Determination of the Parity of the Neutral Pion via Its Four-Electron Decay

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We present a new determination of the parity of the neutral pion via the double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$. Our sample, which consists of 30 511 candidate decays, was collected from $K_L \rightarrow \pi^0\pi^0\pi^0$ decays in flight at the KTeV-E799 experiment at Fermi National Accelerator Laboratory. We confirm the negative π^0 parity and place a limit on scalar contributions to the $\pi^0 \rightarrow e^+e^-e^+e^-$ decay amplitude of less than 3.3% assuming *CPT* conservation. The $\pi^0\gamma^*\gamma^*$ form factor is well described by a momentum-dependent model with a slope parameter fit to the final state phase-space distribution. Additionally, we have measured the branching ratio of this mode to be $B(\pi^0 \rightarrow e^+e^-e^+e^-) = (3.26 \pm 0.18) \times 10^{-5}$.

The parity of the neutral pion has been determined indirectly by studying negative pions captured on deuterium [1,2]. The observed reactions imply that the π^- is a pseudoscalar and that the parities of the π^- and the π^0 are the same. It has long been known that the decay $\pi^0 \rightarrow \gamma\gamma$ in principle offers a direct means of determining the π^0 parity through the polarizations of the photons [3,4]. Given that there are no available methods for measuring the polarization of a high-energy photon, this measurement has never been performed. However, it was soon noted that the double Dalitz decay $\pi^0 \rightarrow e^+e^-e^+e^-$, which proceeds through an intermediate state with two virtual photons (see Fig. 1), is sensitive to the parity of the pion since the plane of a Dalitz pair is correlated with the polarization of the virtual photon [5,6]. This process was studied in a 1962 hydrogen bubble chamber experiment using stopping negative pion capture ($\pi^- p \rightarrow n\pi^0$). That group observed 206 $\pi^0 \rightarrow e^+e^-e^+e^-$ events and reported that the observed distribution of the e^+e^- planes was consistent with a pseudoscalar pion and disfavored a scalar pion at the level of 3.6 standard deviations [7]; this experi-

ment also produced a measurement of the branching ratio of this decay, which remains the most precise result to date.

Using a sample of more than 30 000 $\pi^0 \rightarrow e^+e^-e^+e^-$ decays, we report new precise measurements of the properties of this decay. Our modeling of the decay includes for the first time a proper treatment of the exchange contribu-
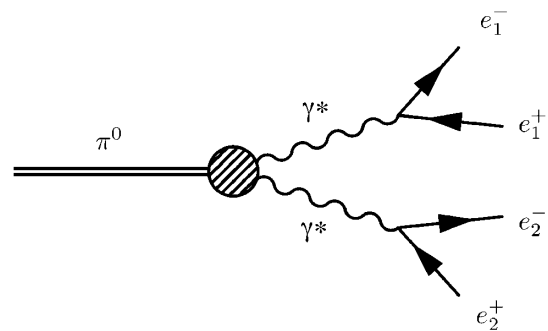

FIG. 1. Lowest order Feynman diagram for $\pi^0 \rightarrow e^+e^-e^+e^-$. The direct contribution is shown; a second diagram exists with e_1^+ and e_2^+ exchanged.

tion to the matrix element and consideration of full $\mathcal{O}(\alpha^2)$ radiative corrections. With these advances, we have tested for a scalar contribution in the $\pi^0 \gamma^* \gamma^*$ coupling with a sensitivity of a few percent. We have also measured for the first time the momentum dependence of the form factor in this decay mode. In addition, we present a new measurement of the $\pi^0 \to e^+ e^- e^+ e^-$ branching ratio, taking into account radiative effects.

The most general interaction Lagrangian for the $\pi^0 \to \gamma^* \gamma^*$ transition can be written as [8]

$$\mathcal{L} \propto C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \Phi, \qquad (1)$$

where $F^{\mu\nu}$ and $F^{\rho\sigma}$ are the photon fields, Φ is the pion field, and the coupling has the form

$$C_{\mu\nu\rho\sigma} \propto f(x_1, x_2)[\cos\zeta \, \epsilon_{\mu\nu\rho\sigma}$$
$$+ \sin\zeta e^{i\delta}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})]. \qquad (2)$$

The first term in $C_{\mu\nu\rho\sigma}$ is the expected pseudoscalar coupling and the second term introduces a scalar coupling with a mixing angle ζ and a phase difference δ. Nuclear parity violation would introduce a nonzero ζ, while *CPT* violation would cause the phase δ to be nonzero. We assume the standard parity-conserving form for the $\gamma^* \to e^+ e^-$ conversion.

