Plasma Currents and Electron Distribution Functions under a dc Electric Field of Arbitrary Strength

S. M. Weng,[1] Z. M. Sheng,[1,2,*] M. Q. He,[1] J. Zhang,[1,2] P. A. Norreys,[3] M. Sherlock,[3] and A. P. L. Robinson[3]

[1]Beijing National Laboratory of Condensed Matter Physics, Institute of Physics, CAS, Beijing 100190, China
[2]Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China
[3]Central Laser Facility, Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, United Kingdom

(Received 4 February 2008; published 5 May 2008)

The currents induced by arbitrarily strong dc electric fields in plasma and the evolution of electron distributions have been studied by Fokker-Planck simulations. We find that the electron distributions evolve distinctly under different fields; especially, the electron distribution is well represented by the sum of a stationary and drifting Maxwellian at the moderate field. A set of hydrodynamiclike equations, similar to Spitzer's but without the weak-field limit, is given for calculating the current. It is more suitable for application in hybrid particle-in-cell simulations and may extend plasma transport theory in models that do not employ a kinetic description of the electrons.

DOI: 10.1103/PhysRevLett.100.185001 PACS numbers: 52.65.Ff, 52.25.Fi, 52.25.Kn, 52.50.Nr

As a basic process, the behavior of plasmas under a dc electric field has been studied for nearly a century. In the weak-field limit, it is found that the plasma current is linearly related to the applied electric field [1] and the electric conductivity follows the well-known $T_e^{3/2}$ law [2]. However, the electron dynamics are quite complex and the observed electric conductivities are usually much lower than that predicted by Spitzer's model in the non-weak fields [3]. Based on some unexamined assumptions, such as a drifting Maxwellian electron distribution function (EDF) [4] or a static Maxwellian background EDF [5], the behavior of electrons in plasmas was studied in electric fields of arbitrary magnitude. However, the conductivity based on the drifting Maxwellian EDF assumption is found about half of Spitzer's in the weak-field limit [6], and the static Maxwellian background EDF assumption will induce an artificial cooling that disturbs the understanding of the actual EDF [7]. To our knowledge, the behavior of electrons in plasmas under an electric field of arbitrary strength is still an open problem. This problem is relevant to many areas of plasma physics and is particularly relevant to the very topical problem of fast electron transport in the fast ignition of fusion targets [8], where usually high dc electric fields as well as return currents are induced as found in hybrid-particle-in-cell (PIC) simulations [9]. The return currents in turn have strong effects on the fast electron transport [10]. In such a simulation, proper treatment of the response of background plasma electrons still needs to be solved.

In this Letter, we study the EDF and the current of plasmas under a dc electric field with a wide strength range by a Fokker-Planck code [11], which includes the full electron-electron (e-e) collision operator. We show that the behaviors of electrons are distinct under electric fields with different strengths, and that the EDF is well represented by the hybrid of a stationary and drifting Maxwellian at the moderate field. A set of hydrodynamic-like equations, which can be used as conveniently as Spitzer's, is derived for plasma current according to a detailed knowledge of EDFs from our Fokker-Planck simulation. Furthermore, it is not restricted by the weak-field limit and offers much better estimation than Spitzer's of the electric field to generate return current during the fast electron transport in the fast ignition targets.

In the presence of a dc electric field, the evolution of the EDF in a homogeneous, fully ionized plasma can be described by the Fokker-Planck equation [7,11–13]

$$\frac{\partial f^e}{\partial t} = \frac{e\mathbf{E}}{m_e} \cdot \nabla_{\mathbf{v}} f^e + C_{ei}(f) + C_{ee}(f), \tag{1}$$

where $\mathbf{E}$ is the applied electric field. Following the note of Ref. [11], the electron-ion (e-i) collision term $C_{ei}(f)$ and e-e collision term $C_{ee}(f)$ can be expressed as

$$C_{ei}(f) = -\nabla_{\mathbf{v}} \cdot (-D_{\theta\theta}^{e|i} \mathbf{e}_\theta \mathbf{e}_\theta \cdot \nabla_{\mathbf{v}} f^e), \tag{2}$$

$$C_{ee}(f) = -\nabla_{\mathbf{v}} \cdot (-\mathbf{D}^{e|e} \cdot \nabla_{\mathbf{v}} f^e + \mathbf{F}^{e|e} f^e). \tag{3}$$

Except that the dc electric field takes the place of the ac electric field, Eq. (1) is the same as the Fokker-Planck equation solved in Ref. [11]. Therefore, the numerical scheme adopted to solve this equation is essentially the same one presented there.

