Energy Relaxation in the Spin-Polarized Disordered Electron Liquid

N. M. Chtchelkatchev[1,2] and I. S. Burmistrov[1,2]

[1]L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 117940 Moscow, Russia
[2]Department of Theoretical Physics, Moscow Institute of Physics and Technology, 141700 Moscow, Russia

(Received 6 August 2007; published 20 May 2008)

Energy relaxation is studied in the spin-polarized disordered electron systems in the diffusive regime. We derive a quantum kinetic equation in which the kernel of the electron-electron collision integral explicitly depends on the electron magnetization. As a consequence, the inelastic scattering rate has a nonmonotonic dependence on the spin polarization of the system.

DOI: 10.1103/PhysRevLett.100.206804 PACS numbers: 73.40.−c, 71.70.Di, 73.20.−r, 74.78.Na

The study of quantum electron kinetics in metallic conductors and, in particular, problems related to inelastic electron scattering represent the basic directions of mesoscopics [1–3]. Most theoretical and experimental research on the quantum transport in mesoscopic systems involves an estimate of the electron inelastic scattering time [or corresponding length scale]. Its comparison with the physical parameters of the system allows us to understand when the concept of electrons with the well-defined phase and energy is relevant for the electron transport description [1,2], and when the nonequilibrium distribution function can be approximated by the Fermi-Dirac distribution with an effective electron temperature [3]. The possibility to manipulate the inelastic scattering rates with the external fields, e.g., by a magnetic field, is one of the fundamental questions of electron kinetics.

The typical main building block of a mesoscopic device is a diffusive normal metal or a heavily doped semiconductor wire connected to massive electrodes acting as reservoirs. Electrons in the device interact with each other. In addition, electrons are coupled to phonons, electromagnetic environment, and so on. Understanding of transport phenomena in mesoscopic devices is based on the ability to solve the quantum kinetic equation (QKE) for the electron density matrix $\hat{f}$ in which interactions are taken into account via scattering integrals [4]. The derivation of the scattering integrals is another fundamental problem of electron kinetics. Electron-electron interaction usually provides the strongest mechanism for energy relaxation in metallic conductors at low temperatures (T) which are typical for experiments. The frequency of the inelastic electron-electron collisions appears in the QKE formalism as the "out-scattering" rate [1,5,6].

Presently, the issue of identifying scattering rates is of much practical importance due to the significant progress achieved in the last decade in the fabrication of devices in which electron distributions can be directly controlled and manipulated [3]. For example, in recent experiments a magnetic field (H) has been used as an effective tool for detailed study of energy relaxation in Cu [7] and Ag [8] mesoscopic wires. Its dependence on the Zeeman splitting can be attributed to inelastic electron-electron scattering mediated by the exchange interaction of the electron with magnetic impurities [9]. However, this interpretation relies on the assumption that the wire contains magnetic impurities. The impurity concentration is determined from a fit of the experimental data rather than from independent measurements [7,8].

In this Letter we derive the QKE for the spin-polarized disordered electron liquid and find that the kernel of the collision integral for the scattering with the electron spin-flip depends strongly on the magnetic field. For the small energy transfer, $|\omega| \ll |\mathbf{m}|/\nu$, this kernel saturates, $P_{\mathbf{m}}^{(\sigma)}(\omega) \propto |\mathbf{m}|^{d/2-2}$, while for $|\omega| \gg |\mathbf{m}|/\nu$, $P_{\mathbf{m}}^{(\sigma)}(\omega) \propto |\omega|^{d/2-2}$. At $\omega = 2|\mathbf{m}|\sigma/\nu$ this kernel has a pole related to electron paramagnetic resonance. Here, $\mathbf{m}$ denotes the average spin density, ν is the thermodynamic density of states per one spin projection, and d is the space dimension. As a consequence, the inelastic scattering rate has a nonmonotonic dependence on the spin polarization of the electron system. Our main result for the out-scattering rate in the presence of the disorder and magnetic field is given in Eq. (7). For a wire, our results imply that the energy relaxation should be sensitive to the magnetization $|\mathbf{m}| \propto (1+\gamma)H$ if $|\mathbf{m}|$ is larger than the Thouless energy. This prediction agrees qualitatively with the experiments [7,8] in which the Thouless energy of the wires corresponds to $H \lesssim 0.1T$ and triplet interaction amplitude $\gamma \approx 0.3$.

