Bound States in the Continuum in Photonics

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With examples of two parallel dielectric gratings and two arrays of thin parallel dielectric cylinders, it is shown that the interaction between trapped electromagnetic modes can lead to scattering resonances with practically zero width. Such resonances are the bound states in the radiation continuum first discovered in quantum systems by von Neumann and Wigner. Potential applications of such photonic systems include: large amplification of electromagnetic fields within photonic structures and, hence, enhancement of nonlinear phenomena, biosensing, as well as perfect filters and waveguides for a particular frequency, and impurity detection.

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Scattering of electromagnetic waves on periodic subwavelength structures has shown a number of interesting phenomena, such as enhanced light transmission through perforated metal films [1–4], total reflection from thin dielectric cylinder arrays [5], total light absorption [6], etc. The detailed mechanisms are still under debate [3,4,7,8]. However, since the observed phenomena manifest themselves as sharp features in the frequency dependence of the scattering matrix, it appears clear that the framework of the resonant scattering theory developed in quantum mechanics [9,10] can be used for their explanation [11,12]. Indeed, there is a close analogy between quantum mechanical and electromagnetic scattering [13], and the analytical properties of the scattering amplitude are expected to be similar in both theories. In quantum mechanics, the resonances reflect the transient trapping of the scattered particle and correspond to the poles of the scattering amplitude on the complex energy plane [13]. In electromagnetism, periodic subwavelength arrays have long-lived trapped electromagnetic field modes. These modes are responsible for the enhanced transmittance and/or reflectance in the very same way as quasistationary states in quantum systems are responsible for resonant peaks in the scattering cross section [11,12].

A long time ago, von Neumann and Wigner showed that there are quantum systems which have bound states above the continuum threshold [14]. Later, physical systems with this remarkable property were found [15]. Finally, it was proved that bound states in the continuum can occur due to the direct and via-the-continuum interaction between quasistationary states [16–18] and can be viewed as resonances with practically infinite lifetimes.

In this Letter, it is demonstrated that a similar phenomenon occurs in Maxwell's theory. An electromagnetic wave of a specific frequency can be trapped forever by a structure that is neither a metal cavity nor a defect in a photonic crystal. As examples, we use simple photonic systems: two arrays of identical dielectric gratings and two arrays of parallel dielectric cylinders as sketched in Figs. 1 and 2, respectively. Each single array has a trapped mode which appears as a scattering resonance. In the double-array structure, the interaction between trapped modes localized on each array leads to the formation of two "molecular" resonances. Their properties depend on a continuous parameter (here, the distance h between the arrays). For a

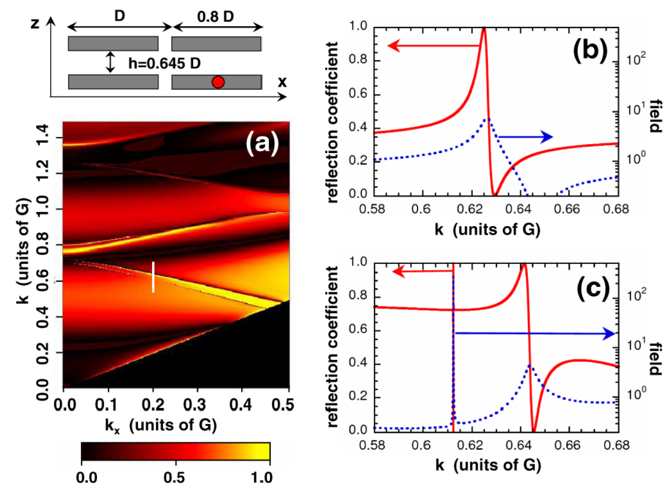

FIG. 1 (color online). Scattering properties of the double grating structure in the vacuum sketched in the upper left panel. D is the period of the grating, and the thickness of the grating is $0.2D$. The grating material (gray) has a dielectric constant $\varepsilon = 4$. (a) shows the specular reflection coefficient $\Re$ as a function of the wave vector k and its component k_x. (b) and (c) show $\Re$ (solid red line) and the electric field (dashed blue line) for the (b) single and (c) double grating structures. The electric field is calculated at the position indicated by the red dot on the sketch of the grating. The amplitude of the incident field is 1. Results are presented as a function of k for the fixed value of $k_x = 0.2G$. Values of k range over a neighborhood of the intersection of the white bar with the maximum of $\Re$ as indicated in (a).