The terms in Eq. (1) can be divided into two types, the first type is the diffusion term, including $\mathbf{D}^{e|e}$ and $\mathbf{D}^{e|i}$, which tends to spread electrons throughout the velocity space; the second is the friction term, including $\mathbf{F}^{e|e}$ and $-e\mathbf{E}/m_e$, which tends to decelerate or accelerate electrons [13]. A general knowledge of the magnitudes of these terms will be helpful for the division of computational regions for the electric field $\mathbf{E}$ and for the comprehension of the evolution of EDFs in these regions. In Fig. 1, we show $F_v^{e|e}$, $D_{\theta\theta}^{e|e}$, and $D_{\theta\theta}^{e|i}$ as functions of $v_\parallel$ at $v_\perp = 0$ for a Maxwellian distribution. Notice that $F_v^{e|e}$ is in units of

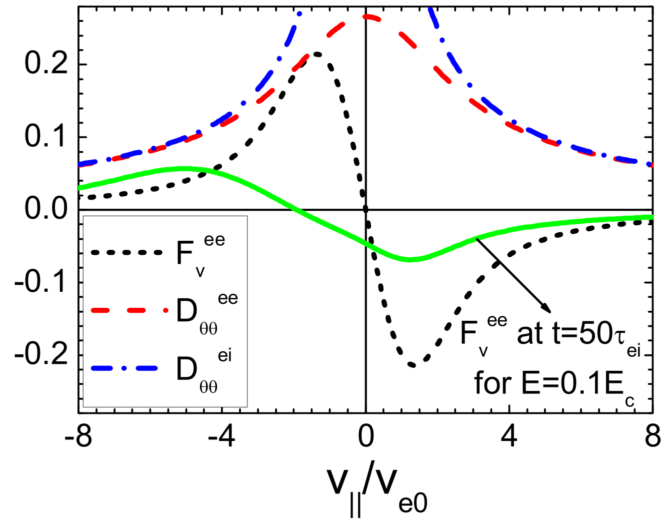

FIG. 1 (color online). $F_v^{e|e}$, $D_{\theta\theta}^{e|e}$, and $D_{\theta\theta}^{e|i}$ of a Maxwellian distribution, and $F_v^{e|e}$ of a Maxwellian distribution affected by an electric field $0.1E_c$ at time $50\tau_{ei}$ as functions of $v_\parallel$ at $v_\perp = 0$. $D_{\theta\theta}^{e|e}$ is in units of $eE_c v_{e0}/Z_i m_e$, $F_v^{e|e}$ in units of $eE_c/Z_i m_e$, and $D_{\theta\theta}^{e|i}$ in units of $eE_c v_{e0}/m_e$.

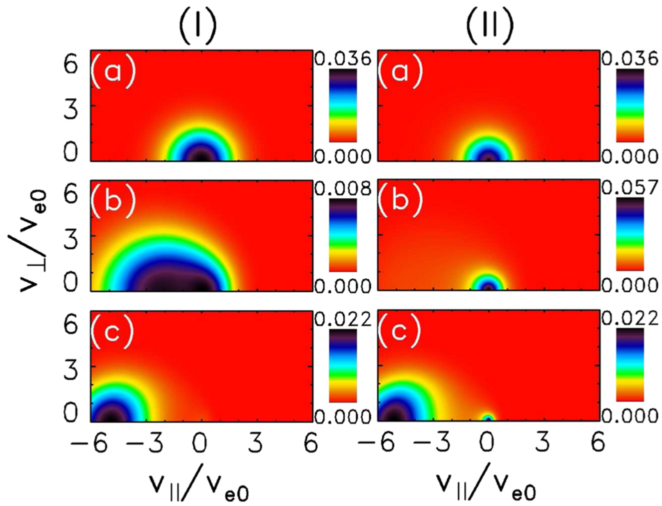

FIG. 2 (color online). Snapshots of the EDFs under electric fields of different strengths: (a) $0.01E_c$ after $500\tau_{ei}$, (b) $0.1E_c$ after $50\tau_{ei}$, and (c) $1.0E_c$ after $5\tau_{ei}$. Column (I) is for plasmas with $Z_i = 1$ and column (II) for $Z_i = \infty$. The EDF is in units of n_e/v_{e0}^3.

