Lasing without Inversion: Counterintuitive Population Dynamics in the Transient Regime

S. Ya. Kilin,[1] Kishore T. Kapale,[2,*] and Marlan O. Scully[3,4]

[1]*Stepanov Institute of Physics, National Academy of Sciences of Belarus, F. Skarina Avenue 70, Minsk 220602, Belarus*
[2]*Department of Physics, Western Illinois University, Macomb, Illinois 61455-1367, USA*
[3]*Institute for Quantum Studies and Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*
[4]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
(Received 28 February 2007; published 29 April 2008)

We explore a three-level Λ scheme to demonstrate that the phenomenon of lasing without inversion (LWI) can be observed in the transient regime. We demonstrate that the pure LWI contribution to the gain of a probe field is distinct from both the resonant absorption (or gain) and the coherent Raman gain (or absorption) by choosing specific initial populations in the dressed-state basis. The suppression of the non-LWI (resonant and Raman) processes is followed by the ''rich get richer'' (capitalistic) effect for the ground-state population dynamics: Initially, the more populated ground state becomes even more populated. The conditions for the observation of the effect are specified.

DOI: 10.1103/PhysRevLett.100.173601 PACS numbers: 42.50.Gy, 42.50.Md, 42.55.Ye

The fundamental concepts of quantum coherence and interference have tremendous practical importance. One of their primary applications is the generation of coherent radiation. Ever since Einstein's consideration of the elementary processes of spontaneous and induced emission [1], tremendous progress has been made in the direction of creation and utilization of different sources of coherent light. The majority of the sources of coherent light, i.e., lasers, are based on the idea of light amplification in a media where the upper level of the radiative transition is more populated compared to the ground level [2], a condition termed as population inversion. Some time ago, it was realized that population inversion is not a necessary condition for lasing [3]. The quantum interference between two different transitions, simultaneously excited via coherent radiation, makes possible principally new kinds of coherent radiation sources—lasers without inversion (LWI). The coherent contributions from the two transitions can interfere to give related coherence effects—LWI [4], coherent population trapping [5], and electromagnetically induced transparency [6]. Several methods have been introduced in the literature to explain LWI: the method of quantum trajectories [7] and methods using combinations of phenomenological approaches and earlier quantum concepts [8], to name a few. The relation between the steady state and the transient regime of LWI has been studied by Harris and Macklin in a V-type three-level atom [9]. In this Letter, we present a simple model to understand this effect through the dressed-state analysis and a special preparation of the atomic initial state.

We consider a three-level Λ-type atomic system with two optically allowed transitions ab and ac (Fig. 1), excited by two resonant fields $E_b(t) = E_b \exp(-i\omega_b t)$ and $E_c(t) = E_c \exp(-i\omega_c t)$, respectively. We intend to consider the absorption at the frequency of field E_b. To avoid possible direct interference influence of field E_c on the studied absorption, the resonant frequencies ω_b and ω_c are taken to differ significantly. In this case, the absorption of the ω_b field is proportional to the imaginary part of the transient dipole moment corresponding to the transition ab, i.e., to $\mathrm{Im}[\rho_{ab}(t)]$ [10], where $\rho_{ab}(t)$ is the off-diagonal element of the atomic density matrix. The master equation governing the time evolution is given by:

$$\dot{\rho} = -i[H_0, \rho]/\hbar + \{\gamma_c[n_{\mathrm{th}}(\mathcal{L}_c + \bar{\mathcal{L}}_c) + \mathcal{L}_c] + \gamma_b \mathcal{L}_b\}\rho, \tag{1}$$

where $H_0 = -\hbar(\Omega_c \sigma_{cx} + \Omega_b \sigma_{bx})$ is the atom-field interaction Hamiltonian with the interaction Rabi frequencies $\Omega_i = \wp_{ai} E_i/\hbar$ ($i = c, b$). The atomic transitions are de-

