Quantum Information Processing with Single Photons and Atomic Ensembles in Microwave Coplanar Waveguide Resonators

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We show that pairs of atoms optically excited to the Rydberg states can strongly interact with each other via effective long-range dipole-dipole or van der Waals interactions mediated by their nonresonant coupling to a common microwave field mode of a superconducting coplanar waveguide cavity. These cavity mediated interactions can be employed to generate single photons and to realize in a scalable configuration a universal phase gate between pairs of single photon pulses propagating or stored in atomic ensembles in the regime of electromagnetically induced transparency.

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Ensembles of trapped atoms or molecules are promising systems for quantum information processing and communications [1]. They can serve as convenient and robust quantum memories for photons, providing thereby an interface between static and flying qubits [2], using, e.g., stimulated Raman techniques, such as electromagnetically induced transparency (EIT) [3]. However, controlled interactions realizing universal quantum logic gates and entanglement in a deterministic and scalable way are difficult to achieve with photonic qubits propagating or stored in atomic ensembles.

A promising scheme for deterministic logic operations between stored photonic qubits was proposed in [4]. It exploits the so-called dipole blockade mechanism, wherein strong dipole-dipole interaction (DDI) between Rydberg atoms suppresses multiple excitations within a certain interaction volume. First proof-of-principle experiments have impressively demonstrated the blockade effect for related van der Waals interactions (VdWIs) between the atoms [5]. The achieved blockade radius of only a few μm is, however, not yet sufficient for implementing logic gates. Furthermore, the scheme of [4] has two principle drawbacks. (i) In free-space, the DDI scales with interatomic distance r_{ij} as r_{ij}^{-3}. The gap in the Rydberg excitation spectrum of an atomic cloud is determined by the smallest DDI between atoms at opposite ends of the cloud. Yet, for closely spaced atoms, the DDI can be very large, which may lead to level crossings with other Rydberg states opening detrimental loss channels. (ii) Complete excitation blockade in an atomic ensemble requires spherical symmetry of the resonant DDI, which severely restricts the choice of suitable Rydberg states.

Here, we put forward an alternative, scalable, and efficient approach untainted by the above difficulties. We first show that superconducting coplanar waveguide (CPW) resonators [6,7], operating in the microwave regime, can mediate long-range controlled interactions between neutral atoms optically excited to the Rydberg states. By appropriate choice of the system parameters, effective resonant DDI or VdWI between pairs of atoms located near the CPW surface [8] can be achieved. These interactions can then be employed to generate single photons and to realize a universal phase gate between pairs of single photon pulses propagating or stored in cold trapped atomic ensembles in the EIT regime.

Consider a pair of atoms i and j optically excited to the Rydberg states $|r\rangle$. The atoms interact nonresonantly with a certain mode of CPW cavity with frequency ω_c via transitions to adjacent Rydberg states $|a\rangle$ and $|b\rangle$ lying, respectively, above and below $|r\rangle$ (Fig. 1). All the other cavity modes are far detuned from the atomic transition frequencies ω_{ar} and ω_{rb} and do not play a role. In the frame rotating with ω_c, the Hamiltonian is given by

$$H = \hbar \sum_{l=i,j} [(\Delta_a \hat{\sigma}^l_{aa} - \Delta_b \hat{\sigma}^l_{bb}) + (g^l_{br}\hat{c}^\dagger \hat{\sigma}^l_{br} + g^l_{ar}\hat{c}\hat{\sigma}^l_{ar} + \text{H.c.})], \quad (1)$$

where $\hat{\sigma}^l_{\mu\nu} = |\mu_l\rangle\langle\nu_l|$ is the transition operator of the lth atom, $\Delta_a = \omega_{ar} - \omega_c$ and $\Delta_b = \omega_{rb} - \omega_c$ are the corresponding detunings, $\hat{c}$ is the cavity field operator, and $g^l_{\mu\nu} = -(\wp_{\mu\nu}/\hbar)\varepsilon_c u(\mathbf{r}_l)$ is the atom-field coupling rate determined by the dipole matrix element $\wp_{\mu\nu}$ of the atomic transition, the field per photon $\varepsilon_c = \sqrt{\hbar\omega_c/\epsilon_0 V_c}$ within the