Below we formulate our main results in detail. For $T\tau \ll 1$ and $|\mathbf{m}|\tau/\nu \ll 1$, the QKE can be written

$$D\Delta\hat{f} - \partial_\tau\hat{f} + \left(e\mathbf{E} + \frac{\nabla(\mathbf{m}\cdot\hat{\mathbf{s}})}{\nu}\right)D\partial_\varepsilon\nabla\hat{f} = I[\hat{f}]. \quad (1)$$

Here, $\mathbf{E}$ is an electric field, D is a diffusion coefficient, τ is an elastic scattering time, and E_F is the Fermi energy. The Pauli matrices $\hat{s}^\alpha$, $\alpha = \overrightarrow{0,3}$ act in the spin space. The collision integral in Eq. (1) is given by a sum of two terms, $I[\hat{f}] = I_0[\hat{f}] + I_{\mathbf{m}}[\hat{f}]$. They describe the processes without ($I_0[\hat{f}]$) and with ($I_{\mathbf{m}}[\hat{f}]$) electron spin flips; see Fig. 1. Choosing $\mathbf{m}$ in the z direction $\hat{f}$ becomes diagonal. The collision integral $I_0[\hat{f}]$ has a magnetic field dependence only due to the Zeeman shift of the electron energy (in the distribution function). In the special case,

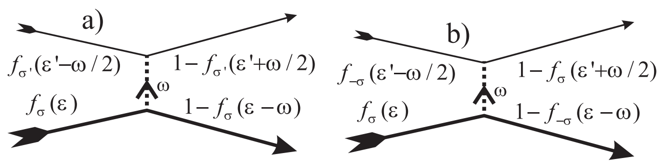

FIG. 1. The diagrammatic illustration of the out-scattering terms in the collision integral: (a) the magnetization-independent spin-conserving scattering and (b) the triplet channel magnetization sensitive scattering with spin-flips. The dashed line denotes the screened *e*-*e* interaction.

$f_{\varepsilon,\sigma} = f_{\varepsilon_\sigma}$ [10], we get $I_0[f_\sigma] = (8/\nu)\int d\omega[P^{(\rho)}(\omega,0) + P^{(\sigma)}(\omega,0)]J_{\sigma,\sigma}(\varepsilon,\omega)$. Here $J_{\sigma,\sigma'}(\varepsilon,\omega) = \int\frac{d\varepsilon'}{2\pi}\times[(1-f_{\varepsilon'_+,\sigma})(1-f_{\varepsilon-\omega,\sigma'})f_{\varepsilon'_-,\sigma'}f_{\varepsilon,\sigma} - f_{\varepsilon'_+,\sigma}f_{\varepsilon-\omega,\sigma'}(1-f_{\varepsilon'_-,\sigma'})(1-f_{\varepsilon,\sigma})]$, $\varepsilon'_\pm = \varepsilon' \pm \omega/2$, and $\varepsilon_\sigma = \varepsilon - \sigma|\mathbf{m}|/\nu$ is an electron energy with respect to E_F. The kernels $P^{(\rho,\sigma)}$ are the scattering probabilities in the singlet and triplet ($S_z = 0$) particle-hole channels and they are independent of $|\mathbf{m}|$.

The collision integral $I_{\mathbf{m}}[\hat{f}]$ describes scattering in the triplet $S_z = \pm 1$ particle-hole channel which is accompanied by a spin flip:

$$I_{\mathbf{m}}[f_\sigma] = \frac{16}{\nu}\int d\omega P^{(\sigma)}(\omega,|\mathbf{m}|\sigma)J_{\sigma,-\sigma}(\varepsilon,\omega). \quad (2)$$

In contrast to the clean Fermi-liquid [11], the probability $P^{(\sigma)}(\omega,|\mathbf{m}|\sigma)$ of scattering with a spin-flip *does depend* on $|\mathbf{m}|$. It can be presented as

$$P^{(a)}(\omega,|\mathbf{m}|) = \sum_{\mathbf{q}}\left|U^{(a)}(q,\omega,|\mathbf{m}|)\mathrm{Re}\frac{1}{Dq^2 - i\omega}\right|^2. \quad (3)$$