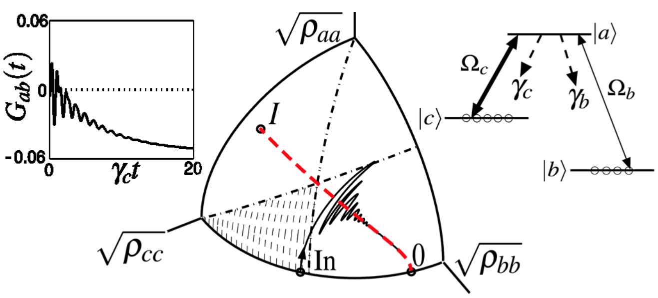

FIG. 1 (color online). Evolution of the population of the states a, b, and c represented as a trajectory on the sphere $(\sqrt{\rho_{aa}})^2 + (\sqrt{\rho_{bb}})^2 + (\sqrt{\rho_{cc}})^2 = 1$ for the initial bare-state mixture with $\rho_{aa}(0) = 0$, $\rho_{bb}(0) = 0.45$, and $\rho_{cc}(0) = 0.55$ (Raman inversion). Point In denotes the initial state. The shaded area represents atomic states satisfying the Raman inversion condition. Dotted-dashed lines mark equally populated states involved in either one of the transitions, while the population of the third level varies between 0 and 1. Points I and 0 represent populations of states $|I\rangle$ and $|0\rangle$. The dashed line (in red) joining point I and 0 corresponds to Eq. (11). Left inset: Integrated probe absorption $G_{ab}(t) = \int_0^t d\tau \mathrm{Im}\rho_{ab}(\tau)$. Here we use $\Omega_c = 1$, $\Omega_b = 0.2$, $\gamma_c = 0.2$, $\gamma_b = 0.05$, and $n_{\mathrm{th}} = 0$.

scribed through the ladder operators $\sigma_{ix} = \sigma_i + \sigma_i^\dagger = |a\rangle\langle i| + |i\rangle\langle a|$. $\mathcal{L}_i$ and $\bar{\mathcal{L}}_i$ are the superoperators acting on the density matrix such that $2\mathcal{L}_i\rho = [\sigma_i, \rho\sigma_i^\dagger] + [\sigma_i\rho, \sigma_i^\dagger]$ and $2\bar{\mathcal{L}}_i\rho = [\sigma_i^\dagger, \rho\sigma_i] + [\sigma_i^\dagger\rho, \sigma_i]$. The relaxation terms in Eq. (1) describe decay in channels ac (rate γ_c) and ab (rate γ_b). Also, $n_{\rm th}$ is the number of thermal photons acting on the ac transition. Inclusion of the thermal photons facilitates the study of the effect of thermal decoherence. Instead of the steady-state excitation, we consider the transient excitation by supposing that atoms come into the interaction region in some prechosen initial state. As we will show, depending on the initial state, a variety of interaction regimes can be realized: absorption of the probe field, Raman-type lasing, and amplification without inversion leading to LWI.

Now we note some general conclusions that can be drawn from the density matrix equations. We consider the steady-state solution of the equation for $\dot{\rho}_{ab}(t)$ from the set of Eqs. (1) for $n_{\rm th} = 0$:

$$\mathrm{Im}\,\bar{\rho}_{ab} = [\Omega_b(\bar{\rho}_{bb} - \bar{\rho}_{aa}) + \Omega_c \mathrm{Re}\bar{\rho}_{cb}]/\gamma_{ab}, \quad (2)$$

where $\bar{\rho}_{ij}$ are the steady-state values of the density matrix elements ρ_{ij} and $2\gamma_{ab} = \gamma_b + \gamma_c$. From Eq. (2), it follows that, for a two-level system ($\Omega_c = 0$), amplification ($\mathrm{Im}\bar{\rho}_{ab} < 0$) requires population inversion $\bar{\rho}_{aa} > \bar{\rho}_{bb}$. Nevertheless, for a three-level system ($\Omega_c \neq 0$), with sufficiently negative $\mathrm{Re}\bar{\rho}_{cb}$, amplification could be obtained without population inversion on the lasing ab transition. However, to throw light on the appropriate conditions for LWI, we look at the equation for $\dot{\rho}_{bb}(t)$, from the set (1), which leads to [11]

$$2\mathrm{Im}\rho_{ab} = (\gamma_b\rho_{aa} - \dot{\rho}_{bb})/\Omega_b. \quad (3)$$