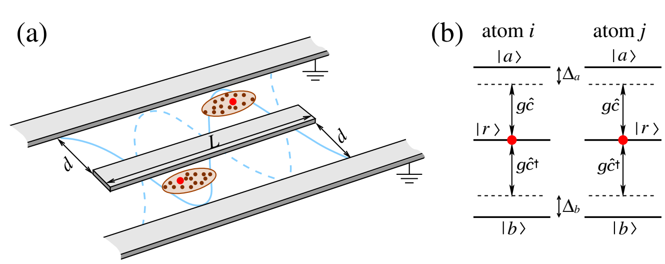

FIG. 1 (color online). (a) CPW cavity with stripline length L and electrode distance d. Ensembles of ground state atoms are trapped near the CPW surface within antinodes of the standing wave field. (b) Level scheme of excited Rydberg atoms i and j interacting nonresonantly with a common cavity mode.

effective cavity volume $V_c = 2\pi d^2 L$, and the cavity mode function $u(\mathbf{r}_l)$ at atomic position $\mathbf{r}_l$ [6,8].

Given an initial configuration $|r_i\rangle|r_j\rangle|0_c\rangle$, with both atoms in state $|r\rangle$ and zero cavity photons $|0_c\rangle$, and large detunings $|\Delta_{a,b}| \gg g^l_{ar}, g^l_{br}, \kappa$, where κ is the cavity mode linewidth, we can use second order perturbation theory to eliminate the nonresonant states $|r_{i,j}\rangle|b_{j,i}\rangle|1_c\rangle$ with a single photon in the cavity. We then obtain that each atom in state $|r_l\rangle$ experiences a cavity-induced level shift $s^l_r = \sqrt{|g^l_{br}|^2 + \Delta_b^2/4} - \Delta_b/2 \simeq |g^l_{br}|^2/\Delta_b$ (ac Stark shift) and small level broadening $\gamma^l_r = \kappa|g^l_{br}|^2/\Delta_b^2 \ll \kappa$. In addition, state $|r_i\rangle|r_j\rangle$ couples to states $|b_{i,j}\rangle|a_{j,i}\rangle$ with rate $D_{ij} = g^i_{br}g^j_{ar}/\Delta_b = g^i_{ar}g^j_{br}/\Delta_b$ via virtual photon exchange between the atoms in the cavity [6–9]. The corresponding interaction Hamiltonian reads

$$V^{(2)}_{ij} = \hbar D_{ij}(\hat{\sigma}^i_{br}\hat{\sigma}^j_{ar} + \hat{\sigma}^i_{ar}\hat{\sigma}^j_{br}) + \text{H.c.} \qquad (2)$$

Note that states $|r_i\rangle|r_j\rangle|0_c\rangle$ and $|b_{i,j}\rangle|a_{j,i}\rangle|0_c\rangle$ are also coupled to state $|b_i\rangle|b_j\rangle|2_c\rangle$ with two photons in the cavity. But due to the large detunings $2\Delta_b$, these transitions yield only small (fourth order) level shifts accounted for below. In second order in $g^l_{\mu\nu}$, the energy offsets of states $|a_{i,j}\rangle|b_{j,i}\rangle$, relative to $|r_i\rangle|r_j\rangle$, are $\hbar(\delta\omega + s^{i,j}_a - s^i_r - s^j_r)$, where $\delta\omega = \Delta_a - \Delta_b = \omega_{ar} - \omega_{rb}$ and $s^{i,j}_a = |g^{i,j}_{ar}|^2/\Delta_b$. If, by an appropriate choice of $\delta\omega$ (with $s^j_a = s^i_a$), the transitions $|r_i\rangle|r_j\rangle \leftrightarrow |b_{i,j}\rangle|a_{j,i}\rangle$ are made resonant, Eq. (2) would describe an effective resonant DDI, or Förster process, between atoms i and j. Then, the eigenstates of (2) form a triplet of states $|\psi^0_{ij}\rangle = \frac{1}{\sqrt{2}}(|b_i\rangle|a_j\rangle - |a_i\rangle|b_j\rangle)$ and $|\psi^\pm_{ij}\rangle = \frac{1}{\sqrt{2}}|r_i\rangle|r_j\rangle \pm \frac{1}{2}(|b_i\rangle|a_j\rangle + |a_i\rangle|b_j\rangle)$ with the corresponding energies 0 and $\pm\hbar\sqrt{2}D_{ij}$ relative to that of state $|r_i\rangle|r_j\rangle$. Note that unlike the free space DDI of [4], here the DDI has very long range as it is mediated by the cavity field extending over $L \sim 1$ cm.

