Quantum Plasma Effects in the Classical Regime

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For quantum effects to be significant in plasmas it is often assumed that the temperature over density ratio must be small. In this paper we challenge this assumption by considering the contribution to the dynamics from the electron spin properties. As a starting point we consider a multicomponent plasma model, where electrons with spin-up and spin-down are regarded as different fluids. By studying the propagation of Alfvén wave solitons we demonstrate that quantum effects can survive in a relatively high-temperature plasma. The consequences of our results are discussed.

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Recently, several studies of quantum plasmas have appeared in the literature [1–15], where the Bohm–de Broglie potential and the Fermi pressure [1–7], spin properties [8–12,16–18], as well as certain quantum electrodynamical effects [13–15,19] are accounted for. The applications range from plasmonics [20] and quantum wells [21], to ultracold plasmas [22], and astrophysics [13,23]. Quantum plasma effects can also be seen in scattering experiments with solid density targets [24]. The usual regime where quantum effects are important involves dense low-temperature plasmas, where either the Fermi pressure is comparable to the thermal pressure or the thermal de Broglie wavelength times the plasma frequency is comparable to the thermal velocity. In recent studies of spin effects in plasmas [8–12], the condition for quantum effects to be important has been found to be somewhat different from the case of nonspin quantum plasmas [1–7], but also here high temperatures tend to make quantum effects small.

In the present Letter we study a weakly collisional high-temperature plasma. In particular, we focus on the case where the temperature over magnetic field ratio is sufficiently high to make spins randomly oriented at thermodynamic equilibrium. Within the one-fluid model, such a condition tends to make the macroscopic spin effects negligible [8–11]. However, here we study a two-electron fluid model, where the different electron populations are defined by their spin relative to the magnetic field. Evaluating this model for the particular case of Alfvén waves propagating along the external magnetic field, it is found that linearly the predictions agree with the one-fluid spin model. Nonlinearly, however, the induced density fluctuations of the spin-ponderomotive force is significantly different for the two-spin populations. As a consequence, the self-nonlinearity of the Alfvén waves gets a large contribution from the spin effects, even for a high-temperature plasma. In general, the conclusion is that spin effects cannot be neglected even in moderate-density high-temperature plasmas that normally are regarded as perfectly classical.

Neglecting spin-spin interactions, the equations of motion are [8–12]

$$\partial_t n_s + \nabla \cdot (n_s \mathbf{v}_s) = 0, \tag{1}$$

where n_s and $\mathbf{v}_s$ are the density and velocity of species s, $s = i, +, -$ enumerates the plasma particle species and $\pm$ denotes the two types of electrons,

$$m_s n_s(\partial_t + \mathbf{v}_s \cdot \nabla)\mathbf{v}_s = q_s n_s(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - c_s^2 \nabla n_s + \frac{2\mu_s n_s}{\hbar} S_s^a \nabla B_a + \frac{\hbar^2 n_s}{2m_s} \nabla\left(\frac{\nabla^2 n_s^{1/2}}{n_s^{1/2}}\right), \tag{2}$$

and

$$(\partial_t + \mathbf{v}_s \cdot \nabla)\mathbf{S}_s = -\frac{2\mu_s}{\hbar}\mathbf{B} \times \mathbf{S}_s, \tag{3}$$

where $\mathbf{S}_s$ is the spin of species s, q_s is the charge of species s, $p_s = p_s(T_s; n_s)$ is the pressure of species s and $c_s = (dp_s/dn_s)^{1/2}$ is the sound speed of species s (where we have assumed an isothermal plasma) containing also contributions due to the Fermi pressure, μ_s is the magnetic moment of species s, and $-\mu_\pm \equiv \mu_B = e\hbar/2m_e$ is the Bohr magneton, e is the magnitude of the electron charge, $\hbar$ is Planck's constant, m_e is the electron rest mass, and c is the speed of light. We note that Einstein's summation convention has been used in (2). In what follows we will neglect the quantum corrections to the ion momentum equation, since from Eq. (2) the quantum terms scales as m_s^{-1}.