The form factor $f(x_1, x_2)$ is expressed in terms of the momentum transfer of each of the virtual photons, or equivalently the invariant masses of the two Dalitz pairs: $x_1 \equiv (m_{e_1^+ e_1^-}/M_{\pi^0})^2$; $x_2 \equiv (m_{e_2^+ e_2^-}/M_{\pi^0})^2$. In calculating the phase-space variables for an individual event, there is an intrinsic ambiguity in assigning each electron to a positron to form a Dalitz pair. Our analysis uses a matrix element model that includes the exchange diagrams and therefore avoids the need to enforce a pairing choice.

The $\pi^0 \gamma^* \gamma^*$ form factor has been studied previously in the decay $\pi^0 \to e^+ e^- \gamma$ [9–11], where the quantity of interest has been the slope parameter a of the first-order Taylor expansion $f(x, 0) = 1 + ax$, with $x \equiv m_{e^+ e^-}^2 / M_{\pi^0}^2$. Here we use a form factor parametrization based on the model of D'Ambrosio, Isidori, and Portolés (DIP) [12], but with an additional constraint that ensures the coupling vanishes at large momenta [13]. In terms of the remaining free parameters, the form factor is

$$f_{\text{DIP}}(x_1, x_2; \alpha) = \frac{1 - \mu(1 + \alpha)(x_1 + x_2)}{(1 - \mu x_1)(1 - \mu x_2)}, \qquad (3)$$

where $\mu = M_{\pi^0}^2 / M_\rho^2 \approx 0.032$. In the limit of small x, this coincides with the Taylor expansion provided $a = -\mu\alpha$.

The parity properties of the decay can be extracted from the angle ϕ between the planes of the two Dalitz pairs in Fig. 1, where pair 1 is defined as having the smaller invariant mass. The distribution of this angle from the dominant direct contribution has the form $d\Gamma/d\phi \sim 1 - A\cos(2\phi) + B\sin(2\phi)$, where $A \approx 0.2\cos(2\zeta)$ and $B \approx 0.2\sin(2\zeta)\cos\delta$. A pure pseudoscalar coupling, therefore, would produce a negative $\cos(2\phi)$ dependence.

The π^0 decays used in this analysis are the result of fully reconstructed $K_L \to \pi^0 \pi^0 \pi^0$ decays in flight collected by the KTeV-E799 experiment at Fermilab. The E799-II experiment and the KTeV detector are described elsewhere [14,15]. This analysis relies on two core systems of the KTeV detector: a drift chamber-based charged particle spectrometer and a cesium iodide (CsI) electromagnetic calorimeter. Electrons are identified as charged particles whose entire energy is deposited in the CsI, while photons are reconstructed from electromagnetic showers in the CsI with no associated charged tracks.

The signal mode, denoted by $K_L \to \pi^0 \pi^0 \pi^0_{DD}$ where π^0_{DD} refers to $\pi^0 \to e^+ e^- e^+ e^-$, has a signature of four charged particles identified as electrons and with a combined invariant mass consistent with the π^0 mass, plus four photons that are compatible with two additional π^0's. Furthermore, the eight-particle state has an invariant mass consistent with the K_L and total momentum vector in the direction of the kaon line of flight.

The branching ratio measurement, which we describe here first, makes use of a normalization mode in which two pions decay via $\pi^0 \to e^+ e^- \gamma$ and the third $\pi^0 \to \gamma\gamma$. This "double single-Dalitz-decay" mode, denoted $K_L \to \pi^0 \pi^0_D \pi^0_D$ where π^0_D refers to $\pi^0 \to e^+ e^- \gamma$, has the same final state particles as the signal mode and is again identified by finding the proper combinations of particles to make three pions with a total momentum consistent with the kaon. The similarity of these modes allows cancellation of most detector-related systematic effects in the branching ratio measurement, but also allows each mode to be a background to the other.

Radiative corrections complicate the definition of the Dalitz decays in general. We define the signal mode $\pi^0 \to e^+ e^- e^+ e^-$ to be inclusive of radiative final states where the squared ratio of the invariant mass of the four electrons to the neutral pion mass $x_{4e} \equiv (M_{4e}/M_{\pi^0})^2$ is greater than 0.9, while events with $x_{4e} < 0.9$ (approximately 6% of the total rate) are treated as $\pi^0 \to e^+ e^- e^+ e^- \gamma$. For normalization, the decay $\pi^0 \to e^+ e^- \gamma$ is understood to include all radiative final states, for consistency with previous measurements of this decay [16]. Radiative corrections in this analysis are taken from an analytic calculation to order $\mathcal{O}(\alpha^2)$ [8].

