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A Test of the Copernican Principle

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The blackbody nature of the cosmic microwave background (CMB) radiation spectrum is used in a modern test of the Copernican principle. The reionized universe serves as a mirror to reflect CMB photons, thereby permitting a view of ourselves and the local gravitational potential. By comparing with measurements of the CMB spectrum, a limit is placed on the possibility that we occupy a privileged location, residing at the center of a large void. The Hubble diagram inferred from lines of sight originating at the center of the void may be misinterpreted to indicate cosmic acceleration. Current limits on spectral distortions are shown to exclude the largest voids which mimic cosmic acceleration. More sensitive measurements of the CMB spectrum could prove the existence of such a void or confirm the validity of the Copernican principle.

Introduction.—The observed accelerating expansion of the Universe [1,2] poses deep questions for cosmology. Is the Universe filled by some new, exotic dark energy, or is one of the basic tenets of the standard model of cosmology invalid? One such tenet is the cosmological principle, the assumption of approximate homogeneity and isotropy of matter and radiation throughout the Universe, which is known to be partly satisfied. The Universe is observed to be very nearly isotropic on our celestial sphere, on the basis of the near-isotropy of the cosmic microwave background (CMB) temperature pattern [3], and is observed to be approximately homogeneous across the distances probed by large-scale structures [4]. Yet, radial homogeneity on scales $\gtrsim 1$ Gpc, a cosmic version of the Copernican principle, remains to be proven. If the assumption of radial homogeneity is relaxed, and if we observe from a preferred vantage point, then it may be possible to explain the apparent cosmic acceleration in terms of a peculiar distribution of matter centered upon our location [5]. In fact, models of the Universe consisting of a spherically symmetric distribution of matter, mathematically described by a Lemaitre-Tolman-Bondi spacetime [6], have been shown to produce a Hubble diagram which is consistent with observations. These models require no dark energy, and locally resemble a matter-dominated low-density Universe or void. The observed near-isotropy constrains us to occupy a very special location, at or near the center of the void, in violation of the Copernican principle. Although the Copernican principle may be widely accepted by *fiat*, it is imperative that such a foundational principle be proven.

We propose to test the Copernican principle, to verify radial homogeneity and thereby constrain nonaccelerating void cosmological models. The test follows a similar argument by Goodman [7], that constrains distortions of the CMB blackbody spectrum produced by anisotropic scattering [8]. The CMB is initially thermal (blackbody), but small inhomogeneities cause variations in the temperature at different locations and along different lines-of-sight that

preserve the blackbody spectrum. However, scattering of this anisotropic radiation into our line-of-sight by ionized gas produces observable spectral distortions.

Here we are interested in anisotropies caused by a large, local void. Such a structure causes ionized gas to move outward, in motion relative to the CMB frame which leads to a Doppler anisotropy in the gas frame. The gravitational potential of such a structure also leads to a Sachs-Wolfe (SW) effect for photons which originate inside of the void and scatter back toward us. The geometry of these effects is illustrated in Fig. 1. A large void, or any other non-Copernican structure, will lead to large anisotropies in other places which will be reflected back at us in the form of spectral distortions. Hence, deviations from a blackbody spectrum can indicate a violation of the Copernican principle. In essence, we use the reionized Universe as a mirror to look at ourselves in CMB light. If we see ourselves in the mirror it is because ours is a privileged location. If we see nothing in the mirror, then the Copernican principle is upheld.

Spectral distortions.—The distortion of the CMB blackbody spectrum due to scattering by anisotropic CMB radiation is [8] $u[\hat{\mathbf{n}}] = \frac{3}{16\pi} \int_0^\infty dz \frac{d\tau}{dz} \int d^2\hat{\mathbf{n}}'(1 + (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}')^2) \times (\frac{\Delta T}{T}[\hat{\mathbf{n}}, \hat{\mathbf{n}}, z] - \frac{\Delta T}{T}[\hat{\mathbf{n}}', \hat{\mathbf{n}}, z])^2$, where $\Delta T/T[\hat{\mathbf{n}}', \hat{\mathbf{n}}, z]$ is the CMB temperature anisotropy in the direction $\hat{\mathbf{n}}'$, as observed at redshift z in the direction $\hat{\mathbf{n}}$ from the central observer, and τ is the optical depth. For cosmic voids extending out to redshifts $z \lesssim 1$, reflections back at us may occur up to $z \lesssim 3$ (see Fig. 1). The optical depth to Thomson scattering is small, so that it is appropriate to consider single scattering. Since the mean CMB temperature is not known *a priori*, but rather is fit to the observations, u is observationally degenerate with the Compton y-distortion parameter according to the relation $u = 2y$. (Compare Refs. [8,9] for details.) Thus observational constraints on $2y$ can be treated as constraints on u.