For temporal variations of the magnetic field faster than the inverse electron cyclotron frequency, spin flips can be induced. Furthermore, particle collisions can also reverse the spin, although it can be seen from the Pauli Hamiltonian [8] that the probability for this typically is far less than unity. Thus, to make sure that spin reversal does not occur, we consider dynamics on a time scale shorter than the inverse spin transition frequency, but longer than the inverse cyclotron frequency. We also note here that the inverse spin transition frequency is at least as long as the inverse collision frequency. For this case, we can replace the spin evolution Eq. (3) with the relation $\mathbf{S}_\pm = \mp(\hbar/2)\hat{\mathbf{B}}$, where $\hat{\mathbf{B}}$ denotes a unit vector in the direction of $\mathbf{B}$, for electrons with spin-up and -down relative to the external magnetic field.

The coupling between the quantum plasma species is mediated by the electromagnetic field. The magnetizations due to the different spin sources are $\mathbf{M}_\pm = -2\mu_B n_\pm \mathbf{S}_\pm/\hbar = \pm\mu_B n_\pm \hat{\mathbf{B}}$. Ampère's law then takes the form

$$c^2\nabla\times\mathbf{B} = c^2\mu_0[\mathbf{j} + \nabla\times(\mathbf{M}_+ + \mathbf{M}_-)] + \partial_t\mathbf{E}, \quad (4)$$

where the free current is denoted $\mathbf{j}$. Moreover,

$$\varepsilon_0\nabla\cdot\mathbf{E} = q_i n_i - e(n_+ + n_-). \quad (5)$$

The system is closed by Faraday's law

$$\nabla\times\mathbf{E} = -\partial_t\mathbf{B}. \quad (6)$$

In previous works [8–12], electrons have been treated as a single population, with a single macroscopic velocity $\mathbf{v}$ and spin vector $\mathbf{S}$. As argued above, for fast dynamics in an approximately collisionless plasma, this is not appropriate, as the populations with positive and negative spins along the magnetic field will not change spins on the short time scales considered, and as seen in Eq. (2) the two populations are described by separate evolution equations. If we describe the plasmas by a single electron population, with a background spin distribution close to thermodynamic equilibrium, the spin effects are limited to a certain extent whenever $\mu_B B_0/k_B T_e \ll 1$. This is due to the thermodynamic Brillouin distribution for spins $\propto \tanh(\mu_B B_0/k_B T_e)$ describing the macroscopic net effect of the spin orientation. Thus within the single electron fluid model, we need low temperatures or very strong magnetic fields for spin effects to be important. By contrast, within the two-fluid electron model, spin effects may be of importance also in a weakly magnetized high-temperature plasma, as will be demonstrated below.

As an example, we consider the nonlinear response to a low-frequency electromagnetic Alfvén wave pulse propagating parallel to an external magnetic field. In linear ideal magnetohydrodynamic (MHD) theory, the magnetic field perturbation thus propagates along the external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ with the Alfvén velocity $c_A = (B_0^2/\mu_0\rho_0)^{1/2}$, where ρ_0 is the unperturbed mass density. Since linearly the Alfvén wave has no density perturbation, the quantum effects of the Bohm–de Broglie potential and the Fermi pressure do not change this result. In what follows, such quantum effects will be omitted. As found in [11], within a single-fluid spin model, the Alfvén velocity is decreased by a factor $1 + (\hbar\omega_{\rm pe}^2/2m_e c^2\omega_{ce}^{(0)}) \times \tanh(\mu_B B_0/k_B T_e)$ due to the spin, where $\omega_{\rm pe}$ is the electron plasma frequency, m_i the ion mass, $\omega_{ce}^{(0)} = e\mu_0 H_0/m_e$ is the electron cyclotron frequency due to the magnetic field $\mu_0 H_0 \equiv B_0 - \mu_0 M_0$, which is the field with external sources only; i.e., the contribution from the spins is excluded (here M_0 is the unperturbed magnetization due to the spin sources). For $\mu_B B_0 \ll k_B T_e$ the correction factor for the Alfvén velocity is close to unity, and the approximation $\mu_0 H_0 \approx B_0$ is a good one. This is the case that will be considered below, and the spin corrections to the linear Alfvén velocity will therefore be omitted in what follows. Furthermore, the envelope of a weakly modulated Alfvén wave will propagate with a group velocity $v_g \simeq c_A$ for frequencies $\omega \ll \omega_{ci}$ [25], where ω_{ci} is the ion cyclotron frequency.

