\omega}^{6/5} N^{-3/5} \qquad (26)$$

$$\Gamma^{(1)} = K_1 \qquad (27)$$

$$\Gamma^{(2)} = \frac{15^{2/5}}{2^{7/5} 7\pi}\left(\frac{\hbar}{M}\right)^{-6/5} a^{-3/5} \bar{\omega}^{6/5} N^{2/5} K_2 \qquad (28)$$

$$\Gamma^{(3)} = \frac{5^{4/5}}{2^{19/5} 3^{1/5} 7\pi^2}\left(\frac{\hbar}{M}\right)^{-12/5} a^{-6/5} \bar{\omega}^{12/5} N^{4/5} K_3. \qquad (29)$$

We first analyze the dependence of squeezing on the initial number of particles, separating for clarity one-, two-, and three-body losses. Figure 1 shows the best squeezing $\xi^2(t_{\rm best})$ as a function of N when only one kind of losses is present. The curve without losses is also shown for comparison. According to Fig. 1, one-body losses do not change qualitatively the picture without losses and we have $\xi^2(t_{\rm best}) \propto N^{-4/15}$ for $N \to \infty$. In the same limit, with two-body losses, $\xi^2(t_{\rm best})$ is independent of N. With three-body losses, $\xi^2(t_{\rm best}) \propto N^{4/15}$ for $N \to \infty$, implying that, for a fixed $\bar{\omega}$, there is a finite optimum number of particles for squeezing.

We now turn to a full optimization of squeezing over $\bar{\omega}$ and N in the simultaneous presence of one-, two-, and three-body losses. To this end, we note that the square brackets in Eq. (23) is an increasing function of C; we can then optimize $\xi^2(t_{\rm best})$ by minimizing C with respect to $\bar{\omega}$. Under the conditions $K_1 \neq 0$ and $K_3 \neq 0$, the minimum of C, $C_{\rm min}$ is obtained for $\Gamma_{\rm sq}^{(3)} = \Gamma_{\rm sq}^{(1)}$, yielding

$$\bar{\omega}^{\rm opt} = \frac{2^{19/12} 7^{5/12} \pi^{5/6}}{15^{1/3}} \frac{\hbar}{M} \frac{a^{1/2}}{N^{1/3}} \left(\frac{K_1}{K_3}\right)^{5/12}. \qquad (30)$$

It turns out that $C_{\rm min}$ is proportional to N and $\xi^2(t_{\rm best}, \bar{\omega}^{\rm opt})$ is a decreasing function of N. The lower bound for ξ^2, reached for $N = \infty$, is then

$$\min_{t,\bar{\omega},N} \xi^2 = \left(\frac{5\sqrt{3}}{28\pi}\frac{M}{\hbar a}\right)^{2/3}\left[\sqrt{\frac{7}{2}(K_1 K_3)} + K_2\right]^{2/3}. \qquad (31)$$

In practice, one can choose $N = N_\eta$ in order to have $\xi^2 = (1 + \eta)\min \xi^2$ (e.g. $\eta = 10\%$), and then calculate the corresponding optimized frequency $\bar{\omega}^{\rm opt}$ with (30). For a suitable choice of the internal state, in an optical trap, the two-body losses can be neglected $K_2 = 0$. One can get in this case very simple formulas for the optimized parameters and squeezing. For $\eta = 10\%$ [13],

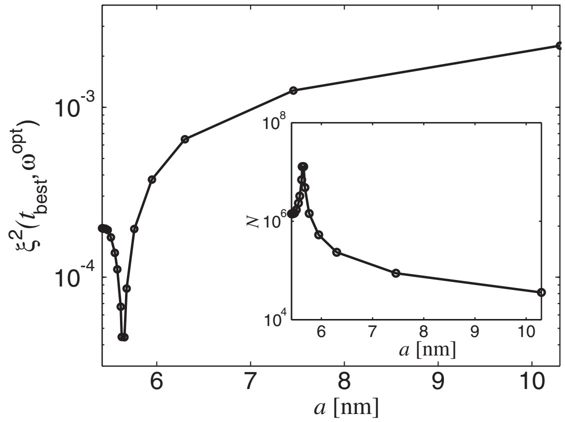

FIG. 2. Spin squeezing $\xi^2(t_{\rm best})$ optimized with respect to $\bar{\omega}$ as a function of the scattering length a, when the magnetic field is varied on the left side of the $B_0 = 1007.4$ G Feshbach resonance of ^{87}Rb. The inset shows the number of particles for each point, calculated for $\eta = 10\%$. We took $a(B) = a_{\rm bg}[1 - \Delta B/(B - B_0)]$ with $a_{\rm bg} = 5.32$ nm, $\Delta B = 0.21$ G. The three-body rate constant $K_3(B)$ is taken from [14], $K_1 = 0.01\ {\rm s}^{-1}$ and $K_2 = 0$.

$$N_\eta \simeq \frac{17.833}{(K_1 K_3)^{1/2}} \frac{\hbar a}{M}, \tag{32}$$

$$t_{\rm best} \simeq 0.277\left(\frac{M}{\hbar K_1}\right)^{2/3}\left(\frac{K_3}{a^2}\right)^{1/3}, \tag{33}$$

$$\xi^2 \simeq 0.356\left(\frac{MK_1}{\hbar}\right)^{1/3}\left(\frac{MK_3}{\hbar a^2}\right)^{1/3}. \tag{34}$$