Therefore, the condition for the amplification of the probe field ($\mathrm{Im}\rho_{ab} < 0$) can also be written as:

$$\dot{\rho}_{bb} > \gamma_b\rho_{aa}. \quad (4)$$

It should be stressed that the gain condition (4) applies to populations only, whereas Eq. (2), while usually used for the same purpose, deals with a combination of populations ρ_{aa} and ρ_{bb} and polarization ρ_{bc}. In the steady state, the gain condition becomes $d_{bb}\rho_{bb} > \gamma_b\rho_{aa}$, in contrast with Eq. (4), where d_{bb} is the incoherent depopulation rate of the state b necessary to achieve steady-state amplification [11]. On the contrary, in the transient regime, without any incoherent ground-state depopulation, the probe-field gain can be observed when the overall transient growth of the ground-state population ($\dot{\rho}_{bb}$) exceeds the number of atoms entering the state per unit time due to the incoherent decay to this state ($\gamma_b\rho_{aa}$).

We introduce a quantity $G_{ab}(t) = \int_0^t d\tau \mathrm{Im}\rho_{ab}(\tau)$ to study the transient amplification conditions. By using Eqs. (3) and (4), it can be seen that the net amplification $[G_{ab}(t) < 0]$ results during the interaction time t iff

$$\rho_{bb}(t) - \rho_{bb}(0) > \gamma_b \int_0^t d\tau \rho_{aa}(\tau). \quad (5)$$

Thus, if an initially noninverted system shows amplification, then Eq. (5) requires that the capitalistic (the rich get richer) population dynamics has to be observed simultaneously; i.e., a more populated ground state should become even more populated under the action of the coherent fields than it would be due to the incoherent decay from the upper state. As we will show later, this explains the LWI phenomenon, while competing coherent Raman and direct amplification processes are suppressed by a special initial-state preparation.

We treat the problem in the case of so-called well-separated components, by assuming that the effective Rabi frequency $\Omega = (\Omega_a^2 + \Omega_c^2)^{1/2}$ is much larger than all of the relaxation rates. Thus, averaging the density matrix equation written in a rotated frame $\rho = \exp(-iH_0t/\hbar)\tilde{\rho}\exp(iH_0t/\hbar)$ over the fast oscillating terms gives averaged master equations for an effective two-level system consisting of states $|0\rangle$ and $|I\rangle$:

$$\dot{\rho}_I = d_{0I}\rho_0 - d_{I0}\rho_I, \qquad \rho_0 + \rho_I = 1. \quad (6)$$

Equation (6) describes the relaxation of the population ρ_0 of a dark state $|0\rangle = \alpha_c|b\rangle - \alpha_b|c\rangle$ and of the sum of populations $\rho_I = \langle +|\rho|+\rangle + \langle -|\rho|-\rangle$ of the states $|\pm\rangle = (\pm|a\rangle + \alpha_b|b\rangle + \alpha_c|c\rangle)/\sqrt{2}$. Here $|0, \pm\rangle$ are the eigenstates of the Hamiltonian H_0 with the eigenvalues 0 and $\pm\hbar\Omega$ (where $\alpha_b = \Omega_b/\Omega$ and $\alpha_c = \Omega_c/\Omega$), respectively. The population and depopulation rates introduced in Eq. (6) can be shown to be $d_{0I} = \alpha_b^2 n_{\rm th}\gamma_c$ and $d_{I0} = [\alpha_b^2(n_{\rm th} + 1)\gamma_c + \alpha_c^2\gamma_b]/2$. The off-diagonal density matrix elements in the dressed-state basis, which are better expressed through $\mathrm{Re}\tilde{\rho}_{+-} = (\langle +|\tilde{\rho}|-\rangle + \langle -|\tilde{\rho}|+\rangle)/2$ and $\mathrm{Re}\tilde{\rho}_{B0} = (\langle B|\tilde{\rho}|0\rangle + \langle 0|\tilde{\rho}|B\rangle)/2$, where $|B\rangle$ is the bright state given by $|B\rangle = (|+\rangle + |-\rangle)/\sqrt{2} = \alpha_b|b\rangle + \alpha_c|c\rangle$, decay exponentially with the rates $4\gamma_{+-} = n_{\rm th}\gamma_c(4\alpha_c^2 + 2) + \gamma_c(\alpha_c^2 + 2) + \gamma_b(\alpha_b^2 + 2)$ and $4\gamma_{B0} = n_{\rm th}\gamma_c(1 + 2\alpha_b^2 + \alpha_c^2) + \gamma_c + \gamma_b$, respectively.