We consider a low-amplitude void embedded in a flat, Einstein–de Sitter (EdS or $\Omega = 1$) matter-dominated
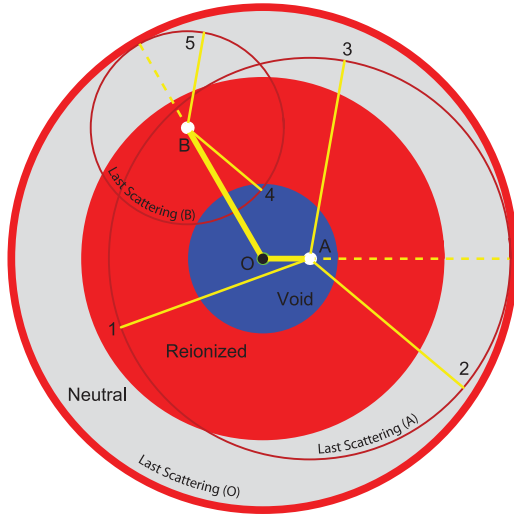
FIG. 1 (color).   Illustrated is a cross-section through a model
Universe with the observer (O) at the center of a void, in
violation of the Copernican principle. CMB photons traveling
in any direction may Thomson scatter off reionized gas. The
observed spectrum will be a mixture of blackbody spectra with
different (anisotropic) tempertures, producing a distorted black-
body. The yellow lines represent: incoming beams of unscat-
tered, primary CMB photons (dashed); incoming beams of
scattered photons (thin), and the observed beams (thick) for
representative scattering centers with last-scattering surfaces
represented by the dark circles. A is in the *Doppler zone*:
Beams 1–3 experience the same SW temperature shift, introduc-
ing no anisotropy. However, gradients in the void gravitational
potential cause the gas to move with respect to the CMB frame,
so A sees a differential Doppler anisotropy, resulting in spectral
distortions. B is in the *reflection zone*: B is at rest with respect to
the CMB frame and sees no Doppler anisotropy. However, some
of the incoming photons, e.g., beam 4, originate inside the void
so there will be an anisotropic SW temperature shift, leading to
spectral distortions.

Universe. The gravitational potential due to the void, $\Phi[\mathbf{x}]$,
is a function of comoving position, $\mathbf{x}$, with Earth near
$\mathbf{x} = 0$. The temperature anisotropy can be divided into a
Sachs-Wolfe and Doppler term $\frac{\Delta T}{T}[\hat{\mathbf{n}}', \hat{\mathbf{n}}, z] = \frac{\Delta T}{T}|_{\text{SW}} +$
$\frac{\Delta T}{T}|_{\text{Doppler}}$ where $\frac{\Delta T}{T}|_{\text{SW}} = \frac{1}{3c^2}(\Phi[\mathbf{x}_{\text{rec}}] - \Phi[\mathbf{x}_{\text{scatter}}])$ and
$\frac{\Delta T}{T}|_{\text{Doppler}} = \frac{2}{3}\hat{\mathbf{n}}' \cdot \nabla_{\mathbf{x}}\Phi[\mathbf{x}_{\text{scatter}}]/cH_0\sqrt{1+z}$, where
$\mathbf{x}_{\text{scatter}} = D_A^{\text{co}}[z]\hat{\mathbf{n}}$, $\mathbf{x}_{\text{rec}} = \mathbf{x}_{\text{scatter}} + (D_A^{\text{co}}[z_{\text{rec}}] - D_A^{\text{co}}[z])\hat{\mathbf{n}}'$,
$D_A^{\text{co}}[z] = 2\frac{c}{H_{e0}}(1 - \frac{1}{\sqrt{1+z}})$. Here D_A^{co} is the comoving
angular-diameter distance, and the redshift of recombina-
tion, z_{rec}, will be approximated by ∞ for simplicity. The
Hubble constant at the present time in the background
cosmology, outside the void, is H_{e0}, whereas $H_0 > H_{e0}$ is
the Hubble constant at the center of the void.

We neglect the integrated Sachs-Wolfe (ISW) effect,
meaning that a CMB photon does not contribute to the u
distortion simply because it passes across the void. This
approximation is justified for a low-amplitude void in the
EdS background where the ISW is a second-order effect.
For an increasingly nonlinear void we expect a larger ISW

contribution to the anisotropy and thus to the spectral
distortion, but we do not expect that the ISW will ever be
the dominant contributor to u for the small voids needed to
mimic an accelerating Universe.

