Higgs Boson Mass in Supersymmetry to Three Loops

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Within the minimal supersymmetric extension of the standard model, the mass of the light *CP*-even Higgs boson is computed to three-loop accuracy, taking into account the next-to-next-to-leading order effects from supersymmetric quantum chromodynamics. We consider two different scenarios for the mass hierarchies of the supersymmetric spectrum. Our numerical results amount to corrections of about 500 MeV, which is of the same order as the experimental accuracy expected at the CERN Large Hadron Collider.

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Introduction.—Supersymmetry (SUSY) is currently the most-studied extension of the standard model (SM) (see, e.g., Ref. [1]). It provides solutions to some profound theoretical problems of the standard model: the fine-tuning of the Higgs mass, the (non)unification of gauge couplings, a mechanism for spontaneous symmetry breaking, and a cold dark matter candidate.

The minimal supersymmetric extension of the standard model (MSSM) is based on a two-Higgs-doublet model with five physical Higgs bosons: two *CP*-even h/H, one *CP*-odd A (also named the "pseudoscalar" Higgs), and two charged scalars $H^{\pm}$. Each particle of this two-Higgs-doublet model receives a SUSY partner of opposite spin statistics, where left- and right-handed components of a standard model Dirac fermion are attributed with separate scalars $\tilde{f}_{L/R}$ which mix to the physical mass eigenstates $\tilde{f}_{1/2}$.

Compared to the standard model, the MSSM Higgs sector is described by two additional parameters, usually chosen to be the pseudoscalar mass M_A and the ratio of the vacuum expectation values of the two Higgs doublets, $\tan\beta = v_2/v_1$. The masses of the other Higgs bosons are then fixed by SUSY constraints. In particular, the mass of the light *CP*-even Higgs boson M_h is bounded from above. At tree level, it is $M_h < M_Z$. Radiative corrections to the Higgs pole masses raise this bound substantially to values that were inaccessible by LEP [2–4]. The large numerical impact is due to a contribution $\sim\alpha_t M_t^2 \sim M_t^4$ coming from top and stop quark loops (M_t is the top quark mass and $\sqrt{\alpha_t}$ is proportional to the top Yukawa coupling).

The one-loop corrections to the Higgs pole masses are known without any approximations [5–8]. They show that the bulk of the numerical effects can be obtained in the so-called effective-potential approach in the limit of vanishing external momentum. Motivated by this observation, all presumably relevant two-loop terms have since been evaluated in this approach (for reviews, see, e.g., Refs. [9,10]). More recently there has been quite a lot of activity in the context of the MSSM with complex parameters which can lead to sizable effects (see, e.g., Ref. [11]). The two-loop results are implemented in the numerical programs FEYNHIGGS [12] and CPSUPERH [13,14] using on-shell particle masses, and in SOFTSUSY [15], SPHENO [16], and SUSPECT [17] using $\overline{\rm DR}$ parameters, that is, dimensional reduction with minimal subtraction. The influence of terms that goes beyond the approximation of vanishing external momentum has been investigated in Ref. [18].

Based mostly on the renormalization scale and scheme dependence, the theoretical uncertainty on the prediction of the light Higgs boson mass M_h has been estimated to 3–5 GeV [10,19]. This is to be compared with the expected experimental uncertainty of a Higgs mass measurement at the CERN Large Hadron Collider (LHC) of the order of 100–200 MeV [20]. At the International Linear Collider, this goes even down to roughly 50 MeV [21]. These numbers clearly show the need for three-loop corrections to the SUSY Higgs boson masses in order to fully exploit the physics potential of these colliders.

In fact, quite recently the leading and next-to-leading logarithmic terms in $\ln(M_{\rm SUSY}/M_t)$ at three-loop level have been obtained, where $M_{\rm SUSY}$ is the typical scale of SUSY particle masses [22]. In this Letter, we want to present the first genuine three-loop calculation of the lightest Higgs boson mass, focusing on a few simplifying limiting cases for the sake of brevity. In particular, we consider effects of order $\alpha_t\alpha_s^2$, keep only the leading terms $\sim M_t^4$, and neglect all mixing effects in the stop sector. More general results and their detailed phenomenological impacts shall be deferred to a later publication.

