Transverse Mass for Pairs of Gluinos

Won Sang Cho,[1] Kiwoon Choi,[1] Yeong Gyun Kim,[1,2] and Chan Beom Park[1]
[1]Department of Physics, KAIST, Daejon 305-017, Korea
[2]ARCSEC, Sejong University, Seoul 143-747, Korea
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We introduce a new observable, "gluino m_{T2}," which is an application of the Cambridge m_{T2} variable to the process where gluinos are pair produced in a proton-proton collision and each gluino subsequently decays into two quarks and one lightest supersymmetric particle, i.e., $\tilde{g}\,\tilde{g} \to qq\tilde{\chi}_1^0 qq\tilde{\chi}_1^0$. We show that the gluino m_{T2} can be utilized to measure the gluino mass and the lightest neutralino mass separately and also the 1st and 2nd generation squark masses if squarks are lighter than the gluino, thereby providing a good first look at the pattern of sparticle masses experimentally.

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The Large Hadron Collider (LHC) at CERN will soon explore the TeV energy scale, where new physics beyond the standard model (SM) likely reveals itself [1,2]. Among various new physics proposals, weak scale supersymmetry (SUSY) [3] is perhaps the most promising candidate. Once SUSY signals are discovered through event excess beyond the SM backgrounds in inclusive search channels, the next step will be the measurements of SUSY particle masses and their physical properties in various exclusive decay chains. Then it might be possible to reconstruct SUSY theory from the experimental information on SUSY particle masses [4].

In this Letter, we introduce a new observable, "gluino m_{T2}," which is an application of the m_{T2} variable [5] to the process where gluinos are pair produced in a proton-proton collision and each gluino subsequently decays into two quarks and one lightest supersymmetric particle: i.e., $pp \to \tilde{g}\,\tilde{g} \to qq\tilde{\chi}_1^0 qq\tilde{\chi}_1^0$, where q stands for the 1st or 2nd generation quark. We show that the gluino m_{T2} can be utilized to measure the gluino mass and the lightest neutralino mass separately and also the 1st and 2nd generation squark masses if squarks are lighter than the gluino, thereby providing a good first look at the pattern of sparticle masses experimentally.

If light enough, gluinos would be pair produced copiously in proton-proton collision ($pp \to \tilde{g}\,\tilde{g}$), and each gluino decays into two quarks and one lightest supersymmetric particle (LSP) ($\tilde{g} \to qq\tilde{\chi}_1^0$) through three-body decay induced by an exchange of an off-shell squark or two-body cascade decay with an intermediate on-shell squark. For each gluino decay $\tilde{g} \to qq\tilde{\chi}_1^0$, the following transverse mass can be constructed:

$$m_T^2(m_{qqT}, m_\chi, \mathbf{p}_T^{qq}, \mathbf{p}_T^{\chi}) = m_{qqT}^2 + m_\chi^2 + 2(E_T^{qq}E_T^{\chi} - \mathbf{p}_T^{qq}\cdot\mathbf{p}_T^{\chi}), \quad (1)$$

where m_{qqT} and $\mathbf{p}_T^{qq}$ are the transverse invariant mass and transverse momentum of the qq system, respectively, while m_χ and $\mathbf{p}_T^{\chi}$ are the trial mass and transverse momentum of the LSP, respectively. Here transverse momenta are measured in the laboratory frame with respect to the proton beam direction. The transverse energies of the qq system and LSP are defined as $E_T^{qq} \equiv \sqrt{|\mathbf{p}_T^{qq}|^2 + m_{qqT}^2}$ and $E_T^{\chi} \equiv \sqrt{|\mathbf{p}_T^{\chi}|^2 + m_\chi^2}$. With two such gluino decays in each event, the gluino $m_{T2}(\tilde{g})$ is defined as

$$m_{T2}^2(\tilde{g}) \equiv \min_{\mathbf{p}_T^{\chi(1)}+\mathbf{p}_T^{\chi(2)}=\mathbf{p}_T^{\text{miss}}} [\max\{m_T^{2(1)}, m_T^{2(2)}\}], \quad (2)$$

where the minimization is performed over all possible splittings of the observed missing transverse momentum $\mathbf{p}_T^{\text{miss}}$ into two assumed transverse momenta $\mathbf{p}_T^{\chi(1)}$ and $\mathbf{p}_T^{\chi(2)}$.