The run of optical depth with redshift is taken from the
unperturbed, background cosmology. We assume a rapid
reionization at $z = z_{\text{rei}}$ such that $\frac{d\tau}{dz} = \tau_{e0}\sqrt{1 + z}\Theta[z_{\text{rei}} - z]$ and $\tau_{e0} = \frac{3H_{e0}\Omega_{b0}\sigma_T c}{8\pi Gm_H}(1 - \frac{1}{2}Y_{\text{He}})$, where $\Theta[x]$ is the
Lorentz-Heaviside step function, σ_T, m_H, Ω_{b0} and Y_{He}
are the Thomson cross-section, the hydrogen mass, the
current baryonic mass density in units of the critical den-
sity, and the helium mass fraction, respectively. We use
$\Omega_{b0}h^2 = 0.022$ ($h \equiv H_{e0}/100$ km/s/Mpc), $Y_{\text{He}} = 0.24$.
For H_0 we use the locally-measured expansion rate:
73 km/s/Mpc (e.g., Refs. [10,11]). Where needed we
use the WMAP3 [12] value, $\tau_{\text{obs}} = 0.09$, for the optical
depth to the surface of last-scattering which in our model
gives $z_{\text{rei}} = 11$. These numbers specify the cosmic evolu-
tion of the density of scatterers.

Void model.—We cannot compute u for every possible
void profile, so we focus our attention on a particularly
simple, two parameter class of spherically symmetric
voids, sometimes known as a *Hubble bubble*: $\Phi[R] = \Phi_0(1 - \frac{R^2}{R_V^2})\Theta[R_V - R]$. The parameters Φ_0, R_V give the
void amplitude and comoving radius. The reason it is
called a Hubble bubble is that the Hubble parameter is
uniform inside and outside the void, but the values differ.
Nonlinear growth leads to the appearance of a shell of mass
overdensity which compensates the underdensity in the
void at the boundary of the outer and inner region. This
compensating shell has a complicated density and velocity
structure, which is safely ignored in linear theory. Away
from the compensating shell this model resembles an open
($\Omega_0 < 1$) FRW cosmology embedded inside a flat EdS
cosmology. Any smooth spherical void which is asymptoti-
cally EdS at large R and has finite density in the center can
be thought of in this way; what differs is the radial profile
of the transition between the two FRW spacetimes. The
Hubble bubble is the limit of a sharp transition between the
interior and exterior regions.

The Hubble bubble amplitude can be expressed in terms
of the present day density parameter, Ω_0, inside the void as
$\Phi_0 = \frac{3}{20}(H_{e0}R_V)^2[\frac{(1-\Omega_0)^{3/2}}{\Omega_0}\frac{H_0}{H_{e0}}]^{2/3}$. Next, the void radius
can be expressed as a function of the redshift at the edge
of the void, z_{edge}. This relationship is complicated by finite
peculiar velocities and nonlinear clustering of the compen-
sating shell, but to first order is simply $R_V = 2(c/H_0) \times (1 - 1/\sqrt{1 + z_{\text{edge}}})$. Finally, the exterior Hubble parame-
ter, H_{e0}, differs from the interior value, H_0. At the same
"time since bang" they are related as $\frac{H_0}{H_{e0}} = \frac{3}{2}\frac{\sqrt{1-\Omega_0} - \Omega_0\sinh^{-1}[\sqrt{\frac{1-\Omega_0}{\Omega_0}}]}{(1-\Omega_0)^{3/2}}$. Note that a small jump in the
Hubble parameter corresponds to a large jump in the
density parameter.

The gas at different redshift must satisfy several criteria in order to contribute to the u distortion. A patch of gas at redshift z must be ionized, on our past light cone, and see an anisotropic ΔT from the void. We refer to the region $z > z_{rei}$ as the *neutral zone* because the gas is not ionized, producing no contribution to u. Even if the gas is ionized, if $z > z_{max} \equiv 3 + 4z_{edge}$ then the gas is not in causal contact with the void so $\Delta T = 0$. We refer to the range $(2\sqrt{z_{max} + 1} - 1)/(\sqrt{z_{max} + 1} - 1)^2 \leq z \leq z_{max}$ as the *reflection zone*; the last scattering surface of gas in this range intersects the interior of the void so that, depending on the scattering angle, some CMB photons will reflect back towards us with anisotropy ΔT_{SW}. Gas which is on our past light cone and within the void will see $\Delta T_{Doppler}$, which we call the *Doppler zone*.