The dynamical equations for the dressed-state quantities could be solved in a straightforward manner. However, further analysis requires transformation of the results back to the bare-state basis; the populations are

$$\rho_{aa} = [\rho_I - 2\mathrm{Re}\tilde{\rho}_{+-}(0)\cos(2\Omega t)e^{-\gamma_{+-}t}]/2, \quad (7)$$

$$\rho_{bb} = \alpha_b^2\rho_B + \alpha_c^2\rho_0 + 2\alpha_b\alpha_c\mathrm{Re}\tilde{\rho}_{B0}(0)\cos(\Omega t)e^{-\gamma_{B0}t}. \quad (8)$$

Here, by solving Eq. (6), the population $\rho_I(t)$ is given by

$$\rho_I = d_{0I}/d + [\rho_I(0) - d_{0I}/d]e^{-dt}, \quad (9)$$

with $d = d_{0I} + d_{I0}$. Further, the dark and bright state populations are $\rho_0 = 1 - \rho_I$ and $\rho_B = \rho_I - \rho_{aa}$, respectively. Now the probe-field absorption can be determined by using Eq. (3), with ρ_{aa} defined by Eq. (7), and $\dot{\rho}_{bb}$, which can be obtained by evaluating the time derivatives of

Eqs. (7) and (8), is given by

$$\dot{\rho}_{bb} = (\alpha_b^2/2 - \alpha_c^2)\dot{\rho}_I - 2\alpha_b^2\Omega_{+-}\mathrm{Re}\tilde{\rho}_{+-}(0) \times \sin(2\Omega t + \theta_{+-})e^{-\gamma_{+-}t} + 2\alpha_b\alpha_c\Omega_{B0}\mathrm{Re}\tilde{\rho}_{B0}(0) \times \sin(\Omega t + \theta_{B0})e^{-\gamma_{B0}t}, \tag{10}$$

where $4\Omega_{+-}^2 = 4\Omega^2 + \gamma_{+-}^2$, $\tan(\theta_{+-}) = \gamma_{+-}/2\Omega$, $\Omega_{B0}^2 = \Omega^2 + \gamma_{B0}^2$, and $\tan(\theta_{B0}) = \gamma_{B0}/\Omega$.

The temporal evolution of the bare-state populations and the probe-field absorption governed by the term $\mathrm{Im}\rho_{ab}(t)$ experience the decaying oscillations (see Figs. 1 and 2). There are three characteristic times for the decay ($1/d > 1/\gamma_{0B} > 1/\gamma_{+-}$) and two periods of oscillations ($2\pi/\Omega$ and π/Ω). At $t \geq 1/d$, the system approaches the steady state with $2\mathrm{Im}\bar{\rho}_{ab} = \gamma_b\bar{\rho}_{aa}/\Omega_b$. Thus, there is no gain in the steady state for our scheme, as $\mathrm{Im}\bar{\rho}_{ab}$ is always positive. However, while approaching the steady state, the probe-field gain and absorption are realized alternatively (see left inset in Fig. 1). The natural question to ask is: What net effect [$G_{ab}(t) = \int_0^t d\tau \mathrm{Im}\rho_{ab}(\tau)$] will be observed when applying the rectangular pulses E_c and E_b of duration t, gain or absorption? The answer strongly depends on both the pulse duration and the initial state of the atoms. We consider two types of initial states.

