Suppression of Molecular Decay in Ultracold Gases without Fermi Statistics

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We study inelastic processes for ultracold three-body systems in which only one interaction is resonant. We show that at ultracold temperatures three-body recombination in such systems leads mainly to the formation of weakly bound molecules. In addition, and perhaps more importantly, we have found that the decay rates for weakly bound molecules due to collisions with other atoms can be suppressed not only without fermionic statistics but also when bosonic statistics applies. These results indicate that recombination in three-component atomic gases can be used as an efficient mechanism for molecular formation, allowing the achievement of high molecular densities.

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In recent years, the efficiency of diatomic molecule formation in ultracold quantum gases and their stability once formed have become key ingredients for many experiments. For example, the remarkable stability of weakly bound molecules in ultracold gases of fermions in two different spin states [1] has greatly helped experimental studies in the Bose-Einstein condensation-BCS crossover regime [2] when tuning the two-body s-wave scattering length a from $a > 0$ to $a < 0$ near a Feshbach resonance. The long lifetimes recently observed for ultracold heteronuclear molecules [3] might also pave the way for future studies on ultracold polar molecules. The stability of weakly bound molecules where at least one atom is a fermion has been interpreted as a result of suppression due to the fermionic statistics governing both atom-molecule and molecule-molecule collisions [4,5]. When bosonic statistics plays a role, however, weakly bound molecules tend to decay rapidly, limiting the experimental possibilities for studies near a Feshbach resonance [5–7]. Obviously, besides stability, efficient production of molecules is also of crucial experimental importance.

In this Letter, we present a series of new predictions that opens up new possibilities for obtaining long molecular lifetimes and that also provides an alternative path for efficient molecular formation. Surprisingly, we have found that weakly bound molecules of two resonantly interacting atoms can still be stable against collisions with other atoms even when bosonic statistics applies. Specifically, inelastic collisions between these weakly bound molecules and *any* atom not resonant with either of the molecule's atoms is suppressed as a^{-1}, therefore allowing long molecular lifetimes near a Feshbach resonance. Note that we will use "resonant" to mean that $|a| \gg r_0$, where r_0 is the characteristic range of the two-body interaction. This suppression in the absence of fermionic statistics has recently been observed in Ref. [3] in an ultracold mixture of weakly bound ^{87}Rb^{40}K molecules and ^{40}K atoms. Only the ^{87}Rb-^{40}K interaction was resonant, and the fermionic ^{40}K were in different spin states. Here we show that the mechanism determining this a^{-1} suppression can be attributed entirely to the Efimov physics governing the atom-molecule interactions [5]. In our picture, the suppression of the molecular decay is due to a repulsive effective atom-molecule interaction that prevents the system from reaching the short distances necessary for inelastic transitions. Physically, the atom-molecule repulsion arises from the increase in kinetic energy that occurs when bringing *any* three particles to short distances. When there is only one resonant interaction, this kinetic energy dominates the attraction due to the interatomic interactions. Two or more resonant interactions, on the other hand, can dominate the kinetic energy, leading to an attractive atom-molecule potential and the Efimov effect. In fact, this picture also applies to systems with identical fermions—Fermi statistics simply enhances the atom-molecule repulsion and, consequently, the suppression of the decay rates [4,5]. Unlike cases with fermionic suppression, our predicted a^{-1} suppression holds for *any* mass ratio between the collision partners. The sole requirement is that only the atoms bound in the molecule interact resonantly.