Five classes of void sizes are identified depending on how the different zones overlap (assuming $z_{rei} > 8$): *small* ($z_{edge} \leq \frac{5}{4}$) whereby the neutral zone, reflection zone, and Doppler zone are all disjoint; *big* ($\frac{5}{4} < z_{edge} \leq \frac{1}{4}(z_{rei} - 3)$) whereby the Doppler zone and reflection zones overlap, but neither overlap the neutral zone; *large* ($\frac{1}{4}(z_{rei} - 3) < z_{edge} \leq z_{rei}$) in which the Doppler and reflection zones overlap, as do the reflection and neutral zones, but the neutral zone does not overlap the Doppler zone; *huge* ($z_{edge} > z_{rei}$) in which the neutral, reflection, and Doppler zones all overlap; and *superhorizon* ($R_V > 2c/H_0$) for which the void encompasses the entire observable Universe. This classification is not restricted to the Hubble bubble void profile, but applies to any void profile with a sharp edge at $z = z_{edge}$. As we shall see it is only the small voids that can explain the current SNe data.

In the linear perturbation approximation for this void model the spectral distortion u, illustrated in Fig. 2, is proportional to $(\frac{1}{3c^2}\Phi_0)^2 \tau_{e0}$ and may be decomposed as $u = \tau_{e0}(\frac{\Phi_0}{3c^2})^2(U_D + U_S + U_{DS})$ where the three terms are, respectively, the contribution from gas where the temperature anisotropies are Doppler only (subscript D), Sachs-Wolfe only (subscript S), and a combination of the two (subscript DS). All of these can be expressed analytically. For small and big voids u does not depend on z_{rei} but only on the dimensionless size parameter $r \equiv \frac{1}{2}\frac{H_0}{c}R_V = 1 - \frac{1}{\sqrt{1+z_V}}$. The general expression for u is long and we do not give it here. For small voids, which are the most relevant, we find $U_D^{small} = \frac{56}{5}\frac{1}{r^3}(1 + \frac{1}{1-r} + \frac{2}{r}\ln[1-r])$ and $U_S^{small} \ll U_D^{small}$ and $U_{DS}^{small} = 0$. Note that the limit $z_{edge} \rightarrow \infty$ corresponds to an open Universe, such that $u \rightarrow 0$. Although this is not generic, being a peculiarity of the Hubble bubble void profile, our main conclusions will not be affected.

The angular-diameter distance D_A is a solution of the Dyer-Roeder [13] equation, $\frac{d}{dz}((1+z)^2 H \frac{d}{dz}D_A) + \frac{3}{2}\Omega H D_A = 0$. In the interior open and exterior flat cosmologies the respective solutions are $D_A[z < z_i] = \frac{2c}{H_0}(2 - \Omega_0(1-z) - (2-\Omega_0)\sqrt{1 + z\Omega_0})/(1+z)^2\Omega_0^2$ and
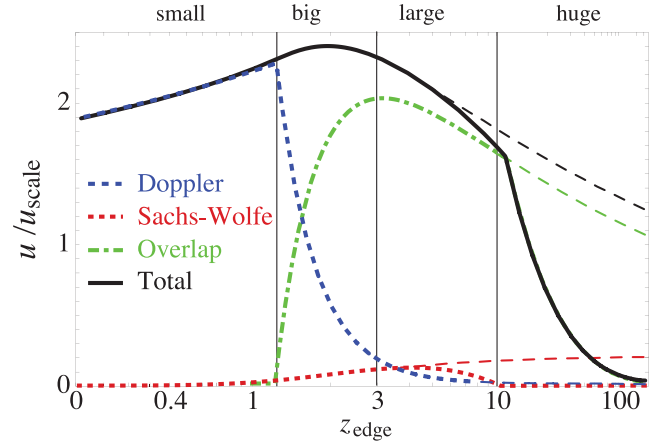

FIG. 2 (color). The dependence of the spectral distortion, u, on the size of a Hubble bubble parameterized by z_{edge}, is shown in units of $u_{scale} \equiv \tau_{e0}(\Phi_0/(3c^2))^2(1 + z_{edge})/(\sqrt{1 + z_{edge}} - 1)$. The thick curves show the various contributions to u. The overlap contribution is due to the Doppler and Sach-Wolfe cross term. The thin curves correspond to the case in which $z_{rei} \rightarrow \infty$. For small voids the Doppler contribution dominates, and the value of z_{rei} is unimportant.