Here, $\mathrm{Re}[Dq^2 - i\omega]^{-1}$ estimates the time that electrons spend in the interaction region of the order of $1/q$ [1]. The dynamically screened Coulomb interaction reads

$$U^{(a)}(q,\omega,|\mathbf{m}|) = \frac{F_0^a(Dq^2 - i\omega)}{D(1+F_0^a)q^2 - i(\omega + \frac{2F_0^a|\mathbf{m}|}{\nu})}, \quad (4)$$

where F_0^a, $a = \{\rho,\sigma\}$ are the standard Fermi-liquid interaction parameters in singlet and triplet channels, respectively. Contrary to naive expectations, $|\mathbf{m}|$ appears only in the denominator of Eq. (4). Our results at $|\mathbf{m}| = 0$ agree with Ref. [1]. Integrating over q in Eq. (3), we obtain

$$P^{(\sigma)}(\omega,|\mathbf{m}|) = \frac{A_d\gamma}{D^{d/2}}\frac{|(1+\gamma)\omega - \frac{2\gamma|\mathbf{m}|}{\nu}|^{d/2} - |\omega|^{d/2}}{(\omega - \frac{2|\mathbf{m}|}{\nu})[(2+\gamma)\omega - \frac{2\gamma|\mathbf{m}|}{\nu}]}, \quad (5)$$

where $A_d^{-1} = [8(4\pi)^{d/2}\Gamma(\frac{d}{2})\sin\frac{\pi d}{2}]$ and $\gamma = -F_0^\sigma/(1+F_0^\sigma)$. The strong dependence of the kernel (5) on $|\mathbf{m}|$ should be taken into account in the solution of the QKE (1). Though the collision integral $I_{\mathbf{m}}[\hat{f}]$ describes the scattering in which electron spin flips, $I[\hat{f}]$ does not lead to a relaxation of spin because the operator of the total spin commutes with the Hamiltonian. The spin density evolves according to the equation: $\partial_\tau\mathbf{m} = D\nabla^2\mathbf{m}$.

To demonstrate that the strong dependence of the kernel (5) on $|\mathbf{m}|$ can significantly modify the energy relaxation, we compute the $|\mathbf{m}|$-dependent contribution to the out-scattering rate, $1/\tau_{\mathrm{out}}^{(\mathbf{m})}(\varepsilon_\sigma,T)$. It can be found [5] from $I_{\mathbf{m}}[f_\sigma]$ by its variation over $f_{\varepsilon,\sigma}$ at the equilibrium when $f_{\varepsilon,\sigma}$ becomes the Fermi-Dirac distribution $f_F(\varepsilon_\sigma)$:

$$\frac{1}{\tau_{\mathrm{out}}^{(\mathbf{m})}} = \int\frac{d\omega}{2\pi\nu}Y(\varepsilon_\sigma,\omega,T)P^{(\sigma)}\left(\omega + \frac{2|\mathbf{m}|\sigma}{\nu},|\mathbf{m}|\sigma\right), \quad (6)$$

where $Y(\varepsilon_\sigma,\omega,T) = \omega[\coth\frac{\omega}{2T} + \tanh\frac{\varepsilon_\sigma-\omega}{2T}]$. The kernel $P^{(\sigma)}$ in Eq. (6) involves well-known diffusion propagators [12] modified by the presence of the magnetic field [13]. For example, at $T = 0$ and for $d = 3$ we find

$$\frac{1}{\tau_{\mathrm{out}}^{(\mathbf{m})}} = \frac{2^{1/2}}{3\pi^2}\frac{|\varepsilon_\sigma|^{3/2}}{\nu D^{3/2}}\frac{\gamma[(1+\gamma)^{3/2} - 1]}{2+\gamma}F_\gamma\left(\frac{2|\mathbf{m}|\sigma}{\nu\varepsilon_\sigma}\right). \quad (7)$$

The function $\mathrm{F}_\gamma(z)$ has the following asymptotics

$$F_\gamma(z) = \begin{cases} 1 - \frac{6+9\gamma+3(\gamma-2)\sqrt{1+\gamma}}{2(2+\gamma)[(1+\gamma)^{3/2}-1]}z & |z| \ll 1, \\ \frac{9\gamma(2+\gamma)}{16[(1+\gamma)^{3/2}-1]}|z|^{-1/2} & |z| \gg 1+\gamma. \end{cases} \quad (8)$$

