Non-Gaussianities in New Ekpyrotic Cosmology

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The new ekpyrotic model is an alternative scenario of the early Universe which relies on a phase of slow contraction before the big bang. We calculate the 3-point and 4-point correlation functions of primordial density perturbations and find a generically large non-Gaussian signal, just below the current sensitivity level of cosmic microwave background experiments. This is in contrast with slow-roll inflation, which predicts negligible non-Gaussianity. The model is also distinguishable from alternative inflationary scenarios that can yield large non-Gaussianity, such as Dirac-Born-Infeld inflation and the simplest curvatonlike models, through the shape dependence of the correlation functions. Non-Gaussianity therefore provides a distinguishing and testable prediction of New Ekpyrotic Cosmology.

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The ekpyrotic scenario is a candidate theory of early-Universe cosmology. Instead of invoking a short burst of accelerated expansion from a hot initial state, as in inflation, the ekpyrotic scenario relies on a cold beginning followed by a phase of very slow contraction. Despite such diametrically opposite dynamics, both models predict a flat, homogeneous, and isotropic Universe, endowed with a nearly scale-invariant spectrum of density perturbations, and are, therefore, equally successful at accounting for all current cosmological observations.

An important drawback of the original ekpyrotic theory [1] is how to avoid the big crunch singularity without introducing ghost instabilities. Moreover, the matching of perturbations through the bounce is ambiguous [2].

Both of these issues have been resolved in the recently proposed New Ekpyrotic scenario [3]. In [3], we derived a fully *nonsingular* bounce within a controlled and ghost-free four-dimensional effective theory using the ghost condensation mechanism [4]. Moreover, the curvature perturbation on uniform-density hypersurfaces, ζ, acquires a scale-invariant spectrum well before the bounce, thanks to an entropy perturbation generated by a second scalar field [3,5,6]. Thus, New Ekpyrotic Cosmology appears to be a consistent alternative to the inflationary scenario.

A distinguishing prediction lies in the tensor spectrum [1]: inflation predicts scale-invariant primordial gravity waves, whereas ekpyrosis does not. Detecting tensor modes from cosmic microwave background (CMB) B-mode polarization could rule out the ekpyrotic scenario, whereas an absence of detection would not discriminate between the two models.

In this Letter, we focus on another key observable: the non-Gaussianity of primordial density perturbations. We show that New Ekpyrotic Cosmology generically predicts large non-Gaussianity, potentially just below current sensitivity levels and detectable by near-future experiments.

We calculate the 3-point and 4-point functions. For typical parameter values, the amplitude of the 3-point function is generically large, with $f_{\rm NL}$ around the current Wilkinson microwave anisotropy probe (WMAP) bound [7]: $-36 < f_{\rm NL} < 100$. That is, assuming all parameters are $\mathcal{O}(1)$, $f_{\rm NL}$ approaches the limits of this bound, depending on the sign of a parameter. These values are well above the expected sensitivity of the Planck experiment: $|f_{\rm NL}| \lesssim 20$. The amplitude of the 4-point function is also generically large: $\tau_{\rm NL} \sim 10^4$, which is again near the estimated WMAP bound and within the reach of Planck: $\tau_{\rm NL} \lesssim 600$ [8].

This is in stark contrast with the highly Gaussian spectrum predicted by slow-roll inflation. Comparably large non-Gaussianity does arise in non-slow-roll models, such as Dirac-Born-Infeld (DBI) inflation [9], and whenever the precursor of density fluctuations is a light spectator field, such as in the curvaton [10,11] or modulon scenarios [12,13]. However, as we will see, New Ekpyrosis predicts a different shape dependence in momentum space for the 3- and/or 4-point spectrum than the simplest such models.

Non-Gaussianity therefore offers a distinguishing prediction of New Ekpyrotic Cosmology, potentially testable in CMB experiments within the next few years.

New ekpyrotic cosmology.—As with inflation, ekpyrosis relies on a scalar field ϕ rolling down a potential $\mathcal{V}(\phi)$. Instead of being flat and positive, however, here $\mathcal{V}(\phi)$ must be steep, negative, and nearly exponential in form. For concreteness, we take

$$\mathcal{V}(\phi) = -V_0 e^{-\phi/\Lambda}, \tag{1}$$

where $\Lambda \equiv \sqrt{\epsilon} M_{\rm Pl}$ and $\epsilon \ll 1$. The Friedmann and scalar field equations then yield a background scaling solution,

$$a(t) \sim (-t)^{2\epsilon}; \qquad \bar{\phi}(t) = \Lambda \log\left(\frac{V_0}{2\Lambda^2(1-6\epsilon)} t^2\right), \tag{2}$$

with Hubble parameter $H = 2\epsilon/t$. Since $\epsilon \ll 1$, this describes a slowly-contracting Universe with rapidly increasing H, again in contrast with the rapid expansion and nearly constant H in inflation.

