Comment on "Intensity Correlations and Mesoscopic Fluctuations of Diffusing Photons in Cold Atoms"

In a recent Letter [1], Assaf and Akkermans claim that the angular correlations of the light intensity scattered by a cloud of cold atoms with internal degeneracy (Zeeman sublevels) of the ground state overcome the usual Rayleigh law. More precisely, they found that they become exponentially large with the size of the sample. In what follows, we will explain why their results are wrong and why, to the contrary, the internal degeneracy leads to lower intensity correlations.

Following the authors' proposed experimental scheme and notation [Eq. (3) of Ref. [1]], the correlation of the transmission coefficient, averaged over the positions and the internal states of the atoms, reads as follows:

$$\overline{T_{ab}T_{a'b'}} = \overline{\sum_{ijkl} A_i^{\{R,m\}} A_j^{\{R,m\}*} A_k^{\{R,m'\}} A_l^{\{R,m'\}*}}. \quad (1)$$

As explained by the authors, the configuration average of the preceding equation leads to two possible pairings among the photon paths, either $i = j$, $k = l$, which corresponds to $\bar{T}_{ab}\bar{T}_{a'b'}$, or $i = l$, $j = k$. In the latter case, the correlation of the transmission coefficient reads

$$\overline{\delta T_{ab}\delta T_{a'b'}} = \sum_{\{m,m'\}} \sum_i A_i^{\{m\}} A_i^{\{m'\}*} \sum_j A_j^{\{m'\}} A_j^{\{m\}*}. \quad (2)$$

The preceding equation corresponds to Eq. (4) of Ref. [1], with the only difference that the average over the internal states is explicitly written [the configuration probabilities $p(\{m\})$, $p(\{m'\})$ are left implicit]. Since the sum over i is just the complex conjugate of the sum over j, Eq. (2) reads

$$\overline{\delta T_{ab}\delta T_{a'b'}} = \sum_{\{m,m'\}} \left| \sum_i A_i^{\{m\}} A_i^{\{m'\}*} \right|^2. \quad (3)$$

Now, for fixed $\{m\}$, $\{m'\}$, applying the Cauchy-Schwartz inequality (i.e., $|\sum_i x_i y_i^*|^2 \le \sum_i |x_i|^2 \sum_j |y_j|^2$) to the sum over i, one obtains

$$\overline{\delta T_{ab}\delta T_{a'b'}} \le \sum_{\{m,m'\}} \sum_i |A_i^{\{m\}}|^2 \sum_j |A_j^{\{m'\}}|^2 = \bar{T}_{ab}\bar{T}_{a'b'}. \quad (4)$$

This inequality proves that the condition $C_{aba'b'} < 1$ is still fulfilled, even in the case of internal degeneracy.

To pinpoint the error in [1], please note that in our Eq. (3), the sum over the internal states is *outside* the modulus square in contradiction with Eq. (5) of Ref. [1], where this sum appears *inside* the modulus square. This results in a completely different mesoscopic situation. From Eq. (3), the intensity correlation arises as an incoherent sum of the square of different correlation diffusons (one for each $\{m\}$, $\{m'\}$ pair), whereas in Eq. (7) of Ref. [1], the authors are calculating the square of a correlation diffuson in which all $\{m\}$, $\{m'\}$ contributions add coherently. Obviously, transforming a sum of intensities into a square of a sum of amplitudes can lead to a very different physical behavior, like an exponential growth with the size of the system.

The preceding points alone are enough to prove that the large intensity fluctuations calculated by the authors are unphysical. Still, we would like to emphasize an additional crucial point: as explained by the authors, the dominant contribution in Eq. (2) is obtained only when the two paths i and j are not sharing any scatterers. In the case of internal degeneracy, this condition actually implies that, along these two paths, the atoms can only undergo Rayleigh transitions (i.e., for which the initial and final internal states are the same), in full agreement with the experimental results of Ref. [2]. Indeed, the fact that both photon paths i and j correspond to the same global configuration of internal states [see Eq. (2)], i.e., $\{m\}$ during the first pulse and $\{m'\}$ during the second pulse, means that if a Raman transition occurs for a given atom along the photon path i, then the same atom must undergo the same Raman transition along the path j: i and j must share, at least, this particular atom. This means that only photon paths along which all the atoms have undergone Rayleigh transitions contribute to the disorder averaged intensity correlations. Because the number of those paths is smaller than the total possible paths, we expect $\overline{\delta T_{ab}\delta T_{a'b'}} < \bar{T}_{ab}\bar{T}_{a'b'}$, i.e., $C_{aba'b'} < 1$. Only when the ground state is not degenerate (i.e., Rayleigh scatterers) can all paths contributing to the average intensity also contribute to the correlation function, leading to $x_i = y_i$, and the Cauchy-Schwartz inequality becomes an equality, implying $C_{abab} = 1$.

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