Consider next the nonresonant case of $|\delta\omega| \gg D_{ij}, s^l_\mu$. Using fourth order perturbation theory, we eliminate states $|r_{i,j}\rangle|b_{j,i}\rangle|1_c\rangle$, $|b_{i,j}\rangle|a_{j,i}\rangle|0_c\rangle$, and $|b_i\rangle|b_j\rangle|2_c\rangle$ connected to the initial state $|r_i\rangle|r_j\rangle|0_c\rangle$ via nonresonant single- and two-photon transitions. This yields an effective cavity mediated VdWI between a pair of Rydberg atoms i and j,

$$V^{(4)}_{ij} = \hbar\hat{\sigma}^i_{rr}W_{ij}\hat{\sigma}^j_{rr}. \qquad (3)$$

The effect of this Hamiltonian is to shift the energy of two atoms simultaneously excited to state $|r\rangle$ by the amount $W_{ij} = 2|g^i_{br}g^j_{br}|^2/\Delta_b^3 - (|g^i_{br}g^j_{ar}|^2 + |g^i_{ar}g^j_{br}|^2)/(\delta\omega\Delta_b^2)$.

Before proceeding, let us estimate the relevant experimental parameters. For atoms placed in the vicinity of CPW field antinodes, the coupling rates $g^l_{\mu\nu}$ are approximately the same, $g^l_{br} \approx g^l_{ar} \equiv g_r$ (see below). Setting $\Delta_{a,b} \simeq g_r f$ ($f \gg 1$) and $\delta\omega = s_r = g_r f^{-1}$, the DDI coefficient is $D_{ij} \simeq D = g_r f^{-1}$. On the other hand, with $\Delta_b \simeq g_r f$ and $\delta\omega = -g_r$ [$\Delta_a \simeq g_r(f-1)$], the VdWI strength is $W_{ij} \simeq W = 4g_r f^{-3}$. The relaxation rate of a Rydberg state $|\mu\rangle$ ($\mu = r, a, b$) is $\Gamma_\mu + \gamma_\mu$, where Γ_μ is a small intrinsic decay and $\gamma_\mu \simeq \kappa f^{-2}$ is the cavity-induced relaxation. To achieve coherent interactions, we require that D, $W \gg \Gamma_\mu + \gamma_\mu$ or $g_r > \Gamma_r f^3$, κf, equivalent to the strong coupling regime of cavity QED [10].

For a CPW cavity with stripline length $L \simeq 1$ cm and electrode distance $d \simeq 15$ μm [Fig. 1(a)], the effective volume is $V_c \simeq 1.4 \times 10^{-11}$ m^3. The mode functions are 1D standing waves $u(z) = \cos(m\pi z/L)$ or $\sin(m\pi z/L)$ with m an even or odd integer and $z \in [-L/2, L/2]$ [6,7]. Choosing, e.g., $m = 5$, the mode wavelength is $\lambda_c = 2L/m \simeq 4$ mm, and there are $m+1$ field antinodes. With effective dielectric constant $\epsilon_r \sim 6$, the mode frequency $\omega_c = 2\pi c/\lambda_c\sqrt{\epsilon_r} \simeq 2\pi \times 30$ GHz. For properly selected Rydberg states $|r\rangle$, $|a\rangle$, $|b\rangle$, the transition frequencies ω_{ar}, $\omega_{rb} \sim \omega_c$, and thereby the values of f, can be precisely adjusted with static electric and magnetic fields [11]. The dipole matrix element between neighboring Rydberg states with principal quantum number n scales as $\wp \propto n^2 a_0 e$, which for $n \sim 50$ yields $g_r \sim 2\pi \times 10$ MHz. In a cavity with quality factor $Q \simeq 10^6$, the photon decay rate is $\kappa = \omega_c/Q \simeq 200$ kHz, while $\Gamma_\mu \lesssim 1$ kHz. Thus, the strong coupling regime with the above stringent condition can be realized for $f < \min[\sqrt[3]{g_r/\Gamma_r}, g_r/\kappa] \sim 40$.