At $|\mathbf{m}| \gg \nu\varepsilon_\sigma$, the $1/\tau_{\mathrm{out}}^{(\mathbf{m})}$-rate is strongly suppressed because the typical interaction region becomes about $\sqrt{\nu D/|\mathbf{m}|}$. For $|\mathbf{m}| \lesssim \nu\varepsilon_\sigma$, the out-relaxation rate is different for the quasiparticle states with spin $\uparrow$ and spin $\downarrow$ due to the presence of the Fermi sea. In the latter case, the rate is a nonmonotonic function of $|\mathbf{m}|$ at a fixed quasiparticle energy $\varepsilon_\downarrow$ and has the maximum at $2|\mathbf{m}|/\nu \sim \varepsilon_\downarrow$ (see Fig. 2). The maximum is most pronounced in the limit $\gamma \to 0$ when the kernel $P^{(\sigma)}$ in Eq. (6) diverges at the transferred energy $\omega = 2|\mathbf{m}|/\nu \equiv \varepsilon_\downarrow - \varepsilon_\uparrow$. The origin of this maximum has much in common with the electron paramagnetic resonance. The interaction F_0^σ flips the quasiparticle spin and acts as an effective magnetic field. The latter appears with frequency dependence due to diffusive electron motion. The noticeable width of the maximum ($\sim$ energy of the electrons, $\varepsilon_\downarrow$) is a result of the diffusive motion of quasiparticles as well. With the increase of the interaction strength, γ (controlled by the electron density), the maximum is suppressed due to screening of the Coulomb interaction in accordance with Eq. (4). For $d = 3$ and $T > 0$ the nonmonotonic behavior of $1/\tau_{\mathrm{out}}^{(\mathbf{m})}$ survives for energies $\varepsilon_\sigma \sim T$. For $d < 3$ and $T > 0$ one should evaluate the integrals in Eq. (6) self-consistently [6].

The most general method to describe a nonequilibrium low-energy dynamics in the disordered interacting electron systems is the Keldysh nonlinear σ-model [14]. For nonzero F_0^σ and $\mathbf{m}$, one can follow the same derivation of the QKE as given in Refs. [14–16] except for complications arising from noncommutativity of the plasmon and gauge fields. Below we highlight the points where the derivation differs from [14–16].

We write the grand partition function of the interacting electrons in a random potential in the coherent state basis: $\mathcal{Z} = \int D\bar{\psi}D\psi\exp\{iS[\bar{\psi},\psi]\}$, where $S[\bar{\psi},\psi] = \int_{\mathcal{C}}dt\{\int_x(\bar{\psi}i\partial_t\psi) - H[\bar{\psi},\psi]\}$. Here, $\mathcal{C}$ is the Keldysh con-

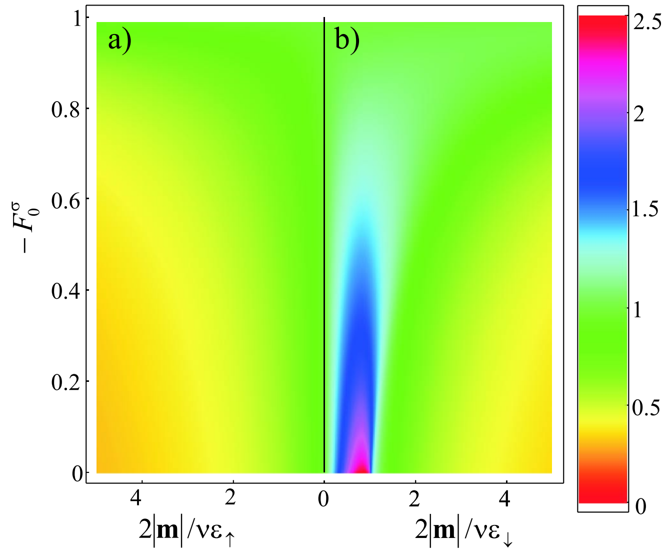

FIG. 2 (color online). The density plot of $\tau_{\rm out}^{(t)}(0)/\tau_{\rm out}^{(t)}(|\mathbf{m}|)$ as a function of F_0^σ and $2|\mathbf{m}|/(\nu\varepsilon_\uparrow)$ for panel (a), and $2|\mathbf{m}|/(\nu\varepsilon_\downarrow)$ for panel (b).