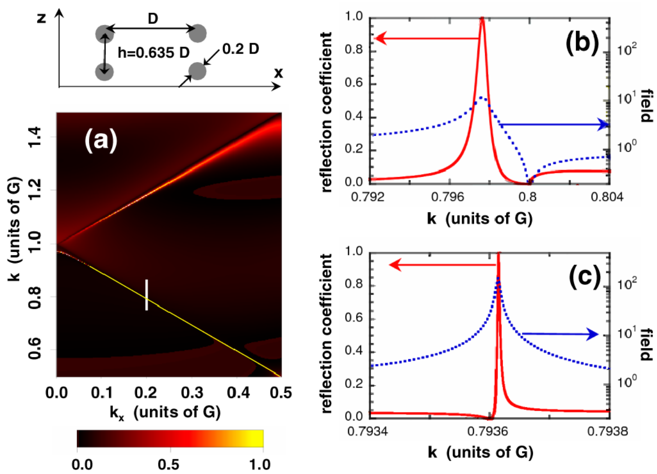

FIG. 2 (color online). Scattering properties of the periodic double array of dielectric cylinders in the vacuum sketched in the upper left panel. The dielectric constant of the cylinders $\varepsilon = 2$. For the detailed description, see the caption of Fig. 1. The electric field [(b),(c)] is calculated at the center of the cylinder for the incident field amplitude of 1 and fixed value of $k_x = 0.2G$.

particular value of the parameter, the width of one of the resonances vanishes. This feature is identical to that of the bound states in the continuum in quantum mechanics. The possibility of changing the resonance width offers the way to control the amplification of the electromagnetic fields in the structure.

The geometry of the studied structures is sketched in Figs. 1 and 2. The coordinate system is set so that the structure is invariant in the y direction and periodic in the x direction (D is the period), and the z axis is normal to the structure. The electric field is along the y axis (TM polarization). The transmission or reflection properties are calculated within the multiple scattering approach [5,19,20] for the cylinder array and via the coupled mode solution of the scattering problem [21] for the dielectric grating.

We first discuss the double dielectric grating. In Fig. 1(a), the specular reflection coefficient $\Re$ is presented as a function of the total wave vector k of the incident radiation and its component along the x axis k_x. The wave vector is measured in units of the primitive reciprocal lattice vector $G = 2\pi/D$. The sharp resonant features clearly visible in Fig. 1(a) correspond to the existence of trapped electromagnetic modes (quasistationary states) in the structure. The dispersion of the trapped modes with k_x can be inferred from the dependence of $\Re$ on k and k_x. A dielectric slab possesses stationary wave-guided modes with frequencies $\omega(k_x)$ below the continuum threshold, i.e., $\omega(k_x) < ck$. When the slab is grated, the wave-guided modes become coupled with the radiative continuum and turn into resonances because of the possibility of the reciprocal lattice vector exchange. In the symmetric case of two equivalent gratings, the trapped modes localized on each grating interact, forming symmetric and antisymmetric quasistationary states. The degeneracy of the resonances is lifted, and two distinct dispersion curves emerge split around the dispersion curve of the single grating. The doubling is clearly seen when comparing Figs. 1(b) and 1(c) (see the discussion in the next paragraph). Below the first diffraction threshold $k < G - k_x$, the trapped modes decay only into the zero diffraction channel (no change in k_x) and thus have the smallest width. In accordance with general scattering theory, $\Re$ reaches 1 in this case [11,12]. When k is such that k_x may change by ℓG, the resonance width increases as more decay channels become available. Here ℓ is an integer denoting the diffraction order (diffraction channel).

