Viscosity Bound and Causality Violation

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In recent work we showed that, for a class of conformal field theories (CFT) with Gauss-Bonnet gravity dual, the shear viscosity to entropy density ratio, η/s, could violate the conjectured Kovtun-Starinets-Son viscosity bound, $\eta/s \geq 1/4\pi$. In this Letter we argue, in the context of the same model, that tuning η/s below $(16/25)(1/4\pi)$ induces microcausality violation in the CFT, rendering the theory inconsistent. This is a concrete example in which inconsistency of a theory and a lower bound on viscosity are correlated, supporting the idea of a possible universal lower bound on η/s for all consistent theories.

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The anti–de Sitter (AdS)/conformal field theory (CFT) correspondence [1–4] has yielded striking insights into the dynamics of strongly coupled gauge theories. Among them is the universality of the ratio of the shear viscosity η to the entropy density s [5–8],

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad (1)$$

for all gauge theories with an Einstein gravity dual in the limit $N \to \infty$ and $\lambda \to \infty$, where N is the number of colors and λ is the 't Hooft coupling. It was further conjectured in [8] that (1) is a universal lower bound [the Kovtun-Starinets-Son (KSS) bound] for all materials. So far, all known substances including water and liquid helium satisfy the bound. The systems coming closest to the bound include the quark-gluon plasma created at RHIC [9–14] and certain cold atomic gases in the unitarity limit (see, e.g., [15]). η/s for pure gluon QCD slightly above the deconfinement temperature has also been calculated on the lattice recently [16] and is about 30% larger than (1) (see also [17]). Furthermore, the leading order α' correction to η/s has been calculated for the dual of type IIB string theory on $AdS_5 \times S^5$ and found to satisfy the bound [18,19]. See [20–24] for other discussions of the bound.

From the point of view of AdS/CFT, an interesting feature of the KSS bound is that it is *saturated* by Einstein gravity. Thus at the linearized order generic small corrections to Einstein gravity violate the bound half of the time. Given that we do expect corrections to Einstein gravity to occur in any quantum theory of gravity, it appears that the bound is in immediate danger of being violated. On the other hand, the correctness of the bound would impose an important constraint on possible higher order corrections to Einstein gravity.

Motivated by the vastness of the string landscape [25], we have explored the modification of η/s due to generic higher derivative terms in the holographic gravity dual [26]. For closely related work, including a plausible counterexample to the KSS bound, see [27]. In particular, for a class of (3 + 1)-dimensional CFTs with Gauss-Bonnet gravity dual, described by the classical action of the form [28] (below $\Lambda = -\frac{6}{L^2}$ and the Gibbons-Hawking term [29] is suppressed)

$$I = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} \Big[R - 2\Lambda + \frac{\lambda_{\rm GB}}{2} L^2 (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) \Big], \quad (2)$$

we found that [26]

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 - 4\lambda_{\rm GB}]. \quad (3)$$

We emphasize that this result is nonperturbative in $\lambda_{\rm GB}$, not just a linearly corrected value. From (3) the KSS bound is violated for $\lambda_{\rm GB} > 0$ and as $\lambda_{\rm GB} \to \frac{1}{4}$, the shear viscosity goes to zero [30].

In this Letter, we will argue that, when $\lambda_{\rm GB} > \frac{9}{100}$, the theory violates microcausality and is inconsistent. Thus, for (3 + 1)-dimensional CFT duals of (4 + 1)-dimensional Gauss-Bonnet gravity, consistency of the theory requires

$$\frac{\eta}{s} \geq \frac{16}{25}\left(\frac{1}{4\pi}\right). \quad (4)$$

This provides a concrete example in which a lower bound on η/s and the consistency of the theory are correlated. The 36% difference from the KSS bound is mysterious, and we discuss two obvious possibilities at the end. Our discussion below will rely heavily on a few technical results derived in [26], to which we refer the readers for details and references.