tour, symbol sp denotes the trace over spin degrees of freedom and $H = H_0 + H_{\rm int}$. The one-particle Hamiltonian H_0 involves the parameters with the standard Fermi-liquid renormalizations [17]. The interacting part $H_{\rm int} = \int d\mathbf{r} d\mathbf{r}'\{\frac{1}{2}\hat{\rho}_\mathbf{r}\Gamma_s(\mathbf{r}-\mathbf{r}')\hat{\rho}_{\mathbf{r}'} + 2\hat{\mathbf{m}}_\mathbf{r}\Gamma_t(\mathbf{r}-\mathbf{r}')\hat{\mathbf{m}}_{\mathbf{r}'}\}$, where $\Gamma_s(\mathbf{q}) = V_0(\mathbf{q}) + F_0^\rho/(2\nu)$ contains the long-range part of the Coulomb interaction V_0 and $\Gamma_t(\mathbf{q}) = F_0^\sigma/(2\nu)$ [12,17]. The charge and spin density operators are given as $\hat{\rho}_\mathbf{r} = \sum_\sigma \bar{\psi}_\sigma(\mathbf{r}t)\psi_\sigma(\mathbf{r}t)$ and $\hat{\mathbf{m}}_\mathbf{r} = \frac{1}{2}\sum_\sigma \bar{\psi}_\sigma(\mathbf{r}t)\hat{\mathbf{s}}_{\sigma\sigma'}\psi_{\sigma'}(\mathbf{r}t)$, respectively.

To derive the nonlinear σ-model we perform the standard steps [14]: (i) we average $\mathcal{Z}$ over the Gaussian, δ-correlated disorder and introduce the $\tilde{Q}$-matrix; (ii) we decouple the four-fermion interaction terms in $H_{\rm int}$ by the Hubbard-Stratonovich transformation using vector field Θ^α, $\alpha = \overrightarrow{0,3}$; (iii) we perform the Keldysh rotation; (iv) finally, we integrate out fermion degrees of freedom:

$$iS[\tilde{Q}, \Theta] = \mathrm{Tr}\ln\left[i\partial_t - \xi + \frac{i}{2\tau}\tilde{Q} + (\Theta_j^\alpha - \phi_j^\alpha)\hat{s}^\alpha\hat{\gamma}_j\right] + \frac{i}{2}\int dt d\mathbf{r}[\Theta^\tau\hat{\Gamma}^{-1}\hat{\sigma}_z\Theta] - \frac{\pi\nu}{4\tau}\mathrm{Tr}(\tilde{Q}^2), \quad (9)$$

where symbol Tr denotes the trace over the spin and Keldysh spaces combined with the time and space integrations. The Pauli matrices $\hat{\sigma}_z$, $\hat{\gamma}_1 = \hat{\sigma}_0$ and $\hat{\gamma}_2 = \hat{\sigma}_x$ operate in the Keldysh space, $\xi = p^2/2m_e - E_F$ where m_e is the electron mass, and $\hat{\Gamma}$ is the diagonal matrix in the 4×4 Θ-space: $\hat{\Gamma} = \hat{F}/(2\nu) = \mathrm{diag}\{\Gamma_s, \Gamma_t, \Gamma_t, \Gamma_t\}$. We assume the presence of the electric potential φ and a static magnetic field H. Then, the classical components $\phi_1^{\alpha=0} = e\varphi$ and $\phi_1^{\alpha>0} = -g\mu_B H_\alpha$, where g and μ_B denote the g factor and the Bohr magneton, respectively. The quantum components are $\phi_2^\alpha \equiv 0$. The low-energy description holds for the following conditions: $\phi_1^\alpha\tau \ll 1$ and $T\tau \ll 1 \ll E_F\tau$. In addition, we assume $T/E_F \lesssim \tau eH/m_e \ll 1$ and ignore, therefore, the Cooper channel and orbital effects.

The Θ-field has nonzero (zero) average for the classical (quantum) components, $[\langle\ldots\rangle = \int(\ldots)\exp(iS)]$:

$$\langle\Theta_1^{\alpha=0}\rangle = -\frac{F_0^\rho}{2\nu}\rho, \qquad \langle\Theta_1^{\alpha>0}\rangle = -\frac{F_0^\sigma}{\nu}\mathbf{m}_\alpha, \qquad \langle\Theta_2^\alpha\rangle = 0, \quad (10)$$

where ρ is the average charge density. Hereafter, we omit $V_0(\mathbf{q})$ for simplicity. This can readily be restored by taking the "unitary" limit, $F_0^\rho \to \infty$ [1]. Next, to improve convergence of the expansion of the logarithm in Eq. (9), we perform the gauge rotation of the Q matrix [14]:

$$\tilde{Q}_{t,t'}(\mathbf{r}) = U(t,\mathbf{r})Q_{t,t'}(\mathbf{r})U^{-1}(t',\mathbf{r}), \quad Q = \begin{pmatrix} R & K \\ Z & A\end{pmatrix}, \quad (11)$$

where $U(t,\mathbf{r}) = \exp\{i\langle\check{k}\rangle\}\exp\{i\delta\check{k}\}$ with $\check{k} \equiv k_i^\alpha\gamma^i\hat{s}_\alpha$.