For $k_x \approx 0.2G$, the lowest frequency resonance narrows to the point that it cannot be resolved in Fig. 1(a). The stabilization of the trapped mode is further illustrated in Figs. 1(b) and 1(c). The resonance located at $k \approx 0.625G$ for the single grating is split into one extremely narrow and one broad resonance for the double grating. The narrow resonance has such a small width that it cannot be resolved in the scale of Fig. 1(c) and appears nearly as a vertical bar. Note that, because of the interference between nonresonant and resonant scattering contributions, the reflection coefficient has the Fano profile [22]. For each resonance, it changes from 1 to 0 on the k scale given by the width as seen for the broader resonance in Fig. 1(c). The narrow resonance has the inverted profile as compared to the broad one. The long-lived quasistationary state corresponds to a large amplification of the resonance field at the structure.

Results for the double array of thin dielectric cylinders are presented in Fig. 2. In contrast to the dielectric grating case, this structure is basically transparent. The scattering is dominated by the resonant contributions, and the reflection coefficient is close to zero except at the resonances located just below the diffraction thresholds [5,20]. The doubling of the resonances is not observed here because the higher frequency state is shifted above the first diffraction threshold and strongly broadened (see Fig. 4). The comparison of the scattering properties of the single [Fig. 2(b)] and double [Fig. 2(c)] cylinder arrays leads to observations similar to those reported for the dielectric grating: (i) The resonance located below the first diffraction threshold is associated with the 100% reflection; (ii) the lowest frequency (symmetric) state of the double structure has a very small width, much smaller than the width of the resonance of the single array; (iii) the long-lived trapped mode is associated with a strong field enhancement.

To qualitatively understand the physics behind the stabilization phenomenon, it is convenient to take the example of thin dielectric cylinders where the narrow resonance contribution dominates the scattering. For a large distance between the two arrays h, the interaction through evanescent fields in closed diffraction channels ($|\ell G - k_x| > k$) can be neglected. If only the zero diffraction channel is open, the transmission amplitude t_2 can be obtained from a Fabry-Perot–type approach:

$$\mathfrak{t}_2 = \frac{\mathfrak{t}_1^2 \exp(ik_z h)}{1 - \{\mathfrak{r}_1 \exp(ik_z h)\}^2}, \tag{1}$$

where $\mathfrak{t}_1$ ($\mathfrak{r}_1$) is the transmission (reflection) amplitude for a single array and k_z is the wave vector component perpendicular to the array. Since the reflection coefficient of the single array reaches 1 at the resonance, $\mathfrak{t}_1$ and $\mathfrak{r}_1$ can be approximated by the Breit-Wigner form [23]:

$$\mathfrak{t}_1 = 1 - \frac{i\Gamma/2}{\omega - \omega_0 + i\Gamma/2}, \qquad \mathfrak{r}_1 = 1 - \mathfrak{t}_1, \tag{2}$$

where $\omega_0 \equiv \omega_0(k_x)$ and $\Gamma \equiv \Gamma(k_x)$ are, respectively, the frequency and width of the resonance. From Eqs. (1) and (2) one obtains the frequencies $\omega_\pm$ and widths $\Gamma_\pm$ of the symmetric (+) and antisymmetric (−) states appearing as the poles of $\mathfrak{t}_2$:

$$\omega_\pm = \omega_0 \pm \Gamma \sin(k_z h)/2, \tag{3}$$

$$\Gamma_\pm = \Gamma[1 \pm \cos(k_z h)], \tag{4}$$

where k_z obeys the energy conservation condition: $\omega_\pm = c\sqrt{k_z^2 + k_x^2}$. The resonances are split because of the interaction through the propagating fields.