Higgs boson mass in the MSSM.—At tree level, the mass matrix of the neutral, *CP*-even Higgs bosons h, H has the following form:

$$\mathcal{M}^2_{H,\rm tree} = \frac{\sin 2\beta}{2}\begin{pmatrix} M_Z^2\cot\beta + M_A^2\tan\beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2\tan\beta + M_A^2\cot\beta \end{pmatrix}. \quad (1)$$

The diagonalization of $\mathcal{M}^2_{H,\rm tree}$ gives the tree-level result for M_h and M_H and leads to the well-known bound $M_h < M_Z$ which is approached in the limit $\tan\beta \to \infty$.

Quantum corrections to the Higgs boson masses are incorporated by evaluating the poles of the Higgs boson propagator at higher orders. As mentioned in the Introduction, the numerically dominant contributions can be obtained in the approximation of zero external momentum (see, e.g., Ref. [23]) which we will adopt in the following. Furthermore, we will only consider corrections of order $\alpha_t\alpha_s^2$. Apart from the quark, squark, and gluino masses, there is another parameter with mass dimension, the trilinear coupling of the soft SUSY breaking terms A_t. Before renormalization, we express it through the stop masses $M_{\tilde{t}_1}$, $M_{\tilde{t}_2}$, the stop mixing angle θ_t, and the bilinear Higgs parameter $\mu_{\rm SUSY}$ as follows:

$$2M_tA_t = (M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)\sin 2\theta_t + 2M_t\mu_{\rm SUSY}\cot\beta. \quad (2)$$

The mass matrix $\mathcal{M}_H^2$ is obtained from the quadratic terms in the Higgs boson potential constructed from the fields ϕ_1 and ϕ_2. They are related to the physical Higgs mass eigenstates via a mixing angle α. Since ϕ_1 does not couple directly to top quarks, it is convenient to perform the calculations of the Feynman diagrams in the (ϕ_1, ϕ_2) basis.

Including higher order corrections, one obtains the Higgs boson mass matrix

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\rm tree}^2 - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}, \quad (3)$$

which again gives the physical Higgs boson masses upon diagonalization. The renormalized quantities $\hat{\Sigma}_{\phi_1}$, $\hat{\Sigma}_{\phi_2}$, and $\hat{\Sigma}_{\phi_1\phi_2}$ are obtained from the self-energies of the fields ϕ_1, ϕ_2, A, evaluated at zero external momentum, as well as from tadpole contributions of ϕ_1 and ϕ_2 (see, e.g., Ref. [9]). Let us remark that if one sets $M_{\tilde{t}_1} = M_{\tilde{t}_2}$ and $A_t = 0$, and evaluates only the leading contribution $\sim M_t^4$, then only $\hat{\Sigma}_{\phi_2} \neq 0$ and the matrix $\mathcal{M}_H^2 - \mathcal{M}_{H,\rm tree}^2$ is diagonal. On the other hand, if we allow for nonzero A_t, also $\hat{\Sigma}_{\phi_1}$ and $\hat{\Sigma}_{\phi_1\phi_2}$ contribute in general.

The calculation of $\hat{\Sigma}_{\phi_2}$ is organized as follows: All Feynman diagrams are generated with QGRAF [24]. In order to properly take into account the Majorana character of the gluino, the output is subsequently manipulated by a PERL script which applies the rules given in Ref. [25]. The various diagram topologies are identified and transformed to FORM [26] with the help of Q2E and EXP [27,28]. The program EXP is also used in order to apply the asymptotic expansion (see, e.g., Ref. [29]) in the various mass hierarchies. The actual evaluation of the integrals is performed with the package MATAD [30], resulting in an expansion in $d-4$ for each diagram, where d is the space-time dimension. The total number of three-loop diagrams amounts to about 16 000.