Other final states of the $K_L \to \pi^0 \pi^0 \pi^0$ decay can become backgrounds to either the signal or normalization mode if one or more photons convert to an $e^+ e^-$ pair in the detector material: $K_L \to \pi^0 \pi^0 \pi^0_D$ where one of the five photons converts, or $K_L \to \pi^0 \pi^0 \pi^0 \to 6\gamma$ where two photons convert. These modes again have the same final state as the signal but can be distinguished statistically since the externally produced pairs tend to have smaller invariant masses than those from internal conversions. The most significant of these backgrounds is $K_L \to \pi^0 \pi^0 \pi^0_D$ with one external conversion in material. The photon must convert upstream of the first drift chamber for the resulting tracks to be reconstructed. The material in this region sums

to 2.8×10^{-3} radiation lengths. With five photons available, the probability of one converting is 1.08%, close to the single-Dalitz branching ratio. The distinguishing characteristic of these events is the small value of the $e^{+}e^{-}$ invariant mass, or similarly, the small value of the opening angle of the pair. Requiring a track separation at the first drift chamber of greater than 2 mm removes 99.74% of the remaining simulated background while preserving 74.3% of signal and 72.7% of normalization events.

The final selection criterion separates $K_{L} \rightarrow \pi^{0}\pi^{0}\pi^{0}_{DD}$ from $K_{L} \rightarrow \pi^{0}\pi^{0}_{D}\pi^{0}_{D}$ events. This is accomplished by a χ^{2} formed of the three reconstructed π^{0} masses. This serves to identify the best pairing of particles for a given decay hypothesis, as well as to select the more likely hypothesis of the two. The event is tagged as the mode with the smaller χ^{2}, which is further required to be less than 12 (with 3 degrees of freedom). This technique correctly identifies more than 99.5% of events (Fig. 2).

The final event sample contains 30 511 signal candidates with 0.6% residual background and 141 251 normalization mode candidates with 0.5% background (determined from the Monte Carlo simulation). The background in the signal sample is dominated by mistagged events from the normalization mode.

The branching ratio is measured from the ratio of reconstructed signal mode events to normalization mode events. This ratio must be corrected by the ratio of acceptances, which has been determined using a detailed Monte Carlo simulation of the beam distribution and detector response. The resulting double ratio is directly related to the branching ratio B_{eeee} of the $\pi^{0} \rightarrow e^{+}e^{-}e^{+}e^{-}$ mode normalized to the square of the branching ratio $B_{ee\gamma}$ of the $\pi^{0} \rightarrow e^{+}e^{-}\gamma$ mode:

$$\frac{B_{eeee} \cdot B_{\gamma\gamma}}{B_{ee\gamma}^{2}} = \frac{N(K_{L} \rightarrow \pi^{0}\pi^{0}\pi^{0}_{DD})}{N(K_{L} \rightarrow \pi^{0}\pi^{0}_{D}\pi^{0}_{D})}$$
$$\cdot \frac{\epsilon(K_{L} \rightarrow \pi^{0}\pi^{0}_{D}\pi^{0}_{D})}{\epsilon(K_{L} \rightarrow \pi^{0}\pi^{0}\pi^{0}_{DD})}, \quad (4)$$

where N is the number of events and ϵ is the combined geometric acceptance and detection efficiency for a given mode.

The statistical error on the ratio in Eq. (4) is 0.62%. Systematic errors on the efficiencies were determined through data studies as well as variations in the parameters of the Monte Carlo simulation. Because the final state particles in the signal and normalization mode are the same, detector-related quantities substantially cancel in the ratio, which is generally insensitive to the details of the simulation. The dominant systematic errors came from variation of the analysis cuts (0.21%) and Monte Carlo simulation statistics (0.25%). Other systematic errors were from uncertainties in the amount of material in the spectrometer (0.15%), uncertainty in the background levels in the two samples (0.15%), modeling of the drift chamber resolutions (0.11%), and radiative corrections (0.04%). The
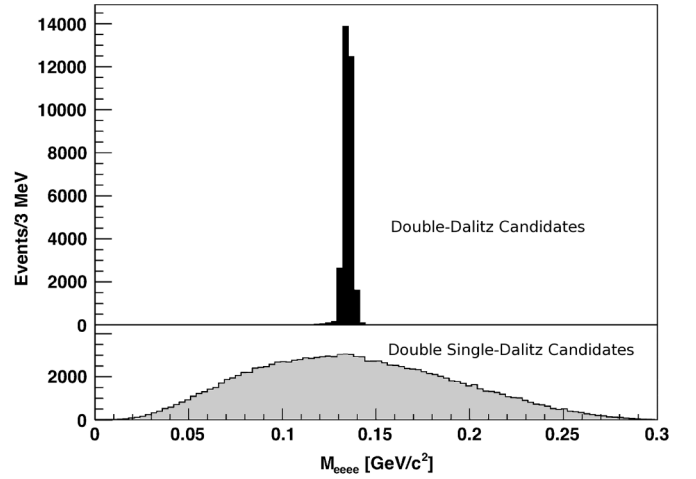