From (2), one finds

$$m_{T2}(\tilde{g}) \le m_{\tilde{g}} \quad \text{for} \quad m_\chi = m_{\tilde{\chi}_1^0}, \quad (3)$$

where m_χ and $m_{\tilde{\chi}_1^0}$ denote the trial LSP mass and the true LSP mass, respectively. Therefore, if $m_{\tilde{\chi}_1^0}$ is known, one can determine the gluino mass $m_{\tilde{g}}$ from the end-point measurement of $m_{T2}(\tilde{g})$ distribution:

$$m_{T2}^{\max}(m_\chi) \equiv \max_{\text{all events}} [m_{T2}(\tilde{g})]. \quad (4)$$

However, $m_{\tilde{\chi}_1^0}$ might not be known in advance, and then $m_{T2}^{\max}(m_\chi)$ can be considered as a function of the trial LSP mass m_χ, satisfying $m_{T2}^{\max}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$.

As we will see, $m_{T2}^{\max}(m_\chi)$ has a different functional form depending upon whether squarks are heavier or lighter than the gluino; thus, we consider the two cases separately. If squark masses $m_{\tilde{q}} > m_{\tilde{g}}$, the gluino will undergo the three-body decay $\tilde{g} \to qq\tilde{\chi}_1^0$ through the exchange of an off-shell squark. In order to see how $m_{T2}^{\max}(m_\chi)$ are determined for generic values of m_χ, we consider two extreme momentum configurations and construct $m_{T2}^{\max}$ associated with each of them.

The first momentum configuration is that two gluinos are produced at rest, and each gluino subsequently decays into two quarks moving in the same direction and one LSP moving in the opposite direction. Furthermore, two sets of gluino decay products are parallel to each other, and all of them are on the transverse plane with respect to the proton beam direction. We then have $m^{(1)}_{qqT} = m^{(2)}_{qqT} = 0$, where the quarks are regarded as massless. The transverse energies and momenta of the qq systems are given by

$$E_T^{qq(1)} = E_T^{qq(2)} = |\mathbf{p}_T^{qq(1)}| = |\mathbf{p}_T^{qq(2)}| = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} \equiv E_T^{qq}(\text{max}), \tag{5}$$

and the corresponding total missing transverse momentum is $|\mathbf{p}_T^{\text{miss}}| = 2E_T^{qq}(\text{max})$.

It has been shown that, for certain momentum configurations, the m_{T2} variable ("balanced solution") can be obtained as the minimum of $m_T^{(1)}$ subject to the following two constraints [5]:

$$m_T^{(1)} = m_T^{(2)}, \qquad \mathbf{p}_T^{\text{miss}} = \mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)}. \tag{6}$$

If one applies this balanced solution approach to the momentum configurations with the above $m^{(1,2)}_{qqT}$, $E_T^{qq(1,2)}$, $\mathbf{p}_T^{qq(1,2)}$, $\mathbf{p}_T^{\text{miss}}$, and still undetermined $\mathbf{p}_T^{\chi(1,2)}$, one finds that the minimum of $m_T^{(1)}(= m_T^{(2)})$ is obtained when $\mathbf{p}_T^{\chi(1)} = \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{\text{miss}}/2$, leading to the following gluino m_{T2}:

$$m_{T2}(\tilde{g}) = E_T^{qq}(\text{max}) + \sqrt{[E_T^{qq}(\text{max})]^2 + m_\chi^2} \tag{7}$$

for generic m_χ. One can show that this $m_{T2}(\tilde{g})$ corresponds to m_{T2}^{max} for $m_\chi \leq m_{\tilde{\chi}_1^0}$ [6]:

$$m_{T2}^{\text{max}}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2} \quad \text{for} \quad m_\chi \leq m_{\tilde{\chi}_1^0}. \tag{8}$$

Note that $m_{T2}^{\text{max}}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$ as required.

