Jet Rates in Electron-Positron Annihilation at $O(\alpha_s^3)$ in QCD

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We compute production rates for two, three, four, and five jets in electron-positron annihilation at the third order in the QCD coupling constant. At this order, three-jet production is described to next-to-next-to-leading order in perturbation theory while the two-jet rate is obtained at next-to-next-to-next-to-leading order. Our results yield an improved perturbative description of the dependence of jet multiplicity on the jet resolution parameter y_{cut}, particularly at small values of y_{cut}.

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Jet observables in electron–positron annihilation play an outstanding role in studying the dynamics of the strong interactions, described by the theory of quantum chromodynamics (QCD, [1]). The initial experimental observation of three-jet events at PETRA [2], in agreement with the theoretical prediction [3], provided the first evidence for the gluon, and thus strong initial support for the correctness of QCD. Subsequently the three-jet rate and related event-shape observables were used for the precise determination of the QCD coupling constant α_s (see [4,5] for a review), and four-jet observables helped substantially to confirm the gauge group structure of QCD by firmly establishing the gluon self-coupling [6].

Jets are defined using a jet algorithm, which describes how to recombine the momenta of all hadrons in an event to form the jets. A jet algorithm consists of two ingredients: a distance measure and a recombination procedure. The distance measure is computed for each pair of momenta to select the pair with the smallest separation. This pair of momenta then is combined according to the recombination procedure into a joint momentum, if its separation is below a predefined resolution parameter y_{cut}. Improving upon the JADE algorithm [7], which uses the pair invariant mass as distance measure, several jet algorithms have been proposed for e^+e^- collisions: Durham [8], Geneva [9], and Cambridge [10]. Among those, the Durham algorithm has been the most widely used by experiments at LEP [11–14] and SLD [15], as well as in the reanalysis of earlier data at lower energies from JADE [16].

The Durham jet algorithm clusters particles into jets by computing the distance measure

$$y_{ij,D} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{E_{\mathrm{vis}}^2} \quad (1)$$

for each pair (i, j) of particles, E_{vis} denotes the energy sum of all particles in the final state. The pair with the lowest $y_{ij,D}$ is replaced by a pseudoparticle whose four-momentum is given by the sum of the four-momenta of particles i and j ('E' recombination scheme). This procedure is repeated as long as pairs with invariant mass below the predefined resolution parameter $y_{ij,D} < y_{\mathrm{cut}}$ are found. Once the clustering is terminated, the remaining (pseudo)-particles are the jets. It is evident that a large value of y_{cut} will ultimately result in the clustering all particles into only two jets, while higher jet multiplicities will become more and more frequent as y_{cut} is lowered. In experimental jet measurements, one therefore studies the jet rates (jet cross sections normalized to the total hadronic cross section) as a function of the jet resolution parameter y_{cut}.

The theoretical prediction of jet cross sections is made within perturbative QCD, where the same jet algorithm is applied to the momenta of final state partons. The QCD description of jet production is either based on a fixed-order calculation, which uses exact parton-level matrix elements (including higher order corrections if available) for a given jet multiplicity, or by a parton shower, which is based on the leading-order matrix element for two-jet production only, and generates higher multiplicities in an iterative manner, thereby accounting only for the leading logarithmic terms from parton-level processes with higher multiplicity. Depending on the jet multiplicity, higher perturbative orders correspond to different powers of the QCD coupling constant: the leading-order prediction for n-jet production is proportional to α_s^{n-2}. So far, fixed-order calculations were available up to next-to-next-to-leading order (NNLO) for two jets [17–19], up to next-to-leading order (NLO) for three [20–22] and four jets [23–26]. For five and more jets, only leading-order calculations were available [27–29]. For jets involving massive quarks, NLO results are available for three-jet final states [30].

Calculations based on parton showers, incorporated in multipurpose event generator programs [31–33], provide a satisfactory description of multijet production rates. Since these programs contain many tunable phenomenological parameters, their predictive power is, however, very limited.

In this Letter, we present the first calculation of NNLO corrections to three-jet production and the next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) corrections to two-jet production in e^+e^- annihilation. Together with the previously available NLO corrections to four-jet production and the leading-order description of five-jet final states, these are used for a fully consistent perturbative description of $e^+e^- \to$ jets at order α_s^3 in perturbative QCD.

The calculation of the α_s^3 corrections for three-jet production is carried out using the newly developed parton-level event generator program EERAD3 which contains the relevant matrix elements with up to five external partons [27,34–36]. Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft. To extract these infrared divergences and combine them with the virtual corrections, the antenna subtraction method [25,37] was extended to NNLO level [38] and implemented for $e^+e^- \to 3$ jets and related event-shape variables [39] into EERAD3. The analytical cancellation of all infrared divergences serves as a very strong check on the implementation.