In general, the gauge k and plasmon Θ fields can be separated into the slow, $\langle k\rangle$ and $\langle\Theta\rangle$, and fast, δk and $\delta\Theta$, contributions. The fast components describe charge and spin fluctuations in the electron system and the slow components are related to the gauge transformations of the external field potentials. Assuming that $E_F \gg 1/\tau \gg T$ and $r_s = e^2/v_F \lesssim 1$, where v_F is the Fermi velocity, we expand the action (9) around the standard saddle-point $Q = \Lambda$ with $Z = 0$, $R = -A = 1$ and arbitrary K. In the equilibrium, the Wigner transform $K_\varepsilon(\mathbf{r},t) = \int dt' K_{t+t'/2,t-t'/2}(\mathbf{r})\exp(i\varepsilon t')$ is equal to $K_\varepsilon^{\rm eq}(\mathbf{r},t) = 2\tanh[(\varepsilon - \varphi - \partial_t\langle k_1^\alpha\rangle\hat{s}^\alpha)/2T]$. As usual, we restrict ourselves to second order in δk, $\delta\Theta$ and gradients of Q within the manifold $Q^2 = 1$. Then

$$iS = i\int dt\int dr[\Theta^\tau\hat{\Gamma}^{-1}\hat{\sigma}_x\Theta + \nu b^\tau\hat{\sigma}_x b] - \frac{\pi\nu}{4}[D\mathrm{Tr}(\partial_r Q)^2 + 4i\mathrm{Tr}(i\partial_t + \check{b})Q]. \quad (12)$$

Here, $\partial_r Q = \nabla Q + i[\check{\mathbf{g}}, Q]_-$, $\check{b} = U^{-1}(-\check{\phi} + \check{\Theta})U + U^{-1}[i\partial_t, U]_-$ and $\check{\mathbf{g}} = -e\mathbf{A}/c + U^{-1}[\mathbf{p}, U]_-$, where $\mathbf{A}$ stands for an external vector potential. It is convenient to write $\check{b} = \langle\check{b}\rangle + \delta^{(1)}\check{b} + \delta^{(2)}\check{b}$ where $\langle\check{b}\rangle = \langle\check{\Theta}\rangle - \check{\phi} - \partial_t\langle\check{k}\rangle$ is of the zeroth order in δk and $\delta\Theta$, $\delta^{(1)}\check{b}$ is the first order term, and $\delta^{(2)}\check{b}$ is the second order one.

The particle and spin densities can be found from Eq. (12) as $(\rho, 2\mathbf{m}) = (i/2)\partial\ln\mathcal{Z}/\partial\phi_2^\alpha$:

$$\begin{pmatrix}\rho \\ 2\mathbf{m}\end{pmatrix} = -\frac{\pi\nu\mathrm{Tr}(\hat{\sigma}_x\hat{s}^\alpha\langle\tilde{Q}\rangle)}{2(1+F_0^\alpha)} - \frac{2\nu}{1+F_0^\alpha}[\phi_1^\alpha + \partial_t\langle k_1^\alpha\rangle], \quad (13)$$

where Eqs. (10) were used and Tr acts in Keldysh and spin spaces. The interaction renormalizations of ρ, $2\mathbf{m}$ in Eq. (13) agree with the Fermi-liquid theory [17].

Although the theory (12) encodes the low-energy dynamics of the electron system, for accurate derivation of the QKE it is enough to consider only the saddle-point configuration $\underline{Q}$ for a given configuration of the plasmon and gauge fields. The saddle-point (Usadel) equation is

$$D\partial_r(\underline{Q}\partial_r\underline{Q}) - [\partial_t - i\check{b}, \underline{Q}]_- = 0. \quad (14)$$