$eE_c/Z_i m_e$, $D_{\theta\theta}^{e|e}$ in units of $eE_c v_{e0}/Z_i m_e$, while $D_{\theta\theta}^{e|i}$ is in units of $eE_c v_{e0}/m_e$, where $E_c = m_e v_{e0} \nu_{ei}/e$, $\nu_{ei} = Z_i \Gamma^{e|e}/v_{e0}^3 = 1/\tau_{ei}$ is the *e-i* collision frequency, and v_{e0} is the initial thermal velocity. Therefore, the *e-e* diffusion and friction terms are comparable to the *e-i* diffusion term for $Z_i = 1$. And the *e-e* collisions may play an important role in the evolution of EDF for low Z_i plasma; however, there are few papers that treat them self-consistently.

Since the diffusion term $D_{\theta\theta}^{e|i}$ is of the magnitude of $0.1eE_c v_{e0}/m$ for the main velocity regime, we select $0.01E_c$, $0.1E_c$, and $1.0E_c$ for E as measures for weak, moderate, and strong fields. The EDFs in these cases are shown in Fig. 2, in which the times are selected to be unequal in order to keep Et equal for all cases. In order to reveal the role of *e-e* collisions, the distributions for $Z_i = 1$ and Lorentz plasma $Z_i = \infty$ are shown in columns (I) and (II), respectively. We note that electric fields with different strengths produce distinct electron distributions. In the weak field, there are almost no shifts of the centers of EDFs from $v = 0$ for both $Z_i = 1$ and $Z_i = \infty$. This means in the weak field the *e-i* collisions themselves are strong enough to prevent electrons from being blown away. In the strong field, the EDFs are almost collectively drifted away from $v = 0$ for both $Z_i = 1$ and $Z_i = \infty$, and electrons seem to be accelerated freely.

The behaviors of electrons and the EDFs deserve more attention in the moderate field region. Since the *e-e* collisions are comparable to the external field for $Z_i = 1$ in this field, the competition between them is so violent that the EDF is torn into a highly distorted one as shown in Fig. 2(b) in column (I). The EDF in this case can be approximately divided into two components, the first satisfies a stationary Maxwellian distribution and the second a drifting Maxwellian distribution, and these two distributions are found to coexist for quite a long time. Actually, the drifting Maxwellian distribution also coexists with the stationary Maxwellian distribution in the strong field limit, but the proportion of the drifting Maxwellian component and the mean drift velocity increase so fast that the drifting Maxwellian distribution is basically dominant. The coexistence of these two distributions could be illuminated as follows. The *e-e* collisions modify themselves following the time evolution of EDF and then reassemble the drifting electrons as a new Maxwellian EDF around the drifting center, while the *e-i* collisions unweariedly capture the electrons as a stationary Maxwellian EDF around $v = 0$. As shown in Fig. 1, the center of $F_v^{e|e}$ at $t = 50\tau_{ei}$ moves to $v_\parallel = -2v_{e0}$, which is also the center of distorted EDF at this time as shown in Fig. 2(b) in column (I), and the effect of $F_v^{e|e}$ seems to be a backward force for the electrons with $v_\parallel > -2v_{e0}$ but a forward force for the electrons with $v_\parallel < -2v_{e0}$. Therefore the drifting electrons can be reassembled around $v_\parallel = -2v_{e0}$ by $F_v^{e|e}$ not just blown away by the electric field as shown in Fig. 2(b) in column (II), where there are no *e-e* collisions. Actually, the *e-e* collisions can be considered as restoring forces to keep electrons assembling as a Maxwellian distribution around a center regardless of the drifting of this center. On the other hand, the *e-i* collisions appear more like restoring forces to keep electrons assembling around $v = 0$, since ions always intend to capture electrons by Coulomb force. And the *e-i* diffusion term $D_{\theta\theta}^{e|i} \propto 1/v$ is so efficient around $v = 0$ that it can compete against the external electric field and then capture some electrons around $v = 0$ even in the strong field as shown in Fig. 2(c) in column (II). But this effect will be weakened by the *e-e* collisions, which tend to bring the "stragglers" back to the drifting Maxwellian distribution as shown in Fig. 2(c) in column (I). Finally, the distribution is presented by the hybrid of a stationary and drifting Maxwellian under the common actions of e-e collisions, e-i collisions, and this moderate field.