(i) Bare-state mixture: $\rho(0) = \sum \rho_{ii}(0)|i\rangle\langle i|$, *where* $i = a, b, c$.—For atoms entering the interaction region in a bare-state mixture, under the noninverted condition $\rho_{aa}(0) < \rho_{bb}(0)$, the initial dynamics of the probe-field absorption is mainly governed by the third term in Eq. (10) that oscillates with the frequency Ω. Its amplitude $-2\alpha_b\alpha_c\Omega_{B0}\mathrm{Re}\tilde{\rho}_{B0}(0) = -2\alpha_b^2\alpha_c^2\Omega_{B0}[\rho_{bb}(0) - \rho_{cc}(0)]$ depends on the initial population difference between states b and c. The second term in Eq. (10), oscillating with the frequency 2Ω, has a smaller contribution to the probe-field absorption because its amplitude $-2\alpha_b^2\Omega_{+-}\mathrm{Re}\tilde{\rho}_{+-}(0) = \alpha_b^2\Omega_{+-}[\rho_{aa}(0) - \alpha_b^2\rho_{bb}(0) - \alpha_c^2\rho_{cc}(0)]$ is approximately twice smaller and also because it becomes zero at instances given by $t_n \approx \pi(2n+1)/2\Omega$, when the third term has the maximum contribution. The first term has a smaller value because of the adopted approximation ($\gamma_c/\Omega \ll 1$). As a result, for the time in the range $t \approx [0, \pi/\Omega]$, the probe-field gain could be realized only for an initially inverted population in the channel cb: $\rho_{bb}(0) < \rho_{cc}(0)$ (compare insets in Figs. 1 and 2). This type of amplification is known as the coherent Raman gain (CRG). In spite of the initial CRG, during the next period ($t \approx [\pi/\Omega, 2\pi/\Omega]$), absorption—the coherent Raman absorption (CRA)—takes place, its value is smaller compared to the initial CRG because of the relaxation. Thus, a net gain could be achieved even when there is no Raman inversion, i.e., $\rho_{bb}(0) > \rho_{cc}(0)$, for the first period of oscillations (compare left insets in Figs. 1 and 2). Besides, there is an indication that when the oscillating terms have decayed, i.e., when off-diagonal elements of the density matrix in a dressed-state representation have vanished, the gain could be restored (left inset in Fig. 2). To investigate this stage of evolution separately, we consider below an initial state of an atom as a dressed-state mixture.

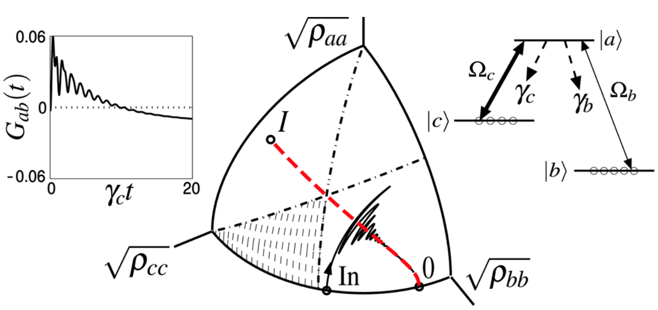

FIG. 2 (color online). The same as Fig. 1, but no initial Raman inversion: $\rho_{aa}(0) = 0$, $\rho_{bb}(0) = 0.55$, and $\rho_{cc}(0) = 0.45$.

(ii) Dressed-state mixture: $\rho(0) = \rho_I(0)(|+\rangle\langle+| + |-\rangle \times \langle-|)/2 + [1 - \rho_I(0)]|0\rangle\langle 0|$.—This mixture has equally populated bright states $|\pm\rangle$ and the remaining population in the dark state, giving an initially diagonal density matrix in the dressed-state representation much as in case (i) after the oscillating terms have decayed. This suggests an experimental method to prepare the aforementioned dressed-state mixture. The populations in the bare-state representation are $\rho_{aa}(0) = \rho_I(0)/2$, $\rho_{bb}(0) = \alpha_c^2 - (2\alpha_c^2 - \alpha_b^2)\rho_{aa}(0)$, and $\rho_{cc}(0) = \alpha_b^2 + (\alpha_c^2 - 2\alpha_b^2)\rho_{aa}(0)$. Moreover, the off-diagonal coherence terms also exist for this initial state as $\rho_{bc}(0) = \alpha_b\alpha_c[\rho_{cc}(0) - \rho_{bb}(0)]/(\alpha_c^2 - \alpha_b^2)$. Furthermore, the oscillating terms in the temporal dependence of the density matrix elements [see Eqs. (7) and (8)] are absent, because for these states $\mathrm{Re}\tilde{\rho}_{B0}(0) = \mathrm{Re}\tilde{\rho}_{+-}(0) = 0$. As a result, both CRG (CRA) and the usual resonant absorption (gain) observable only at $\mathrm{Re}\tilde{\rho}_{B0}(0) \neq 0$ and $\mathrm{Re}\tilde{\rho}_{+-}(0) \neq 0$ do not contribute to the probe field. However, there is still a mechanism ensuring the net gain for the probe field: a specific, relaxation-induced population dynamics demonstrating the effect that initially less populated excited states $|a\rangle$ and $|c\rangle$ become less populated and initially more populated ground state $|b\rangle$ gets even more populated during the interaction as depicted in Fig. 3.