The general solution of Eq. (14) can be written as $\underline{Q} = \langle\underline{Q}\rangle + \delta\underline{Q}$ where $\langle\ldots\rangle$ denotes the average over $\delta\Theta$ fluctuations. The term $\delta\underline{Q}$ involves fluctuations $\delta\Theta$ and δk of the plasmon and gauge fields governed by the small parameter $(E_F\tau)^{-1} \ll 1$. Equation (14) enables finding $\delta\underline{Q}$ to second order in $\delta\Theta$ and δk. The procedure can be simplified demanding after Refs. [14–16] that the Z component of $\delta\underline{Q}$ does not contain terms linear in $\delta\Theta$, δk. This holds if $\delta^{(1)}b_2^\alpha - D\nabla\delta^{(1)}\mathbf{g}_2^\alpha = 0$. The condition $\underline{Q}^2 = 1$ ensures that R- and A-components of $\delta\underline{Q}$ are as as small as the Z component. We get the relation between the classical components of $\delta\Theta$ and δk demanding that the K component of Eq. (14) with $Q \to \langle\underline{Q}\rangle$ should vanish in the linear order in δk and $\delta\Theta$ at $t \to t'$:

$$(D\nabla\delta\mathbf{g} + \delta b)_1^\alpha(\omega) = -2\omega[\hat{S}_\omega^{-1}\hat{B}_\omega]^{\alpha\beta}D\nabla\delta\mathbf{g}_2^\beta(\omega),$$
$$B_\omega^{\alpha\beta} = \frac{\pi}{8\omega}\int\frac{d\varepsilon}{2\pi}\,\mathrm{Tr}(4\hat{s}^\alpha\hat{s}^\beta - \langle K_{\varepsilon_+}\rangle\hat{s}^\beta\langle K_{\varepsilon_-}\rangle\hat{s}^\alpha), \quad (15)$$
$$S_\omega^{\alpha\beta} = \frac{\pi}{4}\int\frac{d\varepsilon}{2\pi}\,\mathrm{Tr}(\hat{s}^\alpha\langle K_{\varepsilon_+}\rangle\hat{s}^\beta - \hat{s}^\beta\langle K_{\varepsilon_-}\rangle\hat{s}^\alpha).$$

In equilibrium and for $\mathbf{m} = 0$, $B_\omega^{\alpha\beta} = \delta^{\alpha\beta}\coth(\omega/2T)$ and $S_\omega^{\alpha\beta} = \omega\delta^{\alpha\beta}$. In general, we find $\hat{S}_\omega = \omega\hat{1} + \hat{\lambda}$, where $\lambda^{\alpha\beta} = \frac{\pi}{4}\int\frac{d\varepsilon}{2\pi}\,\mathrm{Tr}\{[s^\beta, s^\alpha]_-\langle K_\varepsilon\rangle\}$.

Derivation of the QKE becomes less cumbersome in the $\langle k\rangle$-gauge: $\langle\check{b}\rangle = \check{0}$. Then, it implies $\delta^{(1)}\check{\mathbf{g}} = \nabla\delta\check{k}$, $\delta^{(1)}\check{b} = \delta\check{\Theta} - \partial_t\delta\check{k}$ and $\delta^{(2)}\check{b} = i[\delta\check{\Theta} - \frac{1}{2}\partial_t\delta\check{k}, \delta\check{k}]_-$. Then $\lambda^{\alpha\beta} = 2i\varepsilon_{\alpha\beta\gamma}\mathbf{m}^\gamma/\nu$. It is the presence of nonzero $\lambda^{\alpha\beta}$, $\delta^{(2)}\check{b}$ and the matrix structure of $B_\omega^{\alpha\beta}$ that strongly complicates the derivation of the QKE for $\mathbf{m} \neq 0$. Substituting $\underline{Q}$ into Eq. (12), we find expanding to second order in $\delta\Theta$:

$$iS[\delta\Theta] = -i\pi\nu\mathrm{Tr}(\delta^{(2)}b_2\langle K\rangle) + i\int dtd\mathbf{r}[\delta\Theta^\tau\hat{\Gamma}^{-1}\hat{\sigma}_x\delta\Theta + \nu\delta^{(1)}b^\tau\hat{\sigma}_x\delta^{(1)}b] + \frac{\pi\nu D}{4}\mathrm{Tr}[\delta^{(1)}\check{\mathbf{g}}, \Lambda]_-^2. \quad (16)$$