We now ask whether we can use a Feshbach resonance to change the scattering length (but also K_3) to improve the squeezing. In Fig. 2, we plot the squeezing parameter *vs* the scattering length a. Predicted values of K_3, as a function of a, are taken from [14] for ^{87}Rb in the state $|F = 1, m_F = 1\rangle$ and $K_1 = 0.01\ {\rm s}^{-1}$. We calculate $\bar{\omega}^{\rm opt}$ and the number of particles needed for $\eta = 10\%$ for each point in the curve. The dip giving large squeezing corresponds to a strong decrease in K_3 around 1003.5 G ($K_3 \simeq 3 \times 10^{-45}\ {\rm m}^6/{\rm s}$). Close to the Feshbach resonance, the squeezing gets worse as K_3 increases (even if in the figure we do not enter the regime $K_3 \sim \hbar a^4/M$).

Finally, we consider the problem of the survival time of a spin-squeezed state in the presence of one-body losses. We imagine that the system evolves in two periods: for $t < T_1$, the system is squeezed in the presence of interactions ($\chi \neq 0$), one- and three-body losses; and for $t > T_1$, the interaction is stopped ($\chi = 0$), e.g., by opening the trap, and the system only experiences one-body losses. As t can be arbitrarily long, we use the exact solution for $t > T_1$ while for the $t < T_1 \simeq t_{\rm best}$, we use the approximation (13). Then, for $t = T_1 + T_2 > T_1$,

$$\xi^2(t) = \frac{1}{4}\frac{\langle \hat{N}(T_1)\rangle^2}{\langle S_x(T_1)\rangle^2} - \left[\frac{1}{4}\frac{\langle \hat{N}(T_1)\rangle^2}{\langle S_x(T_1)\rangle^2} - \xi^2(T_1)\right]e^{-\gamma^{(1)}T_2}$$
$$\simeq 1 - [1 - \xi^2(T_1)]e^{-\gamma^{(1)}T_2}. \tag{35}$$

This result shows that the spin squeezing can be kept some time after the interactions have been stopped. To give an example, for ^{87}Rb atoms with bare scattering length $a = 5.32$ nm, $K_1 = 0.01\ {\rm s}^{-1}$, $K_2 = 0$, $K_3 = 6 \times 10^{-42}\ {\rm m}^6/{\rm s}$ [15], in optimized conditions (32)–(34) $N = 2.8 \times 10^5$ and $\bar{\omega}^{\rm opt} = 2\pi \times 20.06$ Hz, $\xi^2 = 5.7 \times 10^{-4}$ is reached at $T_1 = t_{\rm best} = 4.4 \times 10^{-2}$ s, and a large amount of squeezing $\xi^2 \simeq 0.01$ is still available after 1 s.

In conclusion, we found the maximum spin squeezing reachable with cold atoms having a S_z^2 Hamiltonian, in presence of decoherence (losses) unavoidably accompanying the elastic interaction among atoms. For an optimized trap frequency, the best squeezing is reached for an atom number $N \to \infty$ and not for a finite value of N. This is important for applications such as spectroscopy where, apart from the gain due to quantum correlations among particles (squeezing), one always gains in increasing N.

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[1] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).
[2] B. Julsgaard, A. Kozhekin, and E. Polzik, Nature (London) **413**, 400 (2001).
[3] A. Sørensen, L. M. Duan, I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001).
[4] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **86**, 4431 (2001).
[5] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A **50**, 67 (1994).
[6] G. Santarelli *et al.*, Phys. Rev. Lett. **82**, 4619 (1999).
[7] J. Hald *et al.*, Phys. Rev. Lett. **83**, 1319 (1999); A. E. Kozhekin, K. Mølmer, and E. Polzik, Phys. Rev. A **62**, 033809 (2000); A. Dantan and M. Pinard, Phys. Rev. A **69**, 043810 (2004).
[8] A. Kuzmich, L. Mandel, and N. Bigelow, Phys. Rev. Lett. **85**, 1594 (2000); J. M. Geremia, J. Stockton, and H. Mabuchi, Science **304**, 270 (2004).
[9] A. M. Rey, L. Jiang, and M. D. Lukin, Phys. Rev. A **76**, 053617 (2007).
[10] The case of two condensates in the same spatial mode and different internal states, in the absence of demixing instability, see [3], can be treated with minor modifications along the same lines.
[11] A. Sinatra and Y. Castin, Eur. Phys. J. D **4**, 247 (1998).
[12] K. Mølmer, Y. Castin, and J. Dalibard, J. Opt. Soc. Am. B **10**, 524 (1993); H. J. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer, New York, 1993).
[13] Our result is valid for min$\xi^2 \ll 1$ as the fraction of lost particles at time $t_{\rm best}$ is $\sim$ minξ^2.
[14] G. Smirne *et al.*, Phys. Rev. A **75**, 020702(R) (2007).
[15] E. A. Burt *et al.*, Phys. Rev. Lett. **79**, 337 (1997).
[16] H. M. J. M. Boesten, A. J. Moerdijk, and B. J. Verhaar, Phys. Rev. A **54**, R29 (1996).