This mechanism cannot be observed for a two-level atom, for which the only state—the equally populated mixture of the two dressed states—represents an absence of coherent oscillations in the case of well-separated components. On the contrary, for a three-level atom a set of mixed states, characterized by a single parameter [for example, by the initial population of bright states $\rho_I(0)$], can evolve in a nonoscillatory manner. This set can be represented by the following equation:

$$[(2\alpha_c^2 - \alpha_b^2)\rho_{cc} + (\alpha_c^2 - 2\alpha_b^2)\rho_{bb}]/(\alpha_c^2 - \alpha_b^2) = 1 \tag{11}$$

[see thick-dashed (red) curve joining points I and 0 in the figures]. Note that the set is limited by two boundary states: the dark state $|0\rangle\langle 0|$ (point 0 on the sphere) and the bright one $|+\rangle\langle+| + |-\rangle\langle-|$ (point I on the sphere). The existence of the set opens up a degree of freedom for relaxation-induced evolution of an atom initially prepared

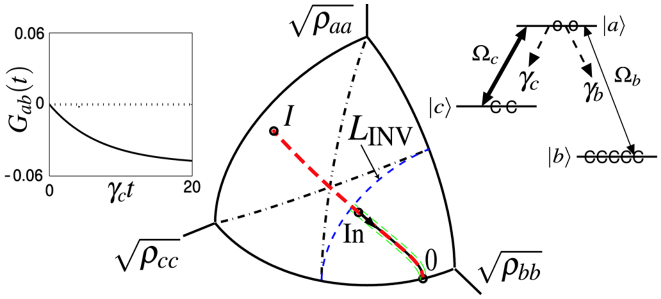

FIG. 3 (color online). Evolution of the population of states a, b, and c for the initial dressed-state mixture (15) with $\rho_{aa}(0) = 0.22$, $\rho_{bb}(0) = 0.5469$, $\rho_{cc}(0) = 0.2331$, and $2\mathrm{Re}\rho_{bc}(0) = -0.1207$. Other atom-field parameters are the same as in Fig. 1. A net gain will be observed for the initial states that belong to the segment of the red $I0$ line marked by two nearby green dashed lines. The thin-dashed (blue) line labeled $L_{\rm INV}$ separates fully inverted states ($\rho_{bb} < 1/2$) to its left from non-inverted ones ($\rho_{bb} > 1/2$) to its right. Open circles on states b and c signify the initial coherence $\rho_{bc}(0)$.

in the special dressed-state mixture. Moreover, the state structure is conserved during the evolution, i.e., $\rho(t) = \rho_I(t)(|+\rangle\langle+| + |-\rangle\langle-|) + [1 - \rho_I(t)]|0\rangle\langle 0|$. Because of this conservation, the population evolution can be represented graphically as a movement along the trajectory (11) down to the dark state (see Fig. 3). This evolution has a specific feature for the dynamics of the bare-state populations where the initially more populated ground state gets more populated with time; this is connected directly to the phenomenon of LWI. A possible value of the probe-field gain depends according to Eq. (3) on the rate of increase of the ground-state, $|b\rangle$, population. If this rate [12] is big enough, a net gain for the probe field can be observed. As follows from Eqs. (7)–(9), the condition (5) for the net amplification can be written, for the considered initial dressed-state mixture, in the form: $K = [(2\alpha_c^2 - \alpha_b^2)d/\gamma_b - 1][\rho_{aa}(0)/\bar{\rho}_{aa} - 1] \geq (1 - e^{-dt})/dt$.