As mentioned above, in the moderate field, we can express the EDF as

$$f(\mathbf{v}) = \delta f_M(\mathbf{v}) + (1-\delta) f_d(\mathbf{v}), \tag{4}$$

where δ is the proportion of the stationary Maxwellian component, the stationary Maxwellian EDF $f_M(\mathbf{v})$ and the drifting Maxwellian EDF $f_d(\mathbf{v})$ are defined by

$$f_M(\mathbf{v}) = \frac{n_e}{(2\pi v_{te1}^2)^{3/2}} \exp\left(-\frac{v^2}{2v_{te1}^2}\right), \tag{5}$$

$$f_d(\mathbf{v}) = \frac{n_e}{(2\pi v_{te2}^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{v_d})^2}{2v_{te2}^2}\right]. \tag{6}$$

For the Maxwellian component, Spitzer's electric conductivity still applies but increases as $\sigma \propto T_e^{3/2}$. Therefore, the current of this component can be calculated as

$$J_M = \delta[1-\exp(-t/\tau_r)]\sigma_0(v_{te1}/v_{e0})^3 E, \tag{7}$$

where $\sigma_0 = \gamma_E 32 n_e e^2/\sqrt{2\pi} m_e \nu_{ei}$ is Spitzer's conductivity [2] for initial temperature $T_0 = m_e v_{e0}^2$, and τ_r is the response time defined as the time when the current is $1/e$ (here e is Euler's number) of the final steady value, which varies from about $3.8\tau_{ei}$ for $Z=1$ to $7.5\tau_{ei}$ for $Z=\infty$. While the current coming from the drifting Maxwellian component can be calculated as

$$\mathbf{J}_d = -(1-\delta)e\int \mathbf{v} f_d(\mathbf{v}) d\mathbf{v} = -(1-\delta) e n_e \mathbf{v}_d. \tag{8}$$

From Eq. (4), we can calculate the parallel temperature $T_\parallel = m_e \int f(\mathbf{v}) v_\parallel^2 d\mathbf{v}$ and the perpendicular temperature $T_\perp = \frac{1}{2} m_e \int f(\mathbf{v}) v_\perp^2 d\mathbf{v}$ as

$$T_\parallel/m_e = \delta v_{te1}^2 + (1-\delta)(v_{te2}^2 + v_d^2), \tag{9}$$

$$T_\perp/m_e = \delta v_{te1}^2 + (1-\delta) v_{te2}^2. \tag{10}$$

Equations (9) and (10) give

$$(1-\delta) v_d^2 = (T_\parallel - T_\perp)/m_e. \tag{11}$$

It is hard to solve v_d from Eq. (11) strictly; however, if $T_\parallel \gg T_\perp$, then the drifting Maxwellian will be much more than the stationary Maxwellian $(1-\delta) \gg \delta$; hence $v_d \simeq [(T_\parallel - T_\perp)/m_e]^{1/2}$. Otherwise, the proportion of the drifting Maxwellian component will be small so the error coming from approximation $v_d \simeq [(T_\parallel - T_\perp)/m_e]^{1/2}$ would not affect the total current obviously. Assuming the drifting Maxwellian EDF and the stationary Maxwellian EDF have the same thermal velocity, we can obtain $v_{te1} = v_{te2} = (T_\perp/m_e)^{1/2}$ from Eq. (10). For simplicity, we assume that the Maxwellian component δ decreases with increasing v_d as $\delta \simeq \exp[-(m_e v_d^2/T_\perp)]$, which is in good agreement with the numerical results. Finally, we get the solution of the total current as