Initial results obtained with EERAD3 on NNLO corrections to event-shape observables were reported in [40] and applied in the extraction of the strong coupling constant from LEP data in [41]. Since the program provides the full kinematical information for each event, it can also be used to simultaneously compute the production cross sections for three, four, and five jets through to $\mathcal{O}(\alpha_s^3)$ for any infrared-safe jet algorithm and as a function of the jet resolution parameter. The jet rates are then defined by normalizing the multijet cross sections to the total hadronic cross section computed at the same order.

The four-jet [23–26] and five-jet rates [27] were known previously to $\mathcal{O}(\alpha_s^3)$. Our major new result is the three-jet rate to this order, which corresponds to NNLO in the perturbative expansion. Figure 1 displays the three-jet rate at LEP1 energy $Q = M_Z$ as function of the jet resolution $y_{\rm cut}$ at LO, NLO, NNLO. At NNLO, the denominator

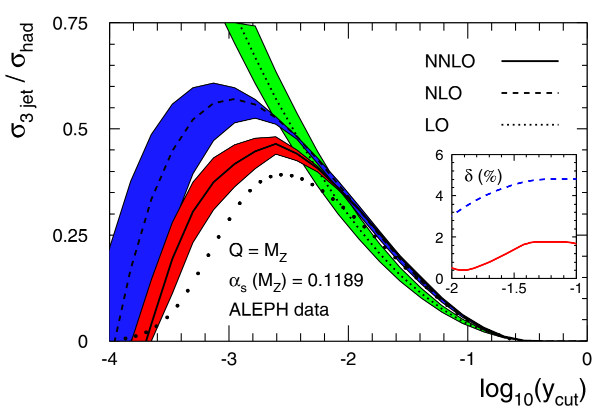

FIG. 1 (color online). Perturbative fixed-order description of the three-jet rate at $Q = M_Z$, compared to data obtained with the ALEPH experiment [11]. Experimental errors are too small to be visible on the figure.

has been expanded, as described in [40] to contain only terms up to $\mathcal{O}(\alpha_s^3)$ in the jet rate. The theoretical uncertainty band is defined by varying the renormalization scale μ in the coupling constant in the interval $M_Z/2 < \mu < 2M_Z$, and the world average value [5] $\alpha_s(M_Z) = 0.1189$ is used, consistently evolved to other scales at each order. The fixed-order theoretical predictions for three-jet rate become negative for small values of $y_{\rm cut}$, where fixed-order perturbation theory is not applicable due to the emergence of large logarithmic corrections at all orders, requiring resummation [8,42]. We therefore restrict our comparison to $y_{\rm cut} > 10^{-4}$, although data at lower jet resolution parameters are available.

For large values of $y_{\rm cut}$, $y_{\rm cut} > 10^{-2}$, the NNLO corrections turn out to be very small, while they become substantial for medium and low values of $y_{\rm cut}$. The maximum of the jet rate is shifted towards higher values of $y_{\rm cut}$ compared to NLO, and is in better agreement with the experimental observation. The theoretical uncertainty is lowered considerably compared to NLO. Especially in the region $10^{-1} > y_{\rm cut} > 10^{-2}$, which is relevant for precision phenomenology, one observes a reduction by almost a factor of 3, down to below 2% relative uncertainty. Since the error band in this region is barely visible in the plot, we display the relative theoretical uncertainty

$$\delta = \frac{\max_{\mu}[\sigma(\mu)] - \min_{\mu}[\sigma(\mu)]}{2\sigma(\mu = M_Z)}$$

at NLO and NNLO as an inset. The relative uncertainty on the LO calculation is constant at 10.2%.

The fixed-order NNLO description is still above the data at low jet resolution, where the convergence of the perturbative series is spoilt by large logarithms of $y_{\rm cut}$ at all orders, and where a resummation should be carried out [8]. Furthermore, the theoretical parton-level prediction is compared to hadron-level data, thereby neglecting hadronization corrections, which may also account for part of the discrepancy.

To compute the jet rates with different multiplicities, it is more appropriate to normalize all jet cross sections to the total hadronic cross section corrected to third order [43] in the QCD coupling constant, $\mathcal{O}(\alpha_s^3)$. We consistently neglect numerically small QCD singlet contributions at this order, which were found to contribute at most 1% [43] to the total coefficient of the $\mathcal{O}(\alpha_s^3)$ correction, and which are equally small in the individual jet multiplicities [23]. The total hadronic cross section is made up from the sum over all jet multiplicities. At $\mathcal{O}(\alpha_s^3)$, this sum runs from two-jet through to five-jet final states, such that the corresponding jet rates must add to unity. Consequently, our calculation yields the N^{3}LO expression for $e^+e^- \to 2$ jets as a by-product. It is interesting to note that some earlier NNLO calculations of the two-jet rate [18,19] were essentially exploiting the same feature at $\mathcal{O}(\alpha_s^2)$.