The ponderomotive force of this envelope will drive low-frequency longitudinal perturbations (denoted by superscript "lf" in what follows) that are second order in an amplitude expansion, and to leading order depend on a single coordinate $\zeta = z - c_A t$. Thus, the dynamics is considered to be slow in a system comoving with the group velocity. Neglecting spin, for a slowly varying magnetic field perturbation of the form $\mathbf{B} = B(z,t)\exp[i(kz - \omega t)]\hat{\mathbf{e}} + \text{c.c.}$ (where $\hat{\mathbf{e}}$ is a unit vector perpendicular to $\hat{\mathbf{z}}$ and c.c. denotes complex conjugate), the low-frequency MHD momentum balance can be written

$$\rho_0\partial_t v_i^{\rm lf} = -\partial_z[|B|^2/\mu_0 + (k_B T_i + k_B T_e)n^{\rm lf}], \quad (7)$$

where index i denotes ions. For simplicity we assume a weak magnetic field, $B_0 \ll [\mu_0\rho_0(k_B T_i + k_B T_e)/m_i]^{1/2}$. Relating the low-frequency perturbations of the density and velocity using Eq. (1), the left-hand side of Eq. (7) is then found to be negligible, and the density depletion is given by

$$n^{\rm lf} = -\frac{|B|^2}{(k_B T_i + k_B T_e)\mu_0}. \quad (8)$$

Moreover, since we have charge neutrality within the MHD approximation, we have here neglected the index i on the density, since the total electron density will be the same for a proton-electron plasma. Next, we add the spin terms in our model. Again neglecting the ion inertia, the MHD low-frequency force balance Eq. (7) is replaced by

$$F_{p+} + F_{p-} = \partial_z[|B|^2/\mu_0 + (k_B T_i + k_B T_e)n^{\rm lf}], \quad (9)$$

where the ponderomotive force contributions $F_{p\pm}$ are low-frequency perturbations due to the terms $2\mu_s n_s S_s^a \nabla B_a/\hbar$ in the electron momentum equations. Including the spin vectors component in the direction of the perturbed magnetic field, the sum of these spin force contributions can be written

$$F_{p+} + F_{p-} \approx \frac{n_0}{2}\frac{\mu_B B_0}{k_B T_e}\frac{e\hbar}{m_e}\frac{\partial}{\partial z}\left(\frac{|B|^2}{B_0}\right), \quad (10)$$

where we have used that the unperturbed density difference $(n_{0+} - n_{0-})$ of the two-spin populations in thermodynamical equilibrium is proportional to $\tanh(\mu_B B_0/k_B T_e) \approx \mu_B B_0/k_B T_e$. Thus the net effect of the spin-ponderomotive force on the ion density as well as the *total* electron and ion density is very small in the regime $\mu_B B_0/k_B T_e \ll 1$, similarly as we would have for a single electron spin model, and, consequently, Eq. (9) is a valid approximation.

The interesting difference between different fluid models comes when we analyze the density perturbations of the two-electron populations separately. The low-frequency momentum balance equations for the different electron species are

$$en_{0\pm}\frac{\partial\Phi^{\rm lf}}{\partial z} - k_B T_e\frac{\partial n_\pm^{\rm lf}}{\partial z} - \left(1 \mp \frac{e\hbar\mu_0 n_{0\pm}}{m_e B_0}\right)\frac{\partial}{\partial z}\left(\frac{|B|^2}{\mu_0}\right) = 0, \tag{11}$$

where we have introduced the electrostatic low-frequency potential $\Phi^{\rm lf}$ (this potential does not appear in the overall momentum balance, as the plasma is quasineutral). By adding the $\pm$ parts of Eq. (11) and integrating we obtain

$$n_+^{\rm lf} + n_-^{\rm lf} = \frac{en_0\Phi^{\rm lf}}{k_B T_e} - \frac{1}{k_B T_e}\left(1 - \frac{\mu_B B_0}{k_B T_e}\frac{\mu_B B_0}{m_i c_A^2}\right)\frac{|B|^2}{\mu_0}, \tag{12}$$