$$J = \sigma_0 E\left[1-\exp\left(-\frac{t}{\tau_r}\right)\right]\exp\left(-\frac{T_\parallel - T_\perp}{T_\perp}\right)\left(\frac{T_\perp}{T_0}\right)^{3/2} + n_e e\left[1-\exp\left(-\frac{T_\parallel - T_\perp}{T_\perp}\right)\right]\left(\frac{T_\parallel - T_\perp}{m_e}\right)^{1/2}. \tag{12}$$

According to the Ohmic heating [4] and the relaxation of the anisotropic temperature [12,14], the parallel temperature $T_\parallel$ and the perpendicular $T_\perp$ can be approximately updated as

$$\frac{dT_\parallel}{dt} = 2JE - 2\nu_{ei}(v_{\rm eff})(T_\parallel - T_\perp), \tag{13}$$

$$\frac{dT_\perp}{dt} = \nu_{ei}(v_{\rm eff})(T_\parallel - T_\perp), \tag{14}$$

where $\nu_{ei}(v_{\rm eff}) = Z_i \Gamma^{e|e}/v_{\rm eff}^3$ is the effective e-i collision frequency, and $v_{\rm eff} = \sqrt{(T_\parallel^2 + 2T_\perp^2)/m_e}$. In the weak-field limit, it satisfies $T_\parallel = T_\perp = T$, and then the Eqs. (12)–(14) will degenerate to Spitzer's model

$$J = \sigma_0 E\left[1-\exp\left(-\frac{t}{\tau_r}\right)\right]\left(\frac{T}{T_0}\right)^{3/2}, \tag{15}$$

$$\frac{dT}{dt} = \frac{2}{3} JE. \tag{16}$$

From hydrodynamiclike Eqs. (12)–(14) with the initial plasma parameters, one can follow the time evolution of the plasma current under a dc electric field without the detailed knowledge of the EDF. Although this set of hydrodynamiclike equations is deduced in the moderate field, it works quite well and agrees with the Fokker-Planck numerical results in a wide range of field strengths while the validity of Spitzer's model is limited in the weak field as

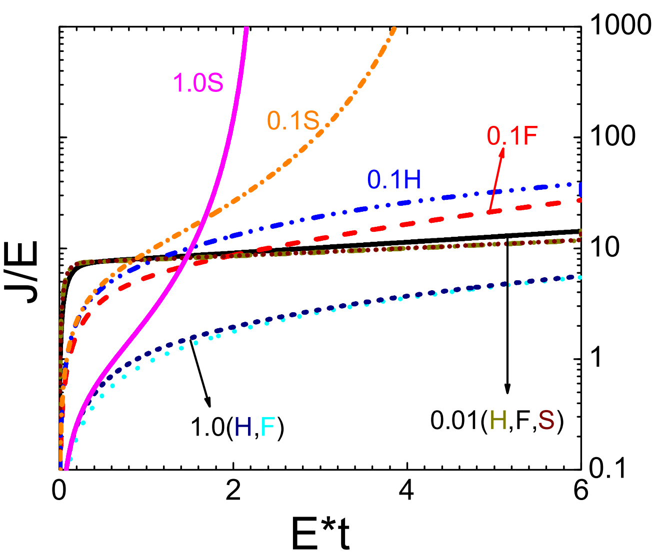

FIG. 3 (color online). Comparison of the currents calculated by our hydrodynamiclike Eqs. (12)–(14) (marked as 0.1 H) with those by Spitzer's (marked as 0.1 S) and those obtained from the Fokker-Planck code (marked as 0.1 F) for $Z_i = 1$ under different electric fields of strengths $0.01E_c$, $0.1E_c$, and $1.0E_c$. Variable Et is in units of $E_c\tau_{ei}$, and J/E is in units of $n_e e^2 \tau_{ei}/m_e$.