Another extreme momentum configuration which would determine $m_{T2}^{\text{max}}(m_\chi)$ for $m_\chi \geq m_{\tilde{\chi}_1^0}$ is that gluinos are pair produced at rest, and, for each gluino decay, two quarks are back to back to each other while LSP is at rest. In addition, all particles are on the transverse plane. In this case, one easily finds

$$m^{(1)}_{qqT} = m^{(2)}_{qqT} = m_{\tilde{g}} - m_{\tilde{\chi}_1^0} \equiv m_{qqT}(\text{max}), \tag{9}$$

and also $E_T^{qq(1)} = E_T^{qq(2)} = m_{qqT}(\text{max})$, with $\mathbf{p}_T^{qq(1)} = \mathbf{p}_T^{qq(2)} = \mathbf{p}_T^{\text{miss}} = 0$.

For the momentum configurations with $m^{(1,2)}_{qqT}$ given by (9) and the above $E_T^{qq(1,2)}$, $\mathbf{p}_T^{qq(1,2)}$ and $\mathbf{p}_T^{\text{miss}}$, $m_T^{(1)}$ is equal to $m_T^{(2)}$ for all possible splitting of $\mathbf{p}_T^{\text{miss}} = 0 = \mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)}$, and the minimum of $m_T^{(1)}(= m_T^{(2)})$ occurs when $\mathbf{p}_T^{\chi(1)} = \mathbf{p}_T^{\chi(2)} = 0$. Then the gluino m_{T2} obtained as a balanced solution is given by

$$m_{T2}(\tilde{g}) = m_{qqT}(\text{max}) + m_\chi \tag{10}$$

for generic values of m_χ. This in fact corresponds to m_{T2}^{max} for $m_\chi \geq m_{\tilde{\chi}_1^0}$ [6]:

$$m_{T2}^{\text{max}}(m_\chi) = (m_{\tilde{g}} - m_{\tilde{\chi}_1^0}) + m_\chi \quad \text{for} \quad m_\chi \geq m_{\tilde{\chi}_1^0}, \tag{11}$$

which again gives $m_{T2}^{\text{max}}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$.

The above momentum configurations leading to (11) have $\mathbf{p}_T^{\text{miss}} = 0$ and, thus, could be eliminated by the event cut imposing a lower bound on $|\mathbf{p}_T^{\text{miss}}|$ in the real data analysis. However, a more detailed study [6] shows that there exist momentum configurations yielding (11) while having a sizable $|\mathbf{p}_T^{\text{miss}}|$ comparable to $m_{\tilde{g}}/2$, for instance, a configuration in which the two quarks from the first gluino move in the same transverse direction, while the other two quarks from the second gluino are back to back. As a result, (11) can be constructed from collider data under a proper cut on $|\mathbf{p}_T^{\text{miss}}|$.

By now, it should be clear that m_{T2}^{max} for $m_\chi < m_{\tilde{\chi}_1^0}$ [Eq. (8)] has a quite different form from m_{T2}^{max} for $m_\chi > m_{\tilde{\chi}_1^0}$ [Eq. (11)]. As required, they should cross at the kink point $m_\chi = m_{\tilde{\chi}_1^0}$. Thus, if the function $m_{T2}^{\text{max}}(m_\chi)$ could be constructed from experimental data, which would identify the kink point, one will be able to determine the gluino mass and the LSP mass simultaneously.