Employing a master equation approach [1], we have simulated the dynamics of the system with above parameters and $f = 10, 20$. Numerical solutions of the density matrix equations for the full system described by the exact Hamiltonian (1) are shown in Fig. 2 and compared to simulations with the corresponding effective Hamil-

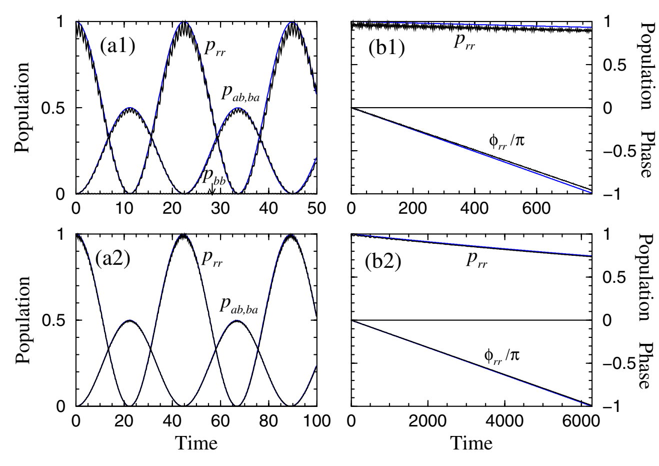

FIG. 2 (color online). Dynamics of two Rydberg atoms i and j coupled through a CPW cavity, initially in state $|r_i\rangle|r_j\rangle|0_c\rangle$. (a1), (a2) correspond to effective DDI, and (b1), (b2) to effective VdWI. All parameters are given in the text, $f = 10$ in (a1), (b1), and $f = 20$ in (a2), (b2). $p_{\mu\nu}$ are populations of states $|\mu_i\rangle|\nu_j\rangle$, and ϕ_{rr} is the phase shift of $|r_i\rangle|r_j\rangle$. Thin (black) lines are numerical solutions of the density matrix equations for the full system using Hamiltonian (1), and thick (blue) lines are solutions for the corresponding reduced model described by Eq. (2) or (3). Time is measured in units of g_r^{-1}.

tonians (2) and (3). As expected, the agreement between the exact and effective models is good for $f = 10$ and excellent for $f = 20$. In the case of DDI, the decay and decoherence are very small on the time scale $(\sqrt{2}D)^{-1}$ of oscillations between states $|r_i\rangle|r_j\rangle$ and $|b_{i,j}\rangle|a_{j,i}\rangle$. In the case of VdWI, the decay is appreciable on the much longer time scale of W^{-1}: At time $T_\pi = \pi/W$, when state $|r_i\rangle|r_j\rangle$ acquires phase shift $\phi_{rr} = \pi$, its population $p_{rr} \simeq 0.92$ for $f = 10$, and $p_{rr} \simeq 0.74$ for $f = 20$. Thus, with realistic experimental parameters and $f = 10$ ($D \simeq 2\pi \times 1$ MHz, $W \simeq 2\pi \times 40$ kHz and $\gamma \simeq 2$ kHz), conditional phase shift of π for a pair of Rydberg atoms can be achieved with fidelity $F > 90\%$. With CPW cavity improvements and parameter optimization, the above fidelity may be further increased.

We envision several quantum information protocols utilizing Hamiltonians (2) and (3) in ensembles of alkali atoms in the ground state. Trapping cold atoms at a distance of 10–20 μm from the surface of a superconducting chip, incorporating the CPW cavity [Fig. 1(a)], is possible with presently available techniques [12]. An elongated volume $V_a \sim d \times d \times \lambda_c/20$ would contain $N \sim 10^6$ atoms at density $\rho_a \sim 2 \times 10^{13}$ cm^{-3}. Each ensemble should be positioned near the cavity field antinode so that the mode function $|u(\mathbf{r})| \approx 1$ is approximately constant throughout the atomic cloud. The corresponding coupling rates $g_{\mu\nu}$ can then be assumed the same for all the atoms in the CPW cavity.