At three-loop level we need to renormalize the top quark mass, the top squark mass, and the stop mixing angle at the two-loop order. In addition, the one-loop counterterm of the gluino mass is needed for the renormalization of the two-loop expression. We implement dimensional reduction with the help of the so-called ϵ scalars which appear for the first time at two loops. The renormalization of the ϵ-scalar mass is performed in the on-shell scheme, requiring that the renormalized mass is equal to zero. In the literature this is referred to as the $\overline{\rm DR}'$ scheme.

The one-loop on-shell counterterms are well known (see, e.g., Refs. [8,31–33]). As far as the two-loop counterterms for the squarks and quarks are concerned, one can find the results in Refs. [34,35]. However, it is rather tedious to extract the results for the mass hierarchies we are interested in. Thus, we recomputed the corresponding corrections.

To our knowledge, the two-loop counterterm for the stop mixing angle is not yet available in the literature. It turns out that in our approximation, where $M_{\tilde{t}_1} = M_{\tilde{t}_2}$ and $A_t = 0$, only the one-loop counterterm of the mixing angle enters the three-loop result.

As a cross-check for our calculation, we recalculated the exact two-loop result (in the limit of vanishing external momentum) and find perfect agreement with the literature [23,36]. Furthermore, the expansion of the exact expressions confirms the limiting cases discussed below. Both the two- and three-loop calculations are performed for a general QCD gauge parameter ξ_S. The independence of the final results on ξ_S serves as another welcome check on the correctness of our result.

We use anticommuting γ_5 which is allowed for fermion traces which involve an even number of γ_5 matrices. It turns out that all traces involving an odd number of γ_5 vanish because they contain less than four γ matrices.

In the following we discuss three different cases for the mass hierarchy. In all cases we set the light quark masses to zero.

(i) *Supersymmetric limit*, i.e., $M_t = M_{\tilde{t}}$ and the gluino and other squarks are massless: $M_{\tilde{g}} = M_{\tilde{q}} = 0$. The quantum corrections to the Higgs boson mass vanish in this case, as required by supersymmetry. Still, the individual diagrams are different from zero and thus the calculation imposes a strong check on our setup.

(ii) *Massless gluino*, $M_{\tilde{g}} = 0$. Expanding in the limit $M_t \ll M_{\tilde{t}} = M_{\tilde{q}} \equiv M_{\rm SUSY}$, we obtain for the leading term of this expansion

$$\begin{aligned}\hat{\Sigma}_{\phi_2} = \frac{3G_FM_t^4}{\sqrt{2}\pi^2\sin^2\beta}\Big\{&L_{tS} + \frac{\alpha_s}{\pi}[-1 - 4L_{tS} + 2L_{tS}^2] \\ &+ \left(\frac{\alpha_s}{\pi}\right)^2\Big[-\frac{593}{27} - \frac{3}{4}L_{\mu t} + \frac{23}{81}\pi^2 + \frac{401}{18}\zeta_3 \\ &+ \left(-\frac{47}{4} - 3L_{\mu t} + \frac{4}{9}\pi^2 - \frac{4}{9}\pi^2\ln 2\right)L_{tS} \\ &+ \left(-\frac{1}{12} + \frac{3}{2}L_{\mu t}\right)L_{tS}^2 + \frac{5}{2}L_{tS}^3\Big]\Big\}, \end{aligned} \quad (4)$$

with $L_{\mu t} = \ln(\mu^2/M_t^2)$ and $L_{tS} = \ln(M_t^2/M_{\rm SUSY}^2)$.

(iii) *Common SUSY mass*. In this scenario we assume $M_t \ll M_{\tilde{t}_1} = M_{\tilde{t}_2} = M_{\tilde{g}} \equiv M_{\rm SUSY} \ll M_{\tilde{q}}$. Even though the top squark masses are equal and thus the mixing angle is zero, it is necessary to introduce a counterterm for θ_t. Since the latter has contributions proportional to $1/(M_{\tilde{t}_2}^2 - M_{\tilde{t}_1}^2)$, we expand the one- and two-loop result in this limit before inserting the counterterms. The cancellation of such terms in the final result provides another check on our calculation.