The experimental feasibility of measuring $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$ through m_{T2}^{max} depends on the systematic uncertainty associated with the jet resolution, since m_{T2}^{max} is obtained mostly from the momentum configurations in which some (or all) quarks move in the same direction. Our Monte Carlo study indicates that the resulting error is not so significant, so that $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$ can be determined rather accurately by the kink structure of m_{T2}^{max}. As a specific example, we have examined a parameter point in the minimal anomaly mediated SUSY-breaking (mAMSB) scenario [7] with heavy squarks, which gives

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \qquad m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

and a few TeV masses for sfermions. We have generated a Monte Carlo sample of SUSY events for a proton-proton collision at 14 TeV by PYTHIA [8]. The event sample corresponds to 300 fb^{-1} integrated luminosity. We have also generated SM backgrounds such as $t\bar{t}$, W/Z + jet $WW/WZ/ZZ$, and QCD events, with less equivalent luminosity [9]. The generated events have been further processed with a modified version of the fast detector simulation program PGS [10], which approximates an ATLAS- or CMS-like detector with reasonable efficiencies and fake rates.

The following event selection cuts are applied to have a clean signal sample for gluino m_{T2}: at least 4 jets with $P_{T1,2,3,4} > 200$, 150, 100, and 50 GeV; missing transverse energy $E_T^{\text{miss}} > 250$ GeV; transverse sphericity $S_T > 0.25$; no b jets and no leptons. The four hardest jets are divided into two groups of dijets as follows. The highest momentum jet and the other jet which has the largest $|p_{\text{jet}}|\Delta R$ with respect to the leading jet are chosen as the two "seed" jets for division. Here p_{jet} is the jet momentum and $\Delta R \equiv \sqrt{\Delta\phi^2 + \Delta\eta^2}$, i.e., a separation in the azimuthal

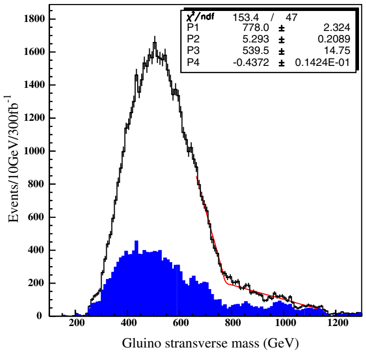

FIG. 1 (color online). The $m_{T2}(\tilde{g})$ distribution with $m_\chi = 90$ GeV for the benchmark point of mAMSB with heavy squarks.

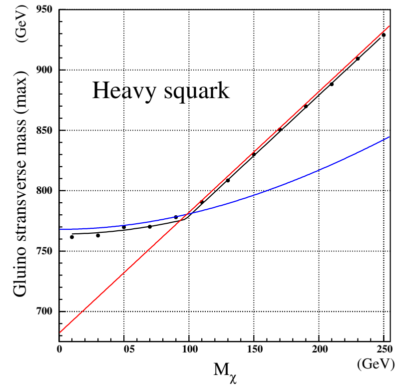

FIG. 2 (color online). $m_{T2}^{\max}$ as a function of the trial LSP mass m_χ for the benchmark point of mAMSB with heavy squarks.

angle and pseudorapidity plane. Each of the remaining two jets is associated to a seed jet which makes the smallest opening angle. Then each group of dijets is considered to originate from the same gluino.

Figure 1 shows the resulting distribution of $m_{T2}(\tilde{g})$ [11] for the trial LSP mass $m_\chi = 90$ GeV. The solid (blue) histogram corresponds to the SM background. By fitting with a linear function with a linear background, we get the end point 778.0 ± 2.3 GeV, which appears as the crossing point of two linear functions. The measured $m_{T2}^{\max}$ as a function of m_χ is shown in Fig. 2. Curved (blue) and straight (red) lines denote the theoretical curves of (8) and (11), respectively, which have been obtained in this Letter from the consideration of extreme momentum configurations. [A rigorous derivation of (8) and (11) is provided in Ref. [6].] By fitting the data points with the curves (8) and (11), we obtain $m_{\tilde{g}} = 776.3 \pm 1.3$ GeV and $m_{\tilde{\chi}_1^0} = 97.3 \pm 1.7$ GeV, which are quite close to the true values $m_{\tilde{g}} = 780.3$ GeV and $m_{\tilde{\chi}_1^0} = 97.9$ GeV. This demonstrates that the gluino m_{T2} can be very useful for measuring the gluino and LSP masses experimentally.