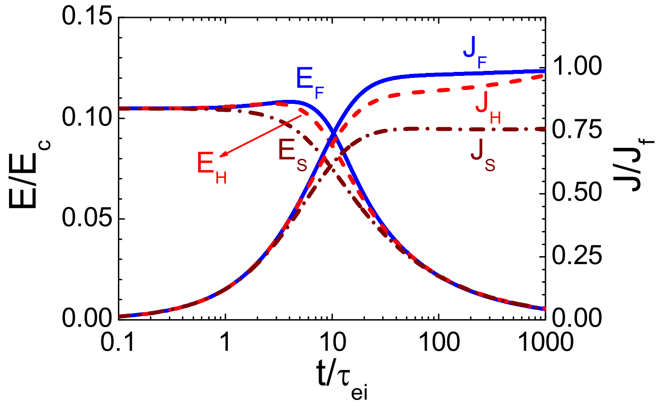

FIG. 4 (color online). The time evolution of the electric field estimated by Eq. (12) with $T_{\parallel}$ and $T_{\perp}$ being updated by Fokker-Planck code (marked as E_F), the electric field updated by hydrodynamiclike Eqs. (12)–(14) (marked as E_H), and the electric field updated by Spitzer's (marked as E_S), as well as the produced currents of these electric fields as a current of fast electrons $J_f = 3.5$ GA with a radius of 20 μm transports into a uniform plasma with initial temperature of 500 eV and density of 5 g cm^{-3}.

shown in Fig. 3. This means this set of hydrodynamiclike equations is free from the weak-field limit and more suitable for applications in hybrid-PIC simulations, and it may also extend plasma transport theory in models that do not employ a kinetic description of the electrons.

As an example, we simulate the generation of return current during the fast electrons transport in the fast ignition scheme. The simulation condition is that a current of fast electrons $J_f = 3.5$ GA with a spot radius of 20 μm is injected into a uniform initial DT plasma with a temperature of 500 eV and density of 10 g cm^{-3}, which is the low density region used in Ref. [9]. For the initial plasma temperature, an electric field of 3.21×10^{10} V/m, which is about $0.1048E_c$, is needed to produce a return current at the same level of the injection current. As shown in Fig. 3, Spitzer's model fails to describe the relation between this moderate electric field and the produced return current. Figure 4 shows the time evolution of the electric fields obtained from three different ways: the electric field estimated by Eq. (12) with $T_{\parallel}$ and $T_{\perp}$ being updated by the Fokker-Planck code, the one updated completely by our hydrodynamiclike Eqs. (12)–(14), and the one updated completely by Spitzer's Eqs. (15) and (16). The produced currents of these electric fields, all obtained from Fokker-Planck simulations, are also shown in Fig. 4. It is found that Spitzer's model underestimates the electric field since it usually overestimates the conductivity. Therefore the produced return current of this electric field is smaller than the injected current. While our hydrodynamiclike equations give a good estimation of the electric field, which can produce the return current compensating the beam current almost completely.

In conclusion, we have shown and explained the distinct behaviors of electrons under electric fields with different strengths. Taking into account the full *e-e* collisions in the Fokker-Planck simulation, the electrons can be approximately divided into two groups, with one satisfying a stationary Maxwellian distribution and another a drifting Maxwellian distribution in the moderate field. According to the EDFs, we derive the hydrodynamiclike Eqs. (12)–(14) to follow the time evolution of plasma current, which can be used as conveniently as Spitzer's model but without the weak-field limit. For fast electron transport in the fast ignition targets, it is found that the return current obtained with our hydrodynamiclike Eqs. (12)–(14) can compensate the beam current almost completely, whereas the one obtained with Spitzer's model cannot.

This work was done partially during S. M. W.'s stay at the STFC Rutherford Appleton Laboratory, U.K. This work was supported by the CAS-STFC collaboration program. Z. M. S. and J. Z. acknowledge the support from the National Nature Science Foundation of China (Grants No. 10425416, No. 10674175, No. 60621063), the National High-Tech ICF Committee in China, and National Basic Research Program of China (Grants No. 2007CB815101 and No. 2007CB815105).

*zmsheng@sjtu.edu.cn

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