In single-field ekpyrosis, fluctuations in ϕ acquire a scale-invariant spectrum. As we review shortly, this traces back to the fact that the above solution satisfies $\bar{\mathcal{V}}_{,\phi\phi} = -2/t^2$. However, this contribution exactly projects out of ζ, leaving the latter with an unacceptably blue spectrum. Since ζ is conserved on super-horizon scales barring entropy perturbations, it is generally expected to match continuously through the bounce, although stringy effects could alter this picture [2].

New Ekpyrotic Cosmology introduces a second field, χ, as the progenitor of the scale-invariant perturbation spectrum [3,5]. This field has no dynamics during the ekpyrotic phase and remains approximately fixed at $\bar{\chi} = 0$. However, as we describe below, its fluctuations generate a scale-invariant spectrum of entropy perturbations, which gets imprinted onto ζ at the end of the ekpyrotic phase.

An essential condition in obtaining a scale-invariant spectrum is that at $\bar{\chi} = 0$, the curvature of the potential be nearly the same along the χ and ϕ directions: $\bar{V}_{,\chi\chi} \approx \bar{V}_{,\phi\phi}$. An example of such a potential is

$$V(\phi, \chi) = \mathcal{V}(\phi)\left(1 + \frac{\chi^2}{2\Lambda^2} + \frac{\alpha_3}{3!}\frac{\chi^3}{\Lambda^3} + \frac{\alpha_4}{4!}\frac{\chi^4}{\Lambda^4} + \ldots\right). \tag{3}$$

The higher-order χ terms are naturally expected to be suppressed by the same scale Λ as the quadratic term, hence the form (3). For simplicity, we take $\alpha_3, \alpha_4, \ldots$ to be constants. While potential (3) yields a slightly blue spectral tilt, a more general potential is presented in [3] which allows for the observed red tilt without altering the conclusions for non-Gaussianity arrived at in this Letter. Note that the required field trajectory lies along an unstable point. However, a pre-ekpyrotic, stabilizing phase can easily create initial conditions so that this trajectory is arbitrarily close to the tachyonic ridge [3].

Power spectrum for χ.—Since our space-time background is nearly static, we ignore gravity in studying χ perturbations. To linear order, the Fourier modes $\delta\chi_k^{(0)}$ around $\bar{\chi} = 0$ satisfy a free field equation with time-dependent mass $\bar{V}_{,\chi\chi} = \bar{V}_{,\phi\phi} = -2/t^2$:

$$\ddot{\delta\chi}_k^{(0)} + \left(k^2 - \frac{2}{t^2}\right)\delta\chi_k^{(0)} = 0. \tag{4}$$

Assuming the usual adiabatic vacuum, we find

$$\delta\chi_k^{(0)} = \frac{e^{-ikt}}{\sqrt{2k}}\left(1 - \frac{i}{kt}\right). \tag{5}$$

On super-Hubble scales, $k(-t) \ll 1$, the power spectrum, defined by $\langle \delta\chi_k^{(0)} \delta\chi_{k'}^{(0)} \rangle = (2\pi)^3\delta^3(\vec{k} + \vec{k}')P_\chi(k)$, is

$$k^3 P_\chi(k) = \frac{1}{2t^2}, \tag{6}$$

which is scale invariant. Including gravity and departing from the pure exponential form (1) results in small deviations from scale invariance. This can yield a small red tilt, consistent with current CMB observations [3].

Evolution of ζ.—We focus for simplicity on the regime where all relevant modes are well outside the horizon, $k \ll aH$. In the small-gradient approximation, the metric can be written as $ds^2 = -\mathcal{N}^2 dt^2 + e^{2\zeta(\vec{x},t)}a^2(t)d\vec{x}^2$ [14], where $\mathcal{N}$ is the lapse function, and ζ is the curvature perturbation. The evolution of ζ on uniform-density hypersurfaces is governed by

$$\dot{\zeta} = 2H\frac{\delta V}{\dot{\bar{\phi}}^2 - 2\delta V}, \tag{7}$$

where $\delta V \equiv V(\phi, \chi) - V(\bar{\phi}, \bar{\chi})$. A key simplification is that $\delta\phi$ has a steep blue spectrum at long wavelengths and, hence, can be neglected. Thus, for the potential (3), we have $\delta V \approx \mathcal{V}(\bar{\phi})\delta\chi^2/2\Lambda^2 + \ldots$.