to first order in the expansion parameter $\mu_B B_0/k_B T_e$. Because of the small factor $\mu_B B_0/k_B T_e$ in front of the last term, the spin effects on the total electron population are small, in agreement with Eq. (10), again justifying the omission of spin effects in Eq. (8). However, solving instead for the density difference between the two-electron populations we find

$$n_+^{\rm lf} - n_-^{\rm lf} = \frac{2}{k_B T_e}\frac{\mu_B B_0}{m_i c_A^2}\frac{|B|^2}{\mu_0}. \tag{13}$$

The importance of the density difference displayed in (13) appears when the nonlinear self-interaction of the Alfvén waves is studied. The momentum equation contains the term $(\mathbf{j}\times\mathbf{B}/n)^{\rm nl}$, where nl denotes the nonlinear part and $\mathbf{j}$ is determined from Eq. (5) with the displacement current neglected. Omitting the terms of higher order in the expansion parameter $\mu_B B_0/k_B T_e$ we find

$$\left[\frac{\mathbf{j}\times\mathbf{B}}{n}\right]^{\rm nl} = -\frac{(i\mathbf{k}\times\mathbf{B}_1)\times\mathbf{B}_0}{\mu_0 n_0}\frac{n^{\rm lf}}{n_0}\left[1 - \left(\frac{2\mu_B B_0}{m_i c_A^2}\right)^2\right], \tag{14}$$

where the last term represent the two-fluid electron spin contribution. This results in a corresponding spin-modification of the self-nonlinearity of the Alfvén waves. Including weakly dispersive effects due to the Hall current, parallel propagating Alfvén waves are described by the so called derivative nonlinear Schrödinger (DNLS) equation [26]. For a quasimonochromatic wave as considered here, the DNLS equation reduces to the usual nonlinear Schrödinger (NLS) equation [27] of the form

$$i\partial_t B_1 + \frac{v_g'}{2}\partial_\zeta^2 B_1 + Q\frac{|B_1|^2}{B_0^2}B_1 = 0. \tag{15}$$

Here $v_g' = dv_g/dk$ is the group dispersion, $\zeta = z - v_g t$ is the comoving coordinate, and v_g is the group velocity. These quantities are determined from the Alfvén wave dispersion relation, which reads $\omega^2 = k^2 c_A^2(1 \pm kc_A/\omega_{\rm ci})$, when weakly dispersive effects due to the Hall current is included [25]. The upper (lower) sign corresponds to right (left) hand circular polarization. The nonlinear coefficient is $Q = Q_c[1 - (2\mu_B B_0/m_i c_A^2)^2]$, where the classical coefficient is $Q_c = kc_A^3/4(c_A^2 - c_s^2) \simeq -kc_A^3/4c_s^2$. The NLS equation has been studied extensively [28], and as is well-known it admit soliton solutions in 1D, and can describe nonlinear self-focusing followed by collapse in higher dimensions. Furthermore, the evolution depends crucially on the sign of the nonlinear coefficent, which may change due to the spin effects. However, our main concern in this context is not the evolution of the Alfvén waves, which were chosen just as an illustration. The fact that interests us here is that spin can modify the dynamics even when the spins are almost randomly distributed due to a moderately high temperature (i.e., when $\mu_B B_0/k_B T_e \ll 1$). The approximately random distribution of spins is shown in the dispersion relation of the linear wave modes, which are more or less unaffected by the spins since linearly the total spin contribution on the electrons cancel to leading order. Nonlinearly, however, the consequences of the different density fluctuations induced in the spin-up and spin-down populations are seen. The unique feature of this quantum effect is that is survives even for a high temperature. By contrast, well-known quantum plasma effects like the Fermi pressure, and the Bohm–de Broglie potential becomes insignificant for high temperatures. This is also true for single-fluid spin effects [8–12].