Let us now consider the case that $m_{\tilde{q}} < m_{\tilde{g}}$. In such a case, the following cascade decay is open: $\tilde{g} \to q\tilde{q} \to qq\tilde{\chi}_1^0$. In this case also, we consider two extreme momentum configurations which are similar to those considered for three-body gluino decay and construct the corresponding $m_{T2}(\tilde{g})$. Here again, we assume that gluinos are pair produced at rest and each gluino decays into a quark and a squark on the transverse plane.

If the quark from squark decay is produced in the same direction as the first quark from gluino decay, and the two sets of gluino decay products are parallel to each other, the transverse energies and transverse momenta of the qq systems are again given by (5), and we have $m_{qqT}^{(1)} = m_{qqT}^{(2)}$ and $|\mathbf{p}_T^{\text{miss}}| = 2E_T^{qq}(\max)$. Then the same procedure to obtain the gluino m_{T2} (7) can be applied to this case, leading to $m_{T2}^{\max}$, which is same as Eq. (8) for $m_\chi \leq m_{\tilde{\chi}_1^0}$.

Now we consider another extreme momentum configuration in which the quark from squark decay is produced in the opposite direction to the first quark from gluino decay. In this case, one easily finds

$$m_{qqT}^{2(1)} = m_{qqT}^{2(2)} = \frac{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)(m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{q}}^2},$$

$$E_T^{qq(1)} = E_T^{qq(2)} = \frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2}\right),$$

$$|\mathbf{p}_T^{qq(1)} = \mathbf{p}_T^{qq(2)}| = \left|\frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) - \frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2}\right)\right|,$$

and $\mathbf{p}_T^{\text{miss}} = -2\,\mathbf{p}_T^{qq(1)}$.

By imposing $m_T^{(1)} = m_T^{(2)}$ and $\mathbf{p}_T^{\text{miss}} = \mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)}$ on the momentum configurations which have $m_{qqT}^{(1,2)}$, $E_T^{qq(1,2)}$, $\mathbf{p}_T^{qq(1,2)}$, and $\mathbf{p}_T^{\text{miss}}$ as above, we obtain the following balanced solution of $m_{T2}(\tilde{g})$ at $\mathbf{p}_T^{\chi(1)} = \mathbf{p}_T^{\chi(2)} = -\mathbf{p}_T^{qq(1)}$:

$$m_{T2}^2(\tilde{g}) = m_{qqT}^{2(1)} + m_\chi^2 + 2\left(E_T^{qq(1)}\sqrt{|\mathbf{p}_T^{qq(1)}|^2 + m_\chi^2} + |\mathbf{p}_T^{qq(1)}|^2\right). \quad (12)$$

Again, one can show that this represents $m_{T2}^{\max}$ for $m_\chi \geq m_{\tilde{\chi}_1^0}$ [6], yielding

$$m_{T2}^{\max} = \left[\frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2}\right)\right] + \sqrt{\left[\frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) - \frac{m_{\tilde{g}}}{2}\left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{q}}^2}\right)\right]^2 + m_\chi^2}. \quad (13)$$

Note that the two functions (8) and (13) cross at $m_\chi = m_{\tilde{\chi}_1^0}$, for which $m_{T2}^{\max} = m_{\tilde{g}}$.