Given Eq. (16), it is easy to find the 2-point correlation function $\mathcal{D}_{ij}^{\alpha\beta}(\mathbf{r}t, \mathbf{r}'t') = i4\nu D\langle\delta^{(1)}\mathbf{g}_i^\alpha(\mathbf{r}t)\delta^{(1)}\mathbf{g}_j^\beta(\mathbf{r}'t')\rangle$ as

$$\hat{\mathcal{D}}_{11}(q,\omega) = \frac{Dq^2}{Dq^2 - i\omega}\{[(1+\hat{F}^{-1})Dq^2 - i\omega] + i\hat{\lambda}\}^{-1},$$
$$\hat{\mathcal{D}}_{12}(q,\omega) = -2i\omega\hat{\mathcal{D}}_{11}(q,\omega)[\hat{B}_\omega - i(Dq^2 - i\omega)(1+\hat{F}^{-1}) \times\hat{S}_\omega^{-1}\hat{B}_\omega + i(Dq^2 + i\omega)\hat{B}_\omega\hat{S}_\omega^{-1}(1+\hat{F}^{-1})] \times\hat{\mathcal{D}}_{22}(q,\omega),$$
$$\hat{\mathcal{D}}_{22}(q,\omega) = [\hat{\mathcal{D}}_{11}(q,\omega)]^\dagger, \quad \hat{\mathcal{D}}_{21}(q,\omega) = 0. \quad (17)$$

Substituting $\underline{Q} = \langle\underline{Q}\rangle + \delta\underline{Q}$ into Eq. (14) and averaging the result over the δk-fluctuations using Eq. (17), we get the QKE (1) for $\hat{f} = [2\hat{s}^0 - \langle K_\varepsilon(\mathbf{r}, t)\rangle]/4$.

In summary, we have derived the QKE that describes the energy relaxation due to electron-electron interaction in the disordered electron systems with the spin polarization. We have found a nonmonotonic behavior of the out-scattering rate as a function of the average electron magnetization. This effect can be used for switching the system from the nonequilibrium to the quasiequilibrium regime and back with increase of the spin polarization. The suppression of the out-scattering rate for large $|\mathbf{m}|$ can be used for decoupling the electron degrees of freedom from the environment.

The research was funded by RFBR (No. 07-02-00998), the Russian President Foundation [Nos. 4401.2007.2, 4445.2007.2], the Dynasty, the Program of RAS "Quantum Macrophysics", CRDF and U.S. Department of Energy Office of Science (No. DE-AC02-06CH11357).

[1] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Conductors*, edited by A. J. Efros and M. Pollack (Elsevier, Amsterdam, 1985).
[2] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, New York, 1997).
[3] F. Giazotto *et al.*, Rev. Mod. Phys. **78**, 217 (2006).
[4] E. M. Lifshitz and L. P. Pitaevskii, *Course in Theoretical Physics* (Pergamon, Oxford, 1981), Vol. 10.
[5] A. Schmid, Z. Phys. **271**, 251 (1974); B. L. Al'tshuler and A. G. Aronov, JETP Lett. **30**, 482 (1979).
[6] Ya. M. Blanter, Phys. Rev. B **54**, 12 807 (1996).
[7] A. Anthore *et al.*, in Proceedings of the XXXVIth Rencontres de Moriond Electronic Correlations: From Meso- to Nano-physics, Les Arcs, France (2001).
[8] A. Anthore *et al.*, Phys. Rev. Lett. **90**, 076806 (2003).
[9] A. Kaminski and L. I. Glazman, Phys. Rev. Lett. **86**, 2400 (2001); G. Göppert *et al.*, Phys. Rev. B **66**, 195328 (2002).
[10] The restriction, $f_{\varepsilon,\sigma} = f_{\varepsilon_\sigma}$, does not affect $\tau_{\mathrm{out}}^{(\mathbf{m})}$ and $I_{\mathbf{m}}$; see also O. V. Dimitrova and V. E. Kravtsov, JETP Lett. **86**, 670 (2007).
[11] V. P. Mineev, Phys. Rev. B **69**, 144429 (2004).
[12] A. M. Finkelstein, *Electron Liquid in Disordered Conductors*, edited by I. M. Khalatnikov, Soviet Scientific Reviews Vol. 14 (Harwood Academic, London, 1990).
[13] C. Castellani *et al.*, Phys. Rev. B **57**, R9381 (1998); G. Zala *et al.*, Phys. Rev. B **65**, 020201 (2001).
[14] A. Kamenev and Y. Gefen, Phys. Rev. B **54**, 5428 (1996); A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).
[15] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).
[16] G. Catelani and I. L. Aleiner, JETP **100**, 331 (2005).
[17] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1963).