FIG. 2. Invariant $e^{+}e^{-}e^{+}e^{-}$ mass for data events identified by the preferred pairing as $K_{L} \rightarrow \pi^{0}\pi^{0}\pi^{0}_{DD}$ (upper panel) and $K_{L} \rightarrow \pi^{0}\pi^{0}_{D}\pi^{0}_{D}$ (lower panel).

total systematic error on the relative branching ratio is 0.41%.

The final result for the ratio of decay rates is

$$\frac{B_{eeee}^{x>0.9} \cdot B_{\gamma\gamma}}{B_{ee\gamma}^{2}} = 0.2245 \pm 0.0014(\text{stat}) \pm 0.0009(\text{syst}).$$
$$(5)$$

The $\pi^{0} \rightarrow e^{+}e^{-}e^{+}e^{-}$ branching ratio can be calculated from the double ratio using the known values $B_{\gamma\gamma} = 0.9880 \pm 0.0003$ and $B_{ee\gamma} = (1.198 \pm 0.032) \times 10^{-2}$ [17]. This yields $B_{eeee}^{x>0.9} = (3.26 \pm 0.18) \times 10^{-5}$, where the error is dominated by the uncertainty in the $\pi^{0} \rightarrow e^{+}e^{-}\gamma$ branching ratio. Using our radiative corrections model [8] to extrapolate to all radiative final states, we find

$$\frac{B_{eeee(\gamma)} \cdot B_{\gamma\gamma}}{B_{ee\gamma}^{2}} = 0.2383 \pm 0.0015(\text{stat}) \pm 0.0010(\text{syst}),$$
$$(6)$$

and $B_{eeee(\gamma)} = (3.46 \pm 0.19) \times 10^{-5}$. Our branching ratio result is in agreement with previous measurements [7].

The parameters of the $\pi^{0}\gamma^{*}\gamma^{*}$ coupling are found by maximizing an unbinned likelihood function composed of the differential decay rate in terms of ten phase-space variables. The first five are $(x_{1}, x_{2}, y_{1}, y_{2}, \phi)$, where x_{1}, x_{2}, and ϕ are described above and the remaining variables y_{1} and y_{2} describe the energy asymmetry between the electrons in each Dalitz pair in the π^{0} center of mass [8]. The remaining five are the same variables but calculated with the opposite choice of $e^{+}e^{-}$ pairings. The likelihood is calculated from the full matrix element including the exchange diagrams and $\mathcal{O}(\alpha^{2})$ radiative corrections.

The fit yields the DIP α parameter and the (complex) ratio of the scalar to the pseudoscalar coupling. For reasons of fit performance, the parity properties are fit to the equivalent parameters κ and η, where $\kappa + i\eta \equiv \tan\zeta e^{i\delta}$.
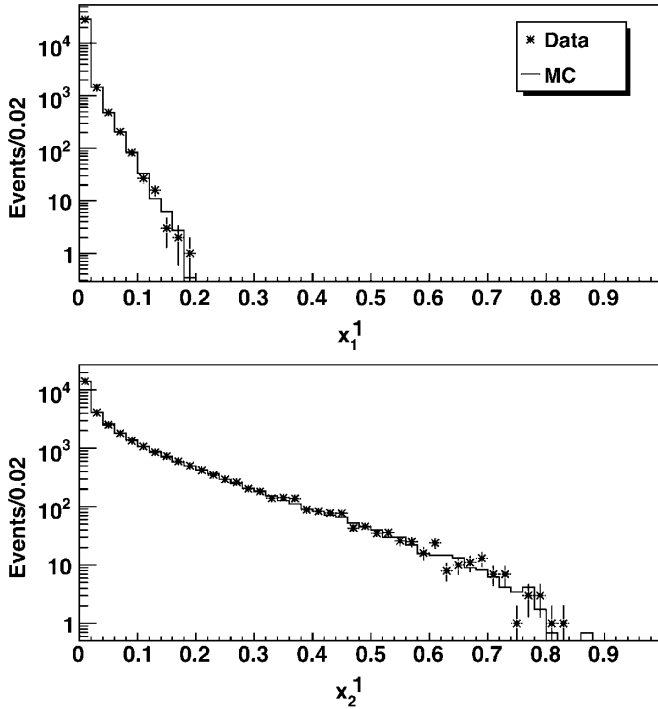
FIG. 3. Distribution of the kinematic variables x_1 and x_2 for signal event candidates (points) and signal Monte Carlo simulation with best fit form factor parameters (histogram).