Employing EIT based light storage techniques [2,3], atomic ensembles in Fig. 1(a) can serve as reversible quantum memories for single photon qubits. Briefly, atoms in the ground state $|g\rangle$ resonantly interact with a quantum field $\hat{\mathcal{E}}$ on the transition $|g\rangle \leftrightarrow |e\rangle$, while a classical driving field with Rabi frequency Ω_d (and wave vector $\mathbf{k}_d$) couples the excited state $|e\rangle$ to the metastable state $|s\rangle$. When the light pulse $\hat{\mathcal{E}}$ (with wave vector $\mathbf{k}$) enters the EIT medium, it is transformed into the so-called dark-state polariton $\Psi = \cos\theta\hat{\mathcal{E}} - \sin\theta\sqrt{N}\hat{\sigma}_{gs}$ [3,13] propagating with reduced group velocity $v_g = c\cos^2\theta$, where $\tan\theta = g_{ge}\sqrt{N}/\Omega_d$ with $g_{ge} = \wp_{ge}\sqrt{\omega/(2\hbar\epsilon_0 V_a)}$. The slowing down of the pulse leads to its spatial compression by a factor of $\cos^2\theta \ll 1 (0 < \theta \lesssim \pi/2)$. Once the pulse has been fully accommodated in the medium, by turning off Ω_d ($\theta = \pi/2$), the photonic excitation is adiabatically mapped onto the collective long-lived atomic excitation represented by state $|s^{(1)}\rangle \equiv 1/\sqrt{N}\sum_{i=1}^N e^{i(\mathbf{k}-\mathbf{k}_d)\cdot\mathbf{r}_i}|g_1, \ldots, s_i, \ldots, g_N\rangle$ which involves a single Raman (spin) excitation, i.e., atom in state $|s\rangle$. At a later time, the photon can be retrieved on demand by turning Ω_d on. With a typical resonant absorption cross section for the alkali atoms $\sigma_0 \simeq 10^{-10}$ cm^2, and the above cited density ρ_a and medium length $L_a = \lambda_c/20 \sim 0.2$ mm, the optical depth of the ensemble $2\sigma_0\rho_a L_a \simeq 80$ is large enough for efficient photon storage [3,13].

Using the cavity-mediated DDI (2), we can implement the dipole blockade [4] of multiple Rydberg excitations in an atomic ensemble. This, in turn, can be used to prepare the collective state $|s^{(1)}\rangle$ and subsequently generate single photon pulses. Consider the level scheme of Fig. 3(a), where laser fields with Rabi frequencies Ω_{gr} and Ω_{sr} and wave vectors $\mathbf{k}_{gr}$ and $\mathbf{k}_{sr}$ resonantly couple the lower atomic states $|g\rangle$ and $|s\rangle$ to the Rydberg state $|r\rangle$. Initially, all the atoms are in state $|g\rangle$, and only Ω_{gr} is on. This field induces the transition from the ground state $|g_1, g_2, \ldots, g_N\rangle \equiv |s^{(0)}\rangle$ to the collective state $|r^{(1)}\rangle \equiv 1/\sqrt{N}\sum_i e^{i\mathbf{k}_{gr}\cdot\mathbf{r}_i}|g_1, \ldots, r_i, \ldots, g_N\rangle$ representing a symmetric single Rydberg excitation of the atomic ensemble. The collective Rabi frequency for transition $|s^{(0)}\rangle \rightarrow |r^{(1)}\rangle$ is $\sqrt{N}\Omega_{gr}$. Once an atom $i(\in \{1, \ldots, N\})$ is transferred to state $|r\rangle$, the excitation of a second atom $j(\neq i)$ is suppressed by the DDI, provided that $D_{ij} \simeq D > \Omega_{gr}$. Indeed, out of the three eigenstates of (2), the unshifted eigenstate $|\psi_{ij}^0\rangle$ is not coupled to state $|r_i\rangle|g_j\rangle$ by Ω_{gr}, while transitions $|r_i\rangle|g_j\rangle \rightarrow |\psi_{ij}^\pm\rangle$ are shifted away from resonance by $\pm\sqrt{2}D_{ij}$ and therefore are inhibited. Hence, a laser pulse of area $\sqrt{N}\Omega_{gr}T_1 = \pi/2$ (an effective π pulse) prepares the state $|r^{(1)}\rangle$. The probability of error due to populating the doubly-excited states $|\psi_{ij}^\pm\rangle$ is found by adding the probabilities of all possible double excitations, $P_{\mathrm{double}} \sim N^{-1}\sum_{i,j}2|\Omega_{gr}|^2/(4D_{ij}^2) \approx N|\Omega_{gr}|^2/(2D^2)$. Additionally, Rydberg state relaxation during the pulse time $T_1 = \pi/(2\sqrt{N}\Omega_{gr})$ causes an error $P_{\mathrm{decay}} \lesssim (\Gamma_r + \gamma_r)T_1$. The total error probability is minimized by choosing $\Omega_{gr} = \sqrt[3]{\pi(\Gamma_r + \gamma_r)D^2/(2N^{3/2})}$. By subsequent application of the second stronger laser with pulse area $\Omega_{sr}T_2 = \pi/2$ (π pulse), state $|r^{(1)}\rangle$ is quickly converted into $|s^{(1)}\rangle \equiv \frac{1}{\sqrt{N}}\sum_i e^{i(\mathbf{k}_{gr}-\mathbf{k}_{sr})\cdot\mathbf{r}_i}|g_1, \ldots, s_i, \ldots, g_N\rangle$, which is precisely the state we need for generating a single photon, as described above and illustrated in Fig. 3(a). For the present parameters and choosing the optimal $\Omega_{gr} \simeq 2\pi \times 100$ Hz, the preparation time of state $|s^{(1)}\rangle$ is $T \sim 2.7$ μs with the fidelity $1 - (P_{\mathrm{double}} + P_{\mathrm{decay}}) \gtrsim 0.98$.