If one could construct (8) and (13) accurately, one would be able to determine $m_{\tilde{g}}$, $m_{\tilde{\chi}_1^0}$, and $m_{\tilde{q}}$ altogether. To see how feasible it is, we examined a parameter point of mirage mediation model [12], providing the following sparticle masses:

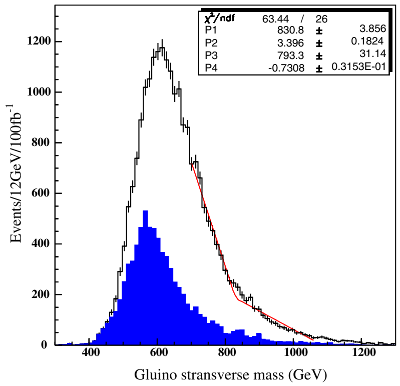

FIG. 3 (color online). The $m_{T2}(\tilde{g})$ distribution with $m_\chi = 350$ GeV for the benchmark point of mirage mediation.

$$m_{\tilde{g}} = 821.4 \text{ GeV}, \qquad m_{\tilde{q}} = 694.0 \text{ GeV},$$
$$m_{\tilde{\chi}_1^0} = 344.2 \text{ GeV}.$$

We have generated a Monte Carlo sample for this benchmark point of mirage mediation, corresponding to 100 fb^{-1} integrated luminosity. After the event selection cuts similar to the case of three-body gluino decay, we obtain Fig. 3, showing the distribution of $m_{T2}(\tilde{g})$ for $m_\chi = 350$ GeV. The edge value $m_{T2}^{\max}$ as a function of m_χ is shown in Fig. 4. By fitting the data points to the curves (8) and (13), we obtain $m_{\tilde{g}} = 799.5 \pm 11.1$ GeV, $m_{\tilde{q}} = 678.2 \pm 7.0$ GeV, and $m_{\tilde{\chi}_1^0} = 316.7 \pm 15.4$ GeV. Though the fitted values are well close to the true sparticle masses, the accuracy is not as good as the three-body decay case, which is mainly due to a mild crossing of two curves. The situation can be improved if we include the information from squark m_{T2} for the process $pp \to \tilde{q}\tilde{q} \to q\tilde{\chi}_1^0 q\tilde{\chi}_1^0$, providing a relation between the edge value of the squark m_{T2} and the trial LSP mass [6] which corresponds to (8) (for all m_χ) with $m_{\tilde{g}}$ replaced by $m_{\tilde{q}}$. By including such information, we get $m_{\tilde{g}} = 803.4 \pm 6.0$ GeV and $m_{\tilde{\chi}_1^0} = 322.4 \pm 7.7$ GeV for the benchmark point. The discrepancy between the fitted mass values and the true mass values may still come from various systematic uncertainties such as the effects of event selection cuts, which is beyond the scope of this Letter.

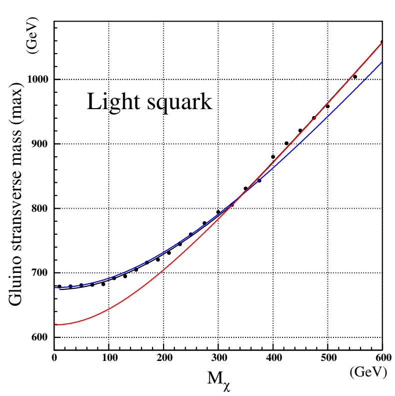

FIG. 4 (color online). $m_{T2}^{\max}$ as a function of the trial LSP mass m_χ for the benchmark point of mirage mediation.

This method can be applied to the SPS1a point of the minimal supergravity model, and we found that sparticle masses can be determined with a similar accuracy.

In general, for given $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$, the kink structure of $m_{T2}^{\max}$ becomes milder if $m_{\tilde{q}}$ is less than $m_{\tilde{g}}$, compared to the case where $m_{\tilde{q}}$ is larger than $m_{\tilde{g}}$. In particular, if the gluino-squark mass difference is smaller than the imposed minimal p_T cut, the kink structure disappears. Also, the contribution from gluino-squark events to the m_{T2} distribution is significant, though the end point is still determined by gluino-gluino events.

To conclude, we have introduced the gluino m_{T2} and shown that it can be used to determine the gluino mass and the lightest neutralino mass separately and also the 1st and 2nd generation squark masses if squarks are lighter than gluino.

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