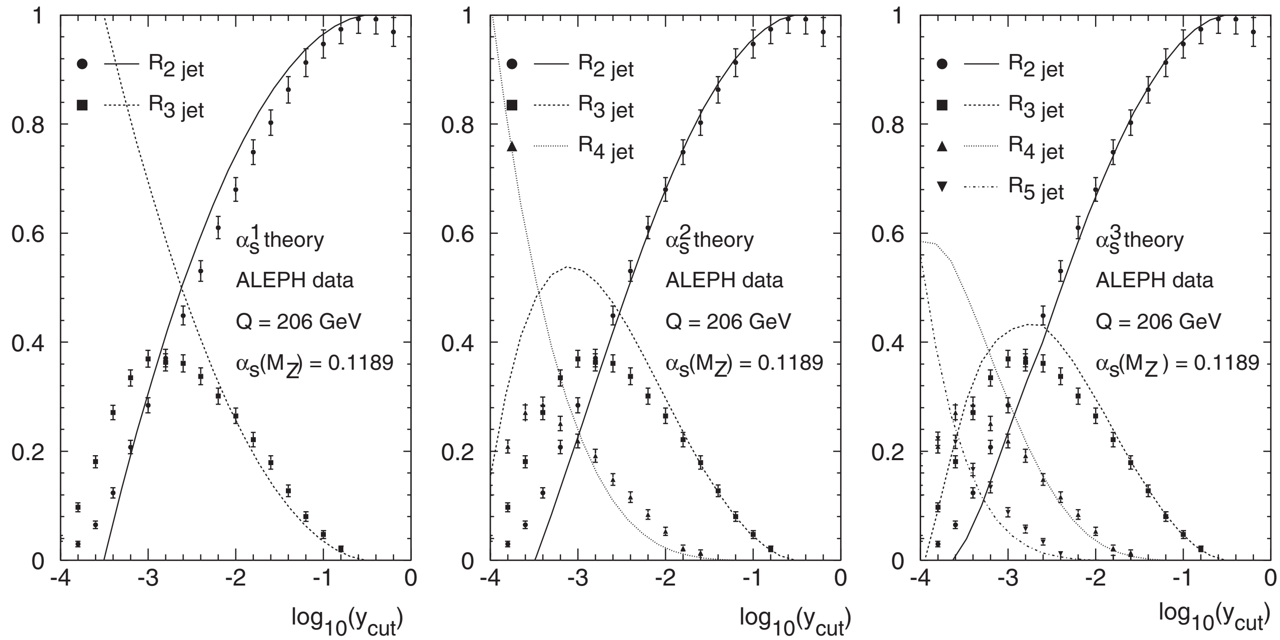

FIG. 2. Jet rates at first, second, and third order in the strong coupling constant, compared with data from ALEPH [11]. The rates are normalized to the total hadronic cross section at that order.

Figure 2 shows the parton-level theoretical predictions for the jet fractions at first, second, and third order in the strong coupling constant, compared to experimental hadron-level data from ALEPH [11].

By comparing the three plots, we observe that there is systematically improved agreement for each of the jet rates as the order of perturbation theory increases. At each order a new multijet channel opens up, e.g., the five-jet rate at $\mathcal{O}(\alpha_s^3)$, which is positive definite and essentially monotonically increasing at small $y_{\rm cut}$. Since all jet rates are normalized to unity, the new five-jet channel has the effect of reducing the contribution to the two-jet, three-jet, and four-jet rates, in the region of $\log_{10}(y_{\rm cut})$ where the five-jet rate contributes. One very clear effect is to cause the turnover in the four-jet rate [which is not present at $\mathcal{O}(\alpha_s^2)$]. A second effect is to add more structure to the shape of the two- and three-jet rates, which lie much closer to the data for $\log_{10}(y_{\rm cut}) < -2.5$. Of course, the effect of the higher order corrections also extends to larger values of $y_{\rm cut}$, due to the different contributions of the two-loop virtual and virtual-radiation graphs to the three- and four-jet rates, as well as the way that the double radiation contribution interacts with the jet algorithm, and through the normalization to the total hadronic cross section. This is visibly less dramatic, but by adding more structure to the theoretical prediction, enables a better description of the data.

Previous experimental studies of multijet production rates compared only with standard leading-order parton shower event generator programs, which yielded a good description of the data at the expense of large hadronization corrections [4,11]. In the light of our new results, this issue should be carefully reexamined within fixed-order perturbation theory.

In this Letter, we reported on the NNLO QCD corrections to the three-jet production rate at parton level in e^+e^- annihilation, which is the first genuine NNLO calculation of a jet production rate at particle colliders. We observed that (hadron-level) experimental three-jet data are described considerably better in shape and normalization, and over a wider range in $y_{\rm cut}$, than at NLO.

At the same order in the strong coupling constant, α_s^3, we describe four-jet production at NLO and five-jet production at LO, reproducing earlier results. By combining those and normalizing to the total hadronic cross section at this order, we obtained the two-jet rate to N^{3}LO in perturbation theory as a byproduct. We observe that with increasing order in the strong coupling constant, the multijet rates are better described over an increasing range of resolution parameters. Our results clearly highlight how perturbative QCD successfully describes jet production rates at the parton level.

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