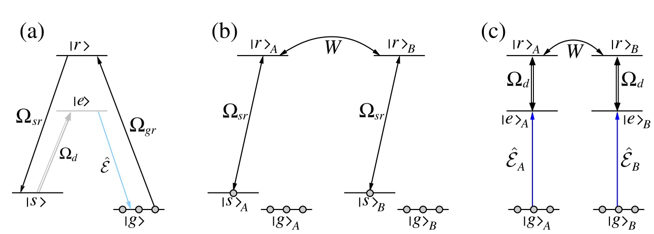

FIG. 3 (color online). (a) DDI between atoms in Rydberg state $|r\rangle$ facilitates generation of single collective spin excitation which can then be converted into a single photon pulse $\hat{\mathcal{E}}$. (b) Pair of atoms excited to state $|r\rangle$ undergo VdWI resulting in a conditional phase shift between ensemble qubits A and B. (c) Single-photon fields $\hat{\mathcal{E}}_A$ and $\hat{\mathcal{E}}_B$ upon entering atomic ensembles A and B are converted into dark-state polaritons undergoing VdWI and acquiring dispersive phase shift.

Note that Rydberg atoms interact also via direct DDI $\bar{D}_{ij} \simeq \wp_{rb}\wp_{ra}/(4\pi\epsilon_0 \hbar r_{ij}^3)$ [4], which, for closely spaced atoms i and j, is much larger than the cavity-mediated DDI D_{ij}. However, $\bar{D}_{ij}$ is a short-range interaction, and already at interatomic distances $r_{ij} \gtrsim 20\ \mu$m, it is smaller than D_{ij}, whose range is given by the CPW cavity size.

Since the cavity-mediated DDI is present between any pair of Rydberg atoms in the cavity, the above technique can be extended to create an entangled state of any two ensembles A and B within the cavity of Fig. 1(a). Thus, by applying Ω_{gr} simultaneously to both ensembles, due to the dipole blockade, only one atom will be excited to state $|r\rangle$. The duration of the pulse should be chosen according to $\sqrt{2N}\Omega_{gr}T_1 = \pi/2$, since it now drives $2N$ atoms. Using the second laser pulse with area $\Omega_{sr}T_2 = \pi/2$ to quickly bring the population of state $|r\rangle$ to the metastable state $|s\rangle$, an entangled state $(|s^{(1)}\rangle_A|s^{(0)}\rangle_B + |s^{(0)}\rangle_A|s^{(1)}\rangle_B)/\sqrt{2}$ of atomic ensembles A and B sharing a single collective spin excitation, will be produced.