The shape of the minimum of the likelihood function indicates that the three parameters α, κ, and η are uncorrelated. Acceptance-dependent effects are included as a normalization factor calculated from Monte Carlo simulations.

Systematic error sources on α and κ are similar to those for the branching ratio measurement. The dominant error is due to variation of cuts, resulting in a total systematic error of 0.9 and 0.011 on α and κ, respectively. For the η parameter, the primary uncertainty results from the resolution on the angle ϕ between the two lepton pairs, which produces an effective flattening of the angular distribution without inducing a phase shift. The fitter interprets this as a small scalar contribution with a phase difference of $90°$, and therefore a larger value of η, particularly for $\eta \approx 0$. This behavior was studied with Monte Carlo simulation, and a correction was calculated. The uncertainty on this correction results in a systematic error of 0.031.

The distributions of x_1 and x_2, overlaid with the Monte Carlo simulation, are shown in Fig. 3. The ϕ distribution is shown in Fig. 4. For plotting the data a unique pairing of the four electrons is chosen such that $x_1 < x_2$ and the product $x_1 x_2$ is minimized: this choice represents the dominant contribution to the matrix element. It is clear that the pseudoscalar coupling dominates, as expected, with no evidence for a scalar component. The distributions of all five phase-space variables agree well with the Monte Carlo simulation.

The final results for the three parameters are $\alpha = 1.3 \pm 1.0(\text{stat}) \pm 0.9(\text{syst})$, $\kappa = -0.011 \pm 0.009(\text{stat}) \pm$
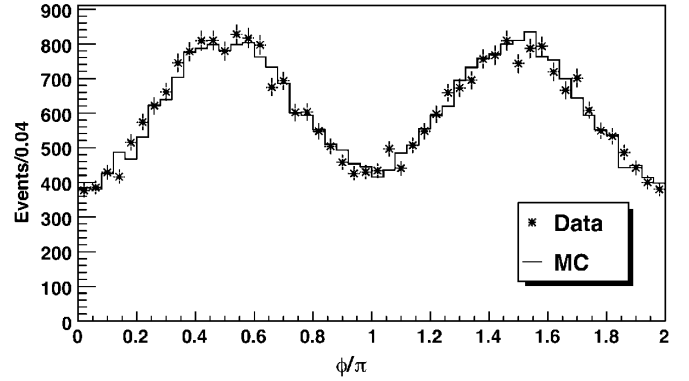

FIG. 4. Distribution of the angle ϕ, in units of π, between the planes of the two e^+e^- pairs. The histogram shows the Monte Carlo expectation for negative parity.

$0.011(\text{syst})$, and $\eta = 0.051 \pm 0.026(\text{stat}) \pm 0.031(\text{syst})$. The DIP α parameter is related to the standard slope parameter by $a = -0.032\alpha$, yielding $a = -0.040 \pm 0.040$. This result is in agreement with recent direct measurements.

The coupling parameters κ and η are transformed into limits on the pseudoscalar-scalar mixing angle ζ under two hypotheses. If CPT violation is allowed, then the limit is set by the uncertainties in η, resulting in $\zeta < 6.9°$ at the 90% confidence level. If instead CPT conservation is enforced, η must be zero, and the limit derives from the uncertainties on κ, resulting in $\zeta < 1.9°$, at the same confidence level. These limits on ζ limit the magnitude of the scalar component of the decay amplitude, relative to the pseudoscalar component, to less than 12.1% in the presence of CPT violation, and less than 3.3% if CPT is assumed conserved. The limits on scalar contributions apply to all π^0 decays with two-photon intermediate or final states.

This analysis confirms the negative parity of the neutral pion with much higher statistical significance than the previous result, and places tight limits on nonstandard scalar and CPT-violating contributions to the $\pi^0 \rightarrow e^+e^-e^+e^-$ decay. We have also measured the momentum-dependent form factor in this decay for the first time and made the first improvement in its branching ratio since 1962. This measurement is limited at present by the current large uncertainty in the branching ratio of the single-Dalitz decays used for normalization, but we expect that uncertainty to be reduced in the near future, at which point the present measurement can be recalculated using the more precise double ratio measurement.

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