It is important to keep $A_t \neq 0$ in the one- and two-loop contributions and to use Eq. (2) before renormalization, because the corresponding counterterms generate terms of order M_t^4 at three-loop level. Setting $A_t = 0$ in the end, we obtain

$$\hat{\Sigma}_{\phi_2} = \frac{3G_F M_t^4}{\sqrt{2}\pi^2 \sin^2\beta}\Big\{L_{tS} + \frac{\alpha_s}{\pi}[-4L_{tS} + 2L_{tS}^2] + \Big(\frac{\alpha_s}{\pi}\Big)^2\Big[\frac{671}{324} + \frac{1}{27}\pi^2 + \frac{1}{9}\zeta_3 + \Big(-\frac{1591}{108} - 3L_{\mu t} + \frac{1}{3}\pi^2 - \frac{4}{9}\pi^2\ln 2 + \frac{55}{18}L_{t\tilde{q}} + \frac{5}{6}L_{t\tilde{q}}^2\Big)L_{tS} + \Big(\frac{13}{18} + \frac{3}{2}L_{\mu t} - \frac{5}{3}L_{t\tilde{q}}\Big)L_{tS}^2 + \frac{53}{18}L_{tS}^3 + \Big(\frac{475}{108} - \frac{5}{9}\pi^2\Big)L_{t\tilde{q}} - \frac{25}{36}L_{t\tilde{q}}^2 - \frac{5}{18}L_{t\tilde{q}}^3 + \mathcal{O}\Big(\frac{M_S^2}{M_{\tilde{q}}^2}\Big)\Big]\Big\}, \quad (5)$$

where $L_{t\tilde{q}} = \ln(M_t^2/M_{\tilde{q}}^2)$. In Eq. (5) we display only the leading term in the $1/M_{\tilde{q}}$ expansion. We actually computed five expansion terms and observe a rapid convergence of the series—even for $M_{\rm SUSY} = M_{\tilde{q}}$. It is interesting to mention that large cancellations occur among the cubic, quadratic, linear, and nonlogarithmic term of $\hat{\Sigma}_{\phi_2}$ at three-loop order. For example, for our default input values the sum of the cubic and quadratic logarithm is negative whereas the complete answer leads to a positive correction for the α_s^2 coefficient of $\hat{\Sigma}_{\phi_2}$.

If we express the result of Eq. (5) in terms of $\overline{\rm DR}'$ parameters, we can compare with the results obtained in Ref. [22]. We find agreement for the cubic logarithm, but the quadratic logarithm disagrees [37].

Numerical results.—In the remainder of this Letter, we discuss the numerical effect of our result, restricting ourselves to $A_t = 0$. We adopt the on-shell scheme for the quark, squark, and gluino masses.

We choose $\mu = M_t$ as the default value for the renormalization scale. First we compute $\alpha_s(M_t)$, defined in the $\overline{\rm DR}$ scheme and the full SUSY theory, from the SM input value $\alpha_s(M_Z) = 0.1189$ [38] which is given within five-flavor QCD. We follow the procedure outlined in Ref. [39] which includes three-loop running and two-loop matching effects. As a result we obtain, e.g., $\alpha_s(M_t) = 0.0926$ for a common SUSY mass $M_{\rm SUSY} = 1$ TeV. The SM input parameters are given as $G_F = 1.166\,37 \times 10^{-5}$ GeV^{-2}, $M_Z = 91.1876$ GeV [40], $M_t = 170.9$ GeV [41]. For the heavy squark mass ($\tilde{q} \neq \tilde{t}$) we use $M_{\tilde{q}} = 2$ TeV.

In order to evaluate the tree-level approximation of the Higgs boson mass we also need the parameters M_A and $\tan\beta$. If not stated otherwise we adopt the values $M_A = 1$ TeV and $\tan\beta = 40$. Since these parameters do not enter the corrections considered in this Letter, they only have minor influence on the plots presented in the following.