To proceed further, one needs an expression for $\delta\chi$ to higher-order than the "free" part $\delta\chi^{(0)}$. To do this, we solve $\ddot{\delta\chi} + \bar{V}_{\chi\chi}\delta\chi = 0$, valid at long wavelengths, perturbatively: $\delta\chi = \delta\chi^{(0)} + \delta\chi^{(1)} + \ldots$. To lowest order, this equation reduces to (4) in the limit $k \to 0$. The next order, $\delta\chi^{(1)}$, satisfies $\ddot{\delta\chi}^{(1)} + \bar{V}_{\chi\chi}\delta\chi^{(1)} + \bar{V}_{,\chi\chi\chi}(\delta\chi^{(0)})^2/2 = 0$. Using (2) and (3), and $\delta\chi^{(0)} \sim 1/t$, we find

$$\delta\chi = \delta\chi^{(0)} + \frac{\alpha_3}{4\Lambda}(\delta\chi^{(0)})^2 + \ldots. \tag{8}$$

Substituting into (7), one can integrate to obtain

$$\zeta_{\rm ek} = \frac{1}{2}\left(\frac{\delta\chi^{(0)}}{M_{\rm Pl}}\right)^2 + \frac{5\alpha_3}{18\sqrt{\epsilon}}\left(\frac{\delta\chi^{(0)}}{M_{\rm Pl}}\right)^3 + \ldots. \tag{9}$$

The ekpyrotic phase must eventually end if the Universe is to undergo a smooth bounce and reheat into a hot big bang phase. This is achieved by adding a feature to the potential (3) which eventually pushes χ away from the tachyonic ridge [3]. Denote the time at which ekpyrosis stops as $t_{\rm end}$. For simplicity, we model this with $V_{,\chi}$ suddenly becoming nonzero and nearly constant at $\chi = 0$. Denote this constant by $V_{,\chi}|$. The exit phase is assumed to last for a time interval Δt which is short compared to a Hubble time: $|H_{\rm end}|\Delta t \ll 1$. This will be the case provided the potential satisfies

$$\epsilon_\chi \equiv \frac{H_{\rm end}^4 M_{\rm Pl}^2}{V_{,\chi}|^2} \lesssim 1. \tag{10}$$

The exit phase generates an additional contribution to ζ. To compute this in the rapid-exit approximation, we can treat the right-hand side of (7) as approximately constant. In evaluating this constant, note that, to leading order in $\delta\chi$, we have $\delta V \approx V_{,\chi}|\delta\chi = \pm H_{\rm end}^2 M_{\rm Pl}\delta\chi/\sqrt{\epsilon_\chi}$. (Higher-order terms in $\delta\chi$ yield small corrections to (9) and are therefore negligible.) Thus, ζ changes from $\zeta_{\rm ek}$ by an amount ζ_c during the exit, given by

$$\zeta_c = \mp 2\sqrt{\epsilon}\beta \frac{\delta\chi(t_{\rm end})}{M_{\rm Pl}}, \tag{11}$$

where $\beta \equiv |H_{\rm end}|\Delta t\sqrt{\epsilon/\epsilon_\chi}$. Noting that to lowest order $\delta\chi \approx \delta\chi^{(0)}$ and substituting (6) evaluated at $t_{\rm end}$, it follows from (11) that the ζ power spectrum is

$$k^3 P_\zeta(k) = \frac{4\epsilon\beta^2}{M_{\rm Pl}^2} k^3 P_\chi(k) = \beta^2 \frac{H_{\rm end}^2}{2\epsilon M_{\rm Pl}^2}. \tag{12}$$

Up to the prefactor β^2, this is identical to the inflationary result, with ϵ playing the role of the usual slow-roll parameter. In the exit mechanism of [3], β denotes the overall change in angle in the field trajectory: $\beta = \Delta\theta$.