An illustration of the regimes where the different quantum plasma effects become significant is provided in Fig. 1. In particular we note that the two-fluid nonlinear spin effects are important for high plasma densities and/or a weak (external) magnetic fields. For comparison, both the Fermi pressure and the Bohm–de Broglie potential need a low-temperature or a very high density to be significant. Single-fluid spin effects can also be significant in this regime, or in the regime of ultrastrong magnetic fields that can occur in astrophysical applications. Especially interesting is that two-fluid nonlinear spin effects can be significant in a high-temperature regime that is normally perceived as classic. While this obviously is an intriguing result, a word of caution is needed. Although our results clearly show that spin effects can be important when $\mu_B B_0/m_i c_A^2$ approaches unity, we note that in a number of applications, spin effects of the kind discussed here can be suppressed even if $\mu_B B_0/m_i c_A^2$ is large. These include: (i) Systems where the dynamics is dominated by compressional effects. In such problems the thermal pressure force

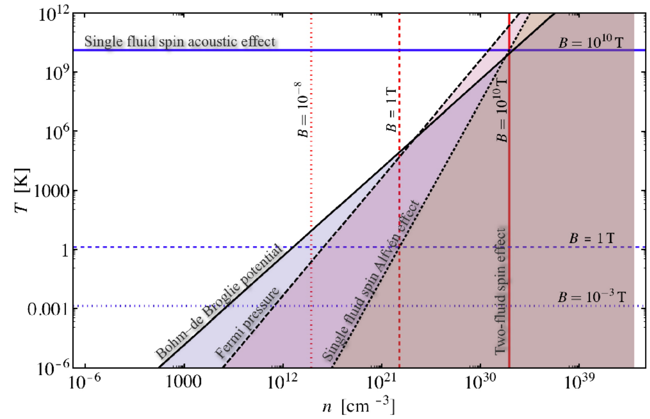

FIG. 1 (color online). Regions of importance in parameter space for various quantum plasma effects. The lines are defined by different dimensionless quantum parameters being equal to unity. The effects included in the figure are the Fermi pressure (dashed black curve), described by the parameter $T_F/T_e \propto \hbar^2 n_0^{2/3}/mk_BT_e$, the Bohm–de Broglie potential (solid black curve), described by the parameter $\hbar\omega_{\rm pe}/k_BT_e \propto n^{1/2}/T_e$, single-fluid spin Alfvén effects (dotted black curve), described by the parameter $\hbar^2\omega_{\rm pe}^2/mc^2k_BT_e$, and single-fluid spin acoustic effects (horizontal blue curves), described by the parameter μ_BB_0/k_BT_e, where three different magnetic field strengths are depicted. The quantum regime corresponds to lower temperatures; i.e., it exists below each of the three horizontal curves. Lastly, the two-fluid spin nonlinear effect derived in this Letter, described by the parameter $\mu_BB_0/m_ic_A^2 \propto n/B_0$, is depicted by the three vertical red curves. The quantum regime corresponds to higher densities; i.e., it exists to the right of each of the three vertical lines.

dominates over the spin force of the electrons, and spin effects are suppressed unless $\mu_BB_0/k_BT_e \to 1$; (ii) High density collisional plasmas. In such cases the spin orientation changes rapidly and different spin populations cannot be sustained; (iii) Systems where the spin forces are negligible as compared to the electrostatic force.

Furthermore, the strong magnetic fields needed for confinement tend to make the two-fluid spin effects studied here negligible for magnetically confined plasmas. Nevertheless, even with the above cases excluded, our discussion shows that there is a large range of plasma problems that traditionally have been dealt with using purely classical plasma equations, but where the electron spin properties give a significant contribution to the dynamics. In general the mechanism can be summarized as follows: for a weakly magnetized initially homogeneous plasma, the spin-up and down populations are (approximately) equal. However, when an electromagnetic perturbation enters the system, the spin-ponderomotive force separates the two populations, which in turn modifies the magnetic field since spin-magnetization no longer cancels. From then on, a two-fluid electron model is needed. In particular, in the region of aligned electron spins, the original magnetic field will be enhanced, and hence mechanisms of this type can play the role of a magnetic dynamo. Suitable plasma conditions for nonlinear two-fluid spin effects to be important may be found in weakly magnetized inertially confined plasmas, near atmospheric pressure plasma dischages, as well as in astrophysical and cosmological plasmas. Exploration of these vast range of problems remain a major research project.

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