$D_A[z > z_e] = \frac{2c}{H_0}(\frac{C_1}{(1+z)} + \frac{C_2}{(1+z)^{3/2}})$, where the coefficients C_1, C_2 are set by the continuity $D_A^{int}[z_i] = D_A^{ext}[z_e]$, and the jump in dD_A/dz as determined by integrating the Dyer-Roeder equation across the delta-function density spike at the void edge. The radial velocity drop, Δv, at the void edge means a double-valued $D_A[z]$ for $z \in [z_e, z_i]$ and $\frac{1+z_i}{1+z_e} = \sqrt{\frac{c+\Delta v}{c-\Delta v}}$. This drop also gives the Doppler anisotropy at the edge. To get z_i and z_e we use the approximations $\frac{\Delta v}{c} = \frac{\Delta T}{T}_{Doppler}[z_{edge}, \hat{n}, \hat{n}]$ and $z_{edge} = \frac{1}{2}(z_i + z_e)$. The luminosity distance versus redshift, also known as the Hubble diagram, is $(1 + z)^2 D_A[z]$.

Constraints.—The u distortion is evaluated according to the procedure described above. We are primarily interested in small and big voids which extend out to $z \sim 1$. Hence our constraints are independent of z_{rei}. The other cosmological parameters only enter into the overall normalization of u through τ_{e0}. What remains are the void size and amplitude: (z_{edge}, Ω_0).

The best current bound on u is due to FIRAS [14–16] which constrains $y < 15 \times 10^{-6}$ or $u < 3 \times 10^{-5}$ at 95% C.L. The corresponding constraint on void parameters are shown in Fig. 3. Also shown are constraints for projected bounds $y < 10^{-6}, 10^{-7}$. The limits are expected to improve [17,18], but a y distortion from the IGM would likely mask the signal discussed here if $u \lesssim 10^{-6}$ [19].

The results rule out large voids with large density contrasts—the most egregious violations of the Copernican principle. The larger the void, the smaller the density contrast must be in order to pass the test. Although not shown, the constraints become weaker for huge (nearly superhorizon sized) voids. Since observationally $\Omega_0 \lesssim 0.3$, only small bubbles with $z_{edge} < 0.9$ are allowed.
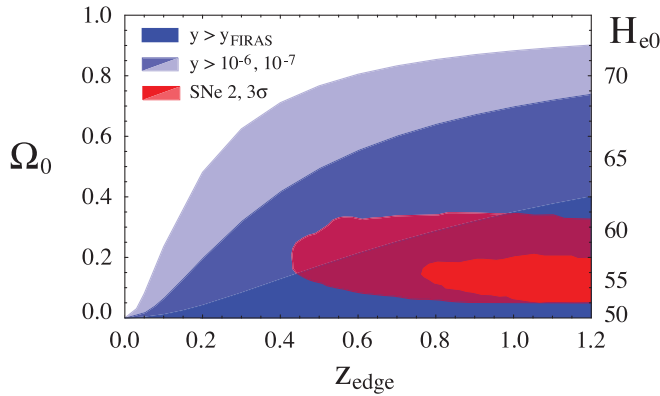
FIG. 3 (color).   The test of the Copernican principle, in terms of constraints on the size and depth of a local, spherically-symmetric void, is shown. The blue shaded regions show the range of parameters excluded by the u-distortion test, whereas the red regions show the range of parameters compatible with the current SNe Hubble diagram data.

Improving the constraint to $y < 10^{-6}$ would lower this bound to $z_{\text{edge}} \lesssim 0.3$ or a radius of 1 Gpc. These constraints are consistent with the very small Hubble bubble proposed in Ref. [20], with $H_0 - H_{e0} \sim 0.1 H_0$ and $z_{\text{edge}} \gtrsim 0.025$.

The observed SNe data can be compared with our model Hubble diagram to further constrain void parameters. Using the SNe data [21,22] compiled in Ref. [23], we computed the likelihood of Ω_0 and z_{edge}. The best-fit parameter combinations give $\chi^2 = 207$ for the 192 SNe magnitudes (within 3σ of the best-fit ΛCDM model based on a $\Delta\chi^2$ test, for a family of models with a sufficient number of parameters to encompass both Λ and the void). Voids which explain the observed Hubble diagram have low density and large size, $z_{\text{edge}} \sim 1$ (radii ~ 2.5 Gpc). However, as shown in Fig. 3, combining the SNe data with current limits on u ($\chi^2 = 225$, 250 for 191 degrees of freedom), we find that nearly all such voids are ruled out. These specific constraints only apply to the Hubble bubble class of models, which also suffers from other flaws not mentioned here. Other models, perhaps with a higher central density or smaller radius, may evade this test. We plan to apply this test to more general and more realistic void profiles.

An improvement in the bound on u by an order of magnitude may confirm or refute a wider variety of such voids as an explanation of the dark energy phenomenon. Yet, the u-distortion test presented here is more general than the question of dark energy. Future pursuit of this test will help improve our view of the Universe on the largest scales.

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