Let us pause to discuss the parameter values that satisfy the CMB constraint $k^3 P_\zeta(k) \approx 10^{-10}$. Although H passes through zero at the bounce, as argued in [3] its magnitude is essentially the same at the beginning of the hot big bang phase as it was at $t_{\rm end}$, the end of the ekpyrotic phase. In other words, $H_{\rm end}$ sets the reheat temperature in the expanding phase. For GUT-scale reheat temperature, we have $H_{\rm end}/M_{\rm Pl} \approx 10^{-6}$. Meanwhile, β is a free parameter whose value depends on the exit dynamics. For the explicit exit mechanism of [3], however, the natural value is $\beta \sim \mathcal{O}(1)$. In this case, setting $k^3 P_\zeta(k) = 10^{-10}$ implies $\epsilon \approx 10^{-2}$. We will henceforth take $\beta = 1$ and $\epsilon = 10^{-2}$ as fiducial parameter values.

Combining (11) with (8) and (9) yields

$$\zeta(x) = \zeta_c(x) + \frac{1}{8\epsilon\beta^2}\zeta_c^2(x) \mp \frac{5\alpha_3}{144\epsilon^2\beta^3}\zeta_c^3(x) + \ldots. \tag{13}$$

The exit from the ekpyrotic phase is followed by a ghost condensate phase which leads to a *nonsingular* bounce and reheating. Meanwhile, χ gets stabilized and further evolution is governed by the single scalar ϕ. It follows that ζ is conserved through the bounce and emerges unscathed in the hot big bang phase.

3-point function.—The 3-point ζ correlation function in New Ekpyrotic Cosmology is given by [3]

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = (2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \frac{6}{5} f_{\rm NL}\{P_\zeta(k_1)P_\zeta(k_2) + \text{perm.}\}. \tag{14}$$

This *local* shape [15] is consistent with $\zeta(x)$ of the form $\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{\rm NL}\zeta_g^2(x)$, where ζ_g is Gaussian. The correlation function is evaluated at $t_{\rm end}$ ignoring gravity.

Thus, the 3-point function is fully specified by $f_{\rm NL}$ [16]. This parameter receives two contributions. To begin with, the non-Gaussianity of $\delta\chi$, due to its cubic interaction in (3), is inherited by ζ through (11). Following Maldacena [17], the $\delta\chi$ 3-point function is given by

$$\langle \delta\chi_1\delta\chi_2\delta\chi_3\rangle = -i\int_{-\infty}^{t_{\rm end}} ds \langle 0|[\delta\chi_1\delta\chi_2\delta\chi_3, \mathcal{H}_{\rm int}(s)]|0\rangle + \text{c.c.}, \tag{15}$$

where $\delta\chi_i \equiv \delta\chi(x_i)$, and $\mathcal{H}_{\rm int}$ is the cubic interaction Hamiltonian from (3): $\mathcal{H}_{\rm int} = \mathcal{V}(\bar\phi)\alpha_3\chi^3/3!\Lambda^3$. An explicit calculation yields the *intrinsic* contribution

$$f_{\rm NL}^{\rm int} = \mp \frac{5}{24}\frac{\alpha_3}{\beta\epsilon}. \tag{16}$$

The $\mp$ sign corresponds to choosing $V_{,\chi}|$ to be $\pm$.

The second contribution comes from the nonlinear relation between $\delta\chi$ and ζ embodied in (11) and (13). Even if $\delta\chi$ were Gaussian, this nonlinearity would make ζ non-Gaussian. This *conversion* contribution to $f_{\rm NL}$ is

$$f_{\rm NL}^{\rm conv} = \frac{5}{24}\frac{1}{\beta^2\epsilon}. \tag{17}$$

Summing (16) and (17) yields a combined $f_{\rm NL}$:

$$f_{\rm NL} \equiv f_{\rm NL}^{\rm int} + f_{\rm NL}^{\rm conv} = \frac{5}{24\beta^2\epsilon}\left(1 \mp \alpha_3\beta\right). \tag{18}$$

Since this is inversely proportional to $\epsilon \ll 1$, non-Gaussianity tends to be large in New Ekpyrotic Cosmology. Related ekpyrotic models [6,18] also give $f_{\rm NL} \sim \epsilon^{-1}$. (A ghost condensate bounce and second scalar field are also invoked in [6], albeit without an explicit conversion mechanism, and while the two-field ekpyrotic phase of [18] is similar to ours, the bounce physics remains unspecified.) This is in sharp contrast with slow-roll inflation, where $f_{\rm NL}$ is *proportional* to the slow-roll parameters and therefore unobservably small. For concreteness, consider our fiducial model with GUT-scale reheating, $\beta = 1$ and $\epsilon = 10^{-2}$. Taking, for example, the $-$ sign in (16) and choosing $2.728 > \alpha_3 > -3.8$ yields $f_{\rm NL}$ within the present WMAP 2σ range: $-36 < f_{\rm NL} < 100$. Thus, $\alpha_3 \sim \mathcal{O}(1)$ yields a non-Gaussian signal near the WMAP bound. Lower reheating temperatures correspond to smaller ϵ and, therefore, larger non-Gaussian signal. Of course, $|f_{\rm NL}|$ can always be made smaller by taking β, ϵ to be larger and/or by suitably choosing α_3.