The static black brane solution for (2) can be written as [31]

$$ds^2 = -f(r)N_\sharp^2 dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}\left(\sum_{i=1}^{3} dx_i^2\right), \quad (5)$$

where

$$f(r) = \frac{r^2}{L^2}\frac{1}{2\lambda_{GB}}\left[1 - \sqrt{1 - 4\lambda_{GB}\left(1 - \frac{r_+^4}{r^4}\right)}\right]. \quad (6)$$

In (5), $N_\sharp$ is an arbitrary constant which specifies the speed of light of the boundary theory. We will take it to be $N_\sharp^2 \equiv \frac{1}{2}(1 + \sqrt{1 - 4\lambda_{GB}})$ to set the boundary speed of light to unity. The horizon is located at $r = r_+$ and the Hawking temperature is $T = N_\sharp \frac{r_+}{\pi L^2}$. Such a solution describes the boundary theory on $\mathbf{R}^{3,1}$ at a temperature T.

The shear viscosity can be computed by studying small metric fluctuations $\phi = h_2^1$ around the black brane background (5). We will take ϕ to be independent of x_1 and x_2 and write

$$\phi(t, \vec{x}, r) = \int \frac{d\omega dq}{(2\pi)^2}\phi(r; \omega, q)e^{-i\omega t + iqx_3}. \quad (7)$$

At quadratic level, the effective action for $\phi(r; \omega, q)$ can be found from (2) as (up to surface terms)

$$S = -\frac{1}{2}C\int dz \frac{d\omega dq}{(2\pi)^2}(K|\partial_z\phi|^2 - K_2|\phi|^2) \quad (8)$$

where C is a constant, and

$$K = z^2\tilde{f}(z - \lambda_{GB}\tilde{f}'), \quad K_2 = K\frac{\tilde{\omega}^2}{N_\sharp^2\tilde{f}^2} - \tilde{q}^2 z(1 - \lambda_{GB}\tilde{f}''). \quad (9)$$

Primes above denote derivatives with respect to z and we have introduced the following notation

$$z = \frac{r}{r_+}, \qquad \tilde{\omega} = \frac{L^2}{r_+}\omega, \qquad \tilde{q} = \frac{L^2}{r_+}q, \quad (10)$$

$$\tilde{f} = \frac{L^2}{r_+^2}f = \frac{z^2}{2\lambda_{GB}}\left(1 - \sqrt{1 - 4\lambda_{GB} + \frac{4\lambda_{GB}}{z^4}}\right). \quad (11)$$

From (8) one can derive the retarded two-point function for the stress tensor component T_{12} in the boundary CFT and read off the shear viscosity (3) in the small q and ω limit. In [26] it was also observed that for $\lambda_{GB} > \frac{9}{100}$ and sufficiently large q, the equation of motion for ϕ admits solutions which can be interpreted as metastable quasiparticles of the boundary CFT. We will now argue that these quasiparticles can travel faster than the speed of light and thus violate causality. We will first display this phenomenon using graviton geodesics.

In a gravity theory with higher derivative terms, graviton wave packets in general do not propagate on the light cone of a given background geometry. The equation of motion following from (8) can be written as

$$\tilde{g}_{\text{eff}}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi = 0, \quad (12)$$

where $\tilde{\nabla}_\mu$ is a covariant derivative with respect to the effective geometry $\tilde{g}_{\mu\nu}^{\text{eff}} = \Omega^2 g_{\mu\nu}^{\text{eff}}$ given by

$$g_{\mu\nu}^{\text{eff}}dx^\mu dx^\nu = f(r)N_\sharp^2\left(-dt^2 + \frac{1}{c_g^2}dx_3^2\right) + \frac{1}{f(r)}dr^2. \quad (13)$$

Here, $\Omega^2 = \frac{K}{\tilde{f}}z(1 - \lambda_{GB}\tilde{f}'')$ and

$$c_g^2(z) = \frac{N_\sharp^2\tilde{f}}{z^2}\frac{1 - \lambda_{GB}\tilde{f}''}{1 - \frac{\lambda_{GB}\tilde{f}'}{z}} \quad (14)$$

can be interpreted as the local "speed of graviton" on a constant r-hypersurface. As pointed out in [26], an important feature of (14) is that c_g can be greater than 1 when $\lambda_{GB} > \frac{9}{100}$ (see Fig. 1). Note that c_g has a maximum $c_{g,\max} > 1$ somewhere outside the horizon and approaches 1 near the boundary at infinity. We will restrict our attention to $\lambda_{GB} > \frac{9}{100}$ for the rest of the Letter.