Depending on the presence or absence of the thermal noise in the ac channel, this condition take two forms. With the thermal noise absent ($n_{\rm th} = 0$), $\bar{\rho}_{aa} = 0$, and the net gain condition becomes [13]

$$\gamma_c > \gamma_b^* = \gamma_b[1 + 1/(\alpha_c^2 - 1/3)], \qquad (12)$$

which means that the decay rate at the ac transition should be larger than that at the ab transition (the inequality $\alpha_c^2 > 1/3$ holds due to the assumption that $\Omega_c > \Omega_b$). Therefore, for $\rho_{bb}(0) > 1/2$ (see line labeled $L_{\rm INV}$ in Fig. 3), a net gain without inversion is obtained when inequality (12) is held. Our study including the thermal noise shows that, below a certain upper limit of the thermal decoherence, we observe gain in the transient regime. The observation of the discussed transient LWI together with the population capitalistic effect can be realized, for example, with a nitrogen vacancy center in strained diamond demonstrating [14] the Λ system with the $m_s = \pm 1$ and $m_s = 0$ ground states. By applying the external electric field [15], it is possible to vary the relaxation rates of the lasing and driving transitions to achieve the net gain condition (12).

In conclusion, we have shown that the transient gain without inversion is possible when the coherent Raman amplification and absorption are canceled by a special initial-state preparation of the three-level atoms. The transient-LWI phenomenon is characterized by a population capitalistic effect: The initially more populated ground state becomes even more populated. The effect is necessary for the net gain, because the transient LWI is generally observable when the overall transient growth of the ground-state population exceeds the number of atoms entering the state per unit time due to the incoherent decay. The net gain inequalities restricting the values for the initial atomic populations and decay rates are formulated. The population capitalistic effect can be used as an effective means for purification of quantum states, which is important for quantum information paradigms.

S. K. thanks EC for support via the EQUIND IST project.

*KT-Kapale@wiu.edu

[1] A. Einstein, Phys. Z. **18**, 121 (1917) English translation in D. Ter Haar, *The Old Quantum Theory* (Pergamon Press, New York, 1967), p. 167.
[2] A. M. Prokhorov, Sov. Phys. JETP **34**, 1658 (1958); A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958).
[3] O. Kocharovskaya and Ya. I. Khanin, JETP Lett. **48**, 581 (1988); S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989); M. O. Scully *et al.*, Phys. Rev. Lett. **62**, 2813 (1989).
[4] A. Nottelmann *et al.*, Phys. Rev. Lett. **70**, 1783 (1993); E. S. Fry *et al.*, Phys. Rev. Lett. **70**, 3235 (1993); A. S. Zibrov *et al.*, Phys. Rev. Lett. **75**, 1499 (1995); G. G. Padmabandu *et al.*, Phys. Rev. Lett. **76**, 2053 (1996).
[5] E. Arimondo, in *Progress in Optics XXXV*, edited by E. Wolf (North-Holland, Amsterdam, 1996).
[6] S. E. Harris *et al.*, Phys. Rev. Lett. **64**, 1107 (1990).
[7] C. Cohen-Tannoudji *et al.*, J. Opt. Soc. Am. B **10**, 2107 (1993); H. J. Carmichael, Phys. Rev. A **56**, 5065 (1997).
[8] A. Imamoglu *et al.*, Phys. Rev. Lett. **66**, 1154 (1991).
[9] S. E. Harris and J. J. Macklin, Phys. Rev. A **40**, 4135 (1989).
[10] But not to the sum $\mathrm{Im}[\rho_{ab}(t)] + \mathrm{Im}[\rho_{ac}(t)]$, as it will be in the case of coincided frequencies ω_a and ω_c.
[11] With a nonzero depopulation rate for state b, d_{bb}, Eq. (3) becomes $2\mathrm{Im}\rho_{ab} = (\gamma_b\rho_{aa} - d_{bb}\rho_{bb} - \dot{\rho}_{bb})/\Omega_b$.
[12] For $\rho_{bb}(0) > 1/2$, this is a population capitalization rate.
[13] Condition (12) is slightly stronger than the LWI condition in the continuous wave regime: $\gamma_c > \gamma_b$.
[14] C. Santori *et al.*, Phys. Rev. Lett. **97**, 247401 (2006).
[15] Ph. Tamarat *et al.*, Phys. Rev. Lett. **97**, 083002 (2006).