4-point function.—The connected 4-point function,

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = (2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \cdot [T(k_1, k_2, k_3, k_4) + T'(k_1, k_2, k_3, k_4)], \tag{19}$$

involves two different shape functions, evaluated at $t_{\rm end}$:

$$T = \frac{1}{2}\tau_{\rm NL}\{P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_{14}) + 23\text{perm.}\};$$
$$T' = \kappa_{\rm NL}\{P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3) + 3\text{perm.}\}, \tag{20}$$

where $\vec{k}_{ij} \equiv \vec{k}_i + \vec{k}_j$. Thus, T and T' are specified, respectively, by the $\tau_{\rm NL}$ and $\kappa_{\rm NL}$ parameters. (Note that $\kappa_{\rm NL}$ is proportional to the f_2 parameter of [8].) Equations (19) and (20) are consistent with $\zeta(x)$ of the form $\zeta(x) = \zeta_g(x) + \frac{\sqrt{\tau_{\rm NL}}}{2}\zeta_g^2(x) + \frac{\kappa_{\rm NL}}{6}\zeta_g^3(x)$, where ζ_g is Gaussian. Note that we can obtain $\tau_{\rm NL}$ immediately by simply comparing its definition with that of $f_{\rm NL}$:

$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2 = \frac{1}{16\beta^4\epsilon^2}\left(1 \mp \alpha_3\beta\right)^2. \tag{21}$$

This was also checked by explicit computation.

Meanwhile, $\kappa_{\rm NL}$ receives two contributions. The first contribution arises from cubic and quartic terms in χ in the potential (3). An explicit calculation gives

$$\kappa_{\rm NL}^{\rm int} = \frac{2\alpha_4 + 3\alpha_3^2}{40\beta^2\epsilon^2}. \tag{22}$$

The second contribution is encoded in the ζ_c^2 and ζ_c^3 terms in (13). Comparing with (19), we obtain

$$\kappa_{\rm NL}^{\rm conv} = \mp \frac{5\alpha_3}{24\beta^3\epsilon^2}. \tag{23}$$

Combining the above results, we find

$$\kappa_{\rm NL} \equiv \kappa_{\rm NL}^{\rm int} + \kappa_{\rm NL}^{\rm conv} = \frac{\alpha_3(9\alpha_3\beta \mp 25) + 6\alpha_4\beta}{120\beta^3\epsilon^2}. \tag{24}$$

Both $\tau_{\rm NL}$ and $\kappa_{\rm NL}$ are proportional to ϵ^{-2} and therefore also tend to be relatively large. Note that $\tau_{\rm NL}$ is always positive, whereas $\kappa_{\rm NL}$ can have either sign. For instance, our fiducial parameter values for GUT-scale reheating with α_3, $\alpha_4 \sim \mathcal{O}(1)$ yield $\tau_{\rm NL} \sim 10^4$, which is around the estimated WMAP bound [8]. Lower non-Gaussianity can again be achieved by taking larger β, ϵ and/or by a suitable choice of α_3 and α_4.

Discussion.—The simplest inflationary models, consisting of one or more slowly-rolling scalar fields, all predict negligible 3-point and higher-order correlation functions. Non-Gaussianity therefore offers a robust test to distinguish New Ekpyrotic Cosmology from slow-roll inflation.

Significant inflationary non-Gaussianity can be obtained in non-slow-roll models, such as DBI, albeit with a distinguishable 3-point shape dependence [15].

Large non-Gaussianity may also be achieved in the curvaton scenario. While the 3-point function is also local, there is an essential difference at the 4-point level. In the simplest curvaton model, the progenitor of density perturbations is a free field. Thus, $\kappa_{\rm NL} \sim f_{\rm NL}$ [11]. In New Ekpyrosis, however, both $\tau_{\rm NL}$ and $\kappa_{\rm NL}$ are $\sim f_{\rm NL}^2$, leading to a distinguishable 4-point shape dependence. More intricate curvaton models can also yield large $\kappa_{\rm NL}$. Similarly, for general modulon scenarios [13].

Near-future non-Gaussianity observations will, therefore, test the new ekpyrotic paradigm and can potentially distinguish it from its inflationary alternatives.

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