From standard geometrical optics arguments [32], in the large momentum limit, a localized wave packet of a graviton should follow a null geodesic $x^\mu(s)$ in the effective graviton geometry (13). More explicitly, write the wave function (7) in the form $\phi = e^{i\Theta(t,r,x_3)}\phi_{en}(t, r, x_3)$, where Θ is a rapidly varying phase and ϕ_{en} denotes a slowly varying envelope function. Inserting into (12), we find at leading order

$$\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}g_{\mu\nu}^{\text{eff}} = 0, \quad (15)$$

with the identification $\frac{dx^\mu}{ds} \equiv g_{\text{eff}}^{\mu\nu}k_\nu \equiv g_{\text{eff}}^{\mu\nu}\nabla_\nu\Theta$. Given translational symmetries in the t and x_3 directions, we can interpret ω and q as conserved integrals of motion along the geodesic,

$$\omega = \left(\frac{dt}{ds}\right)fN_\sharp^2, \qquad q = \left(\frac{dx_3}{ds}\right)fN_\sharp^2\frac{1}{c_g^2}. \quad (16)$$

Assuming $q \neq 0$ and rescaling the affine parameter as $\tilde{s} = qs/N_\sharp$, we get from (15) and (16)

$$\left(\frac{dr}{d\tilde{s}}\right)^2 = \alpha^2 - c_g^2, \qquad \alpha \equiv \frac{\omega}{q}. \quad (17)$$

This describes a one-dimensional particle of energy α^2

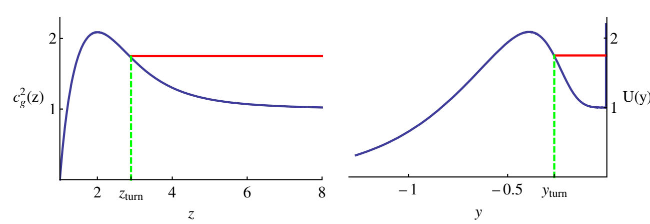

FIG. 1 (color online). Left: $c_g^2(z)$ as a function of z for $\lambda_{GB} = 0.245$. c_g^2 has a maximum $c_{g,\max}^2$ at $z_{\max}$. As λ_{GB} is increased from $\lambda_{GB} = \frac{9}{100}$ to $\lambda_{GB} = \frac{1}{4}$, $c_{g,\max}^2$ increases from 1 to 3. $c_g^2(z)$ also serves as the classical potential for the 1D system (17). The horizontal line indicates the trajectory of a classical particle. Right: $U(y)$ [defined in (25)] as a function of y for $\lambda_{GB} = 0.245$.

moving in a potential given by c_g^2. As is clear from Fig. 1, geodesics starting from the boundary can bounce back to the boundary, with a turning point $r_{\rm turn}(\alpha)$ given by

$$\alpha^2 = c_g^2(r_{\rm turn}). \qquad (18)$$

In contrast, for $\lambda_{\rm GB} \le \frac{9}{100}$, $c_g(z)$ is a monotonically increasing function of z and there is no bouncing geodesic. For a null bouncing geodesic starting and ending at the boundary, we then have

$$\Delta t(\alpha) = 2\int_{r_{\rm turn}(\alpha)}^{\infty} \frac{\dot t}{\dot r} dr = \frac{2}{N_\sharp}\int_{r_{\rm turn}(\alpha)}^{\infty} \frac{\alpha}{f\sqrt{\alpha^2 - c_g^2}} dr, \qquad (19)$$

$$\Delta x_3(\alpha) = 2\int_{r_{\rm turn}(\alpha)}^{\infty} \frac{\dot x_3}{\dot r} dr = \frac{2}{N_\sharp}\int_{r_{\rm turn}(\alpha)}^{\infty} \frac{c_g^2}{f\sqrt{\alpha^2 - c_g^2}} dr, \qquad (20)$$

where dots indicate derivatives with respect to $\tilde s$.