Whenever $k_z h \approx n\pi$, with n being an integer, the transmission coefficient as a function of k shows a rapid change from 1 to zero associated with a narrow resonance close to $k = \omega_0/c$. When $k_z h = n\pi$ holds exactly, (i) the symmetric or antisymmetric state has the vanishing width, and (ii) both ω_+ and ω_- are equal to ω_0 (the states are degenerate) and the transmission drops to zero. Since at ω_0 the single array is 100% reflective, the situation is analogous to the resonator made of perfectly reflective mirrors where the modes trapped between two arrays are decoupled from the continuum.

For small h, the interaction through evanescent fields in closed channels permanently lifts the degeneracy of the states. By using the formalism of [21] for one open and one closed channel, we obtain:

$$\omega_\pm = \omega_0 \pm \Gamma \sin(k_z h)/2 \mp \Gamma k_z \exp(-\alpha h)/2\alpha, \tag{5}$$

$$\Gamma_\pm = \Gamma[1 \pm \cos(k_z h)], \tag{6}$$

where $\omega_\pm = c\sqrt{k_z^2 + k_x^2} = c\sqrt{(k_x - G)^2 - \alpha^2}$. Whenever $k_z h = n\pi$, the symmetric or antisymmetric state has zero width, similar to the large distance case. The other resonance acquires the double width.

The calculated dependence of the frequencies and widths of the resonances on the distance h is presented in Fig. 3 (Fig. 4) for the case of dielectric gratings (cylinders). The splitting between symmetric and antisymmetric resonances with decreasing h is clearly visible. In the case of dielectric cylinders, for $h \sim 2.75D$ the antisymmetric state is shifted above the first diffraction threshold $k = G - k_x$.

Clearly, for well-defined values of h, the width of the symmetric (antisymmetric) mode vanishes while the other

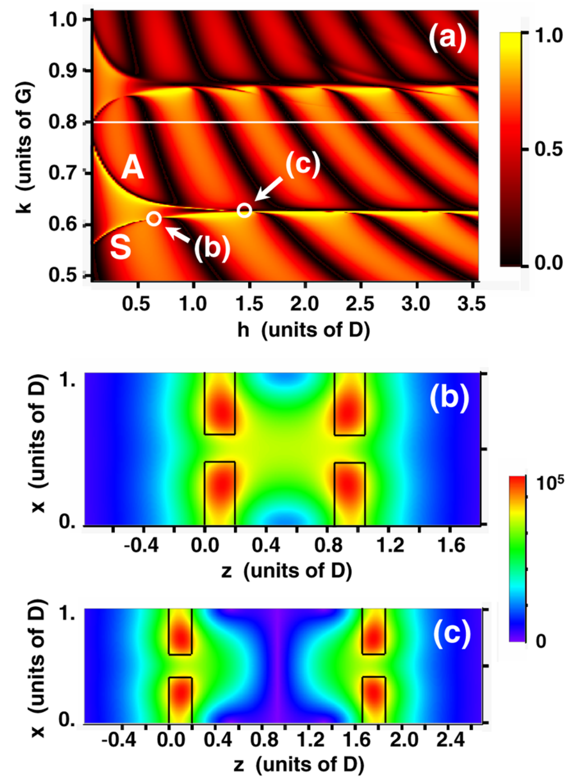

FIG. 3 (color online). Double grating structure. (a) shows the specular reflection coefficient as a function of the wave vector k of the incident radiation and the distance h between the gratings for the fixed value of $k_x = 0.2G$. S (A) stands for the symmetric (antisymmetric) mode. The white horizontal line indicates the opening of the first diffraction channel. (b) and (c) show the absolute value of the electric field for the (b) symmetric and (c) antisymmetric trapped modes. Results are presented as functions of the x (z) coordinate parallel (perpendicular) to the structure. The amplitude of the incident field is 1. The fields are obtained for values of (k, h) indicated by white circles in (a). The grating position is indicated with black lines.