In Figs. 1 and 2 we discuss the difference between the Higgs boson mass evaluated with i-loop approximation and the tree-level result,

$$\Delta M_h^{(i)} = M_h^{(i\ \rm loop)} - M_h^{\rm tree}. \quad (6)$$

Figure 1 shows $\Delta M_h^{(i)}$ for $i = 1$ (dotted line), $i = 2$ (dashed line), and $i = 3$ (solid line) as a function of $M_{\rm SUSY}$ in the range between 200 GeV and 2 TeV. As is well known, the one-loop corrections are large, increasing M_h by up to 46 GeV. The two-loop effects are negative, reducing the size of the overall corrections by about 30% with respect to the one-loop result.

The three-loop terms are much smaller and clearly stabilize the perturbative behavior. At $\mu = M_t$, for example, they lead to a further reduction of $\Delta M_h^{(i)}$ by about 400 MeV for $M_{\rm SUSY} = 300$ GeV and an enhancement of about 500 MeV for $M_{\rm SUSY} = 2$ TeV. Note that the numerical impact is larger than the precision on the lightest Higgs boson mass as expected at the LHC.

In order to estimate the size of the higher order corrections, we consider the dependence of the result on the choice of the renormalization scale. In Fig. 2 we plot $\Delta M_h^{(i)}$ as a function of μ which is varied from 50 to 500 GeV. The two-loop results show a variation of more than 1 GeV over this range. The error band derived in this way nicely covers the three-loop result, which itself varies by less than 35 MeV. For other values of $M_{\rm SUSY}$ the variation can reach up to 100 MeV. The three-loop curve in Fig. 2 shows a shallow minimum close to $\mu = M_t$ which in turn is close to the intersection point of the two- and three-loop result. This justifies the choice $\mu = M_t$ as default value.

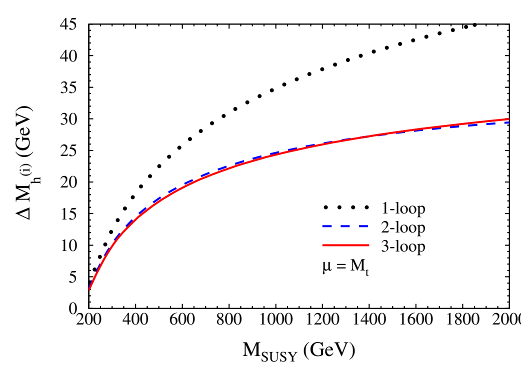

FIG. 1 (color online). ΔM_h as a function of $M_{\rm SUSY}$ at one-, two-, and three-loop level. The renormalization scale is set to $\mu = M_t$.

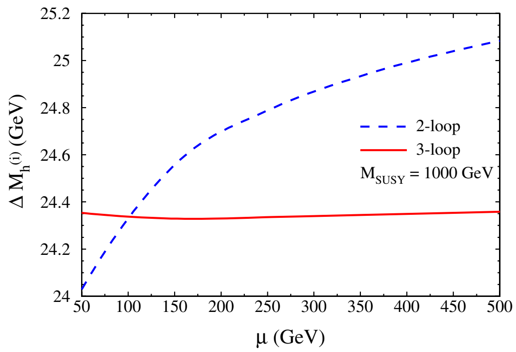

FIG. 2 (color online). ΔM_h as a function of the renormalization scale μ at two- and three-loop level, where $M_{\rm SUSY} = 1$ TeV has been chosen.

Conclusions.—To summarize, in this Letter the three-loop corrections to the lightest Higgs boson mass have been computed in three different limits of the SUSY parameter space. For the phenomenologically interesting case where the gluino and top squarks have about the same mass and the remaining squarks are heavier, we observe effects of approximately 500 MeV. The dependence of the three-loop result on the renormalization scale indicates that the residual theoretical uncertainty matches the expected accuracy for a Higgs mass measurement at the LHC and possibly even at a future linear collider.

It remains to say that the calculational setup which was used to obtain the results of this Letter is not restricted to the specific MSSM parameter points considered here. A more comprehensive study is in preparation and will be presented elsewhere.

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