We now describe possible uses of the cavity-mediated VdWI (3). As detailed above, atomic ensembles can serve as reversible quantum memories for photonic qubits. Conversely, individual ensembles can encode qubits in the corresponding superposition $|\psi\rangle = \alpha|s^{(0)}\rangle + \beta|s^{(1)}\rangle$ of collective states. Consider two such ensemble qubits A and B in the cavity of Fig. 1(a). A resonant π-pulse applied to the transition $|s\rangle \to |r\rangle$ in both atomic ensembles, $\Omega_{sr}T_1 = \pi/2$, converts state $|s^{(1)}\rangle_{A,B}$ of each ensemble to the state $|r^{(1)}\rangle_{A,B}$ with single Rydberg excitation [see Fig. 3(b)]. Since any two atoms in state $|r\rangle$ interact via VdWI with strength W, during time $T_\pi = \pi/W$, they will accumulate a phase shift π. Only if both ensembles were initially in state $|s^{(1)}\rangle$, the above phase shift would occur since otherwise there is only zero or one atom in state $|r\rangle$, and the VdWI (3) will not take place. A second π-pulse $\Omega_{sr}T_2 = \pi/2$ then converts state $|r^{(1)}\rangle_{A,B}$ of each ensemble back to the original state $|s^{(1)}\rangle_{A,B}$. Thus, the universal CPHASE gate $|s^{(x)}\rangle_A|s^{(y)}\rangle_B \to (-1)^{xy}|s^{(x)}\rangle_A|s^{(y)}\rangle_B$ $(x,\ y \in [0, 1])$ [1] is implemented between ensemble qubits A and B.

The CPHASE gate can be implemented directly between two photonic qubits. A possible setup is shown in Fig. 3(c), where single photon fields $\hat{\mathcal{E}}_A$ and $\hat{\mathcal{E}}_B$ propagate in the atomic ensembles A and B under the conditions of EIT in the ladder configuration [14]: the quantum fields act on transition $|g\rangle \to |e\rangle$, while transition $|e\rangle \to |r\rangle$ is driven by resonant classical field with Rabi frequency Ω_d. Upon entering the medium, each quantum field is converted into the dark-state polariton $\Psi_{A,B} = \cos\theta\hat{\mathcal{E}}_{A,B} - \sin\theta\sqrt{N}\hat{\sigma}_{gr}^{A,B}$ whereby part of the photonic excitation is temporally transferred to the atomic excitation. For $\theta \lesssim \pi/2$, the polariton propagating with group velocity $v_g = c\cos^2\theta \ll c$ is mainly an atomic excitation. Thus, each field $\hat{\mathcal{E}}_{A,B}$ containing a single photon creates in the corresponding ensemble an atom in the Rydberg state $|r\rangle$. These atoms interact via the cavity mediated VdWI (3) resulting in the cross-phase modulation between the two quantum fields. If both fields enter the corresponding ensembles simultaneously, the interaction time is $T_{\rm int} = L_a/v_g$. Calculations similar to those in [15] show that the nonlinear phase shift for a pair of single photons is given by $\phi = \sin^4\theta W L_a/v_g$. We can express the group velocity as $v_g = |\Omega_d|^2/(\sigma_0\rho_a\gamma_{ge})$, where $\gamma_{ge} \simeq 15$ MHz is the coherence relaxation rate on the transition $|g\rangle \to |e\rangle$. With the other parameters given above and choosing $\Omega_d = 2\pi \times 1.1$ MHz ($\sin^2\theta \simeq 1$), we obtain $\phi = \pi$. Thus, the two-photon input state $|1\rangle_A|1\rangle_B$ acquires a conditional phase shift of π, and more generally, the CPHASE gate $|x\rangle_A|y\rangle_B \to (-1)^{xy}|x\rangle_A|y\rangle_B$ $(x,\ y \in [0, 1])$ between two photonic qubits is realized.

To summarize, ensembles of cold atoms trapped in the vicinity of a microwave CPW cavity can strongly interact with each other via cavity mediated virtual photon exchange between optically exited atomic Rydberg states. This system can serve as an efficient and scalable platform to realize various quantum information processing protocols with ensemble qubits and single photons.

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