In the boundary CFT we have local operators which create bulk disturbances at infinity that propagate on graviton geodesics sufficiently deep inside the bulk ($r \lesssim \omega$) [33]. In particular, we expect microcausality violation in the boundary CFT if there exists a bouncing graviton geodesic with $\frac{\Delta x_3(\alpha)}{\Delta t(\alpha)} > 1$ [34]. Now, as $r_{\rm turn} \to r_{\rm max}$ ($\alpha \to c_{g,\rm max}$), a geodesic hovers near $r_{\rm max}$ for a long time, propagating with a speed $c_{g,\rm max}$ in x_3-direction. Indeed, the integrals in (19) and (20) are dominated by contributions near $r_{\rm max}$. In such a limit, the ratio of the integrand in $\Delta x_3(\alpha)$ to that in $\Delta t(\alpha)$ near $r_{\rm max}$ is $c_{g,\rm max}$. Thus, $\frac{\Delta x_3(\alpha)}{\Delta t(\alpha)} \to c_{g,\rm max} > 1$, violating causality.

We will now show explicitly that the superluminal graviton propagation described above corresponds to superluminal propagation of metastable quasiparticles [35] in the boundary CFT with $\frac{\Delta x_3}{\Delta t}$ identified as the group velocity of the quasiparticles. For this purpose, we rewrite the full wave equation (12) in a Schrödinger form

$$-\partial_y^2\psi + V(y)\psi = \tilde\omega^2\psi \qquad (21)$$

with ψ and y defined by

$$\frac{dy}{dz} = \frac{1}{N_\sharp \tilde f(z)}, \qquad \psi = B\phi, \qquad B = \sqrt{\frac{K}{\tilde f}}, \qquad (22)$$

and

$$V(y) = \tilde q^2 c_g^2(z) + V_1, \qquad V_1(y) = \frac{N_\sharp^2 \tilde f^2}{B}\left(B'' + \frac{\tilde f'}{\tilde f}B'\right). \qquad (23)$$

In the above primes denote derivatives with respect to z. Note that $y(z)$ is a monotonically increasing function of z with $y \to 0$ as $z \to \infty$ (boundary) and $y \to -\infty$ as $z \to 1$ (horizon). $c_g^2(z)$ is given by (14). V_1 is a monotonically increasing function of y (for $\lambda_{\rm GB} > 0$) with $V_1(y = -\infty) = 0$ and $V_1 \sim y^{-2}$ as $y \to 0$.

Since c_g^2 is monotonically decreasing for $r > r_{\rm max}$, for large enough $\tilde q$, $V(y)$ develops a well and admits metastable states (see Fig. 2). The wave functions of such metastable states are normalizable at the AdS boundary and have an in-falling tail at the horizon, corresponding to quasiparticles in the boundary CFT [35].

Now consider the limit $\tilde q \to \infty$. Since V_1 is independent of $\tilde q$, the dominant contribution to the potential is given by $\tilde q^2 c_g^2(z)$ except for a tiny region $y \gtrsim -\frac{1}{\tilde q}$. Thus in this limit, we can simply replace $V_1(y)$ by $V_1(y) = 0$ for all $y < 0$ and $V_1(0) = +\infty$. Equation (21) can then be written as

$$-\hbar^2\partial_y^2\psi + U(y)\psi = \alpha^2\psi, \qquad \hbar \equiv \frac{1}{\tilde q} \to 0, \qquad (24)$$

where α was introduced in (17) and (see Fig. 1)

$$U(y) = \begin{cases} c_g^2(y) & y < 0 \\ +\infty & y = 0 \end{cases}. \qquad (25)$$