mode broadens. The mode stabilization occurs when the resonance frequency of the double layer structure closely matches the frequency of the standing wave in the structure [black depletions in Figs. 3(a) and 4(a)]. The electric field of the trapped mode grows with decreasing resonance width. Figures 3 and 4 show the examples where the 10^5-fold field enhancement was obtained at the resonance. Eventually, the electric field diverges when the resonance turns into a bound state. At the same time, this state is decoupled from the continuum and cannot be populated by the incoming light. It is noteworthy that the bound state corresponds to the mode propagating along the structure without radiation losses. Thus, such a basically transparent system as a double array of thin dielectric cylinders can serve as a perfect waveguide.

How general is this phenomenon? A typical subwavelength periodic array has a reflectance or transmittance resonance with a lifetime determined by the structure geometry and material. Suppose that there are two such

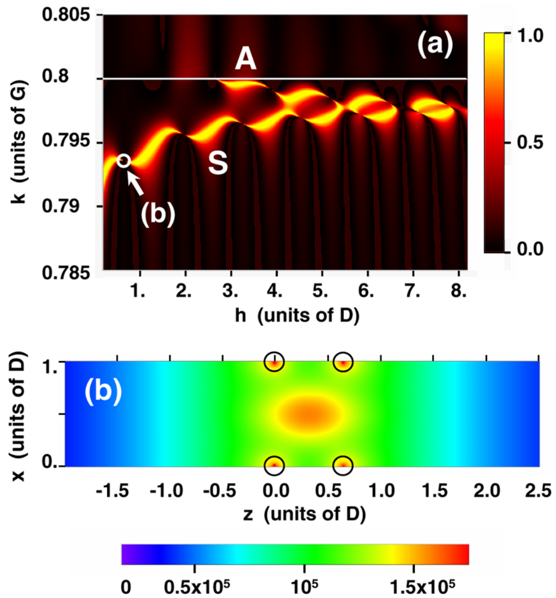

FIG. 4 (color online). Double dielectric cylinder array. (a) shows the specular reflection coefficient. (b) shows the absolute value of the electric field for the symmetric trapped mode. For further details, see the caption of Fig. 3.

identical structures. The quasistationary states of the joint system result from the interaction between the degenerate resonances localized on each structure. The interaction depends continuously upon the distance between the arrays. If one demands that each resonance decays through only one open channel (e.g., the zero-order diffraction), then the resulting system is identical to the quantum system considered in Refs. [16,17]. Given the similarity of Maxwell and Schrödinger's stationary equations, the Feshbach projection formalism [9] can be used to prove that, for a given value of the parameter, one resonance of the joint structure has the vanishing width. It becomes a bound state in the continuum. It is noteworthy that the above phenomenon should be at the origin of the recently reported unusually strong optical interactions between the scatterers in waveguides [24]. It should also occur for the plasmonic gratings [25].

In summary, it has been shown that two interacting resonances in subwavelength periodic arrays may produce a scattering resonance with vanishing width. It represents a bound state in the radiation continuum, where the electromagnetic field is trapped by the structure for infinite time. Photonic nanostructures with bound states in the radiation continuum may have a variety of potential applications. Here are a few. First, controlling the width of the scattering resonance or the lifetime of the corresponding trapped mode offers a possibility of a strong enhancement of electromagnetic fields within subwavelength arrays. Therefore, such a photonic structure can be used for efficient biosensing as well as to study various nonlinear phenomena. Second, a structure with a bound state in the radiation continuum may serve as a perfect filter or waveguide for the resonant frequency. Finally, it is worth noting that a nanophotonic structure tuned to have a bound state in the continuum can be used as a detector for impurities within the structure. The reason is that the impurity-induced scattering leads to the decay of the bound state.

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