In the $\hbar \to 0$ limit, we can apply the WKB approximation. The leading WKB wave function $e^{i\Theta(t,r,x_3)}$ is just the rapidly varying phase of the geometric optics approximation. The real part of α^2 satisfies the Bohr-Sommerfeld quantization condition (with n some integer)

$$\tilde q \int_{y_{\rm turn}}^{0} dy\sqrt{\alpha^2 - c_g^2(y)} = \left(n - \frac{1}{4}\right)\pi. \qquad (26)$$

The above equation determines ω as a function of q for each given n. Taking the derivative with respect to q on both sides of (26), we find that the group velocity of the quasiparticles is given by

$$v_g = \frac{d\omega}{dq} = \frac{\Delta x_3(\alpha)}{\Delta t(\alpha)}, \qquad (27)$$

where $\Delta t(\alpha)$ and $\Delta x_3(\alpha)$ are given by (19) and (20) respectively. Thus as argued in the paragraph below (20), v_g approaches $c_{g,\rm max} > 1$ as $\alpha \to c_{g,\rm max}$, violating causality. In this limit the WKB wave function is strongly peaked near $r_{\rm max}$, reflecting the long time the geodesic spends there. One can also estimate the imaginary part of α^2 (or ω), which has the form $e^{-h(\alpha)\tilde q}$ with $h(\alpha)$ given by the standard WKB formula. Thus in the $\tilde q \to \infty$ limit the quasiparticles become stable. Presumably local boundary operators that couple primarily to the long-lived quasiparticles can be constructed by following [33].

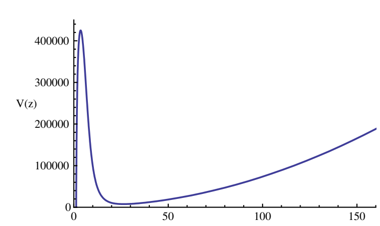

FIG. 2 (color online). $V(z)$ as a function of z for $\lambda_{\rm GB} = 0.2499$ and $\tilde q = 500$.

To summarize, we have argued that signals in the boundary theory propagate outside the light cone. In a boosted frame disturbances will propagate backward in time. Since the boundary theory is nongravitational, these are unambiguous signals of causality violation and hence inconsistency.

Here we observe causality violation in the high momentum limit. This is in agreement with the expectation that causality should be tied to the local, short-distance behavior of the theory. Also, a sharp transition from causal to acausal behavior as a function of $\lambda_{\rm GB}$ is possible because of the limiting procedure $\tilde{q} \to \infty$ needed in our argument. A more rigorous derivation of these phenomena using the full spectral function obtained from the Schrödinger operator would be desirable.

We argued that, for a $(4+1)$-dimensional Gauss-Bonnet gravity, causality requires $\lambda_{\rm GB} \le \frac{9}{100}$. Thus, consistency of this theory requires

$$\frac{\eta}{s} \ge \frac{16}{25}\left(\frac{1}{4\pi}\right). \tag{28}$$

This still leaves rooms for a violation of the KSS bound. We see two possibilities.

First, it could be that Gauss-Bonnet theory with $\lambda_{\rm GB} \le \frac{9}{100}$ is consistent and appears as a classical limit of a consistent theory of quantum gravity, somewhere in the string landscape (see [27] for a plausible counterexample to the KSS bound). Maybe this is how nature works and the KSS bound can be violated, at least by 36%.

Alternatively, it could be that there is a more subtle inconsistency in the theory within the window of $0 < \lambda_{\rm GB} \le \frac{9}{100}$. These issues deserve further investigation.

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[35] Quasiparticles in the boundary CFT correspond to poles in the retarded Green function which are sufficiently close to the real axis in the complex ω-plane. Such poles in turn correspond to solutions of the equation of motion (12) which are normalizable near the AdS boundary and in-falling at the horizon [36].

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