Besides the suppression mentioned above, we will show that three-component systems display other collisional properties favorable for producing weakly bound molecules. In contrast, three-body recombination in the one- and two-component bosonic gases and in the Bose-Fermi mixtures studied to date can lead to a substantial fraction of deeply bound molecules in addition to the weakly bound ones normally produced [5,7,8] In both cases, the binding energy is converted to enough kinetic energy to free the molecule and atom from typical traps. If the scattering length is large enough, however, weakly bound molecules can remain in the trap but will quickly decay due to collisions with other atoms and molecules, leading again to loss [9]. Weakly bound molecules fare somewhat better in mixtures of fermions in two different spin states; their collisional relaxation rate is proportional to $a^{-3.33}$ [4,5], and their formation rate via recombination scales as Ta^6, which dominates recombination into deeply bound mole-

cules ($\propto Ta^{2.455}$) [5]. However, because recombination scales as Ta^6, efficient molecule formation is restricted to relatively high temperatures or large scattering lengths [10]. The three-component gas that we consider, where only one of the three interspecies interactions is resonant, retains the advantages of these fermionic systems with the additional advantage of having a nonzero recombination rate at $T \to 0$. These systems thus support efficient formation of long-lived weakly bound molecules even at ultracold temperatures, which is important to explore the quantum degenerate regime.

We study these ultracold three-body collisions by solving the Schrödinger equation in the adiabatic hyperspherical representation. The three-body inelastic collision rates are determined by solving the hyperradial Schrödinger equation given by (in atomic units)

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2} + W_\nu\right]F_\nu + \sum_{\nu' \neq \nu} V_{\nu\nu'}F_{\nu'} = EF_\nu. \quad (1)$$

Here μ is the three-body reduced mass, E is the total energy, F_ν is the hyperradial wave function, and ν is a collective index that represents all quantum numbers necessary to label each channel. Since the hyperradius R gives the overall size of the system, this equation describes the collective radial motion under the influence of the effective potential W_ν, with inelastic transitions driven by the nonadiabatic couplings $V_{\nu\nu'}$ [5]. For our numerical calculations [Figs. 1(a) and 1(b)], we first solve the hyperangular part of the three-body Schrödinger equation, determining both effective potentials W_ν and couplings $V_{\nu\nu'}$, and then extract the scattering observables from the S matrix obtained from a direct solution of Eq. (1) (see Ref. [11] for details).

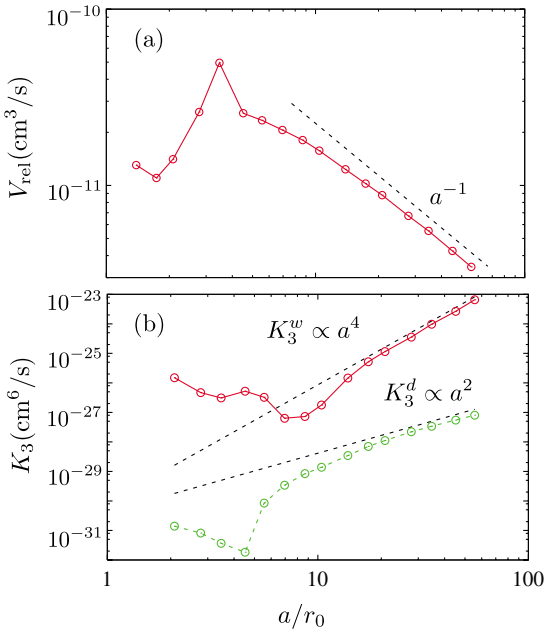

FIG. 1 (color online). (a) Vibrational relaxation rate from weakly bound XY^* molecules and (b) three-body recombination rates into weakly (K_3^w) and deeply (K_3^d) bound molecules.

In the regime where the two-body interactions are resonant, the effective potentials W_ν become universal, and several analytical properties of ultracold three-body systems can be derived. In fact, in our framework [5], both the a and the E dependence of the three-body collision rates can be determined by simply identifying the attractive or repulsive character of W_ν as determined by the Efimov physics [5]. Previous work in ultracold three-body collisions [4,5,7] has analyzed the influence of Efimov physics for systems where at least *two* of the pairwise two-body interactions are resonant. In this Letter, we will show that Efimov physics still has an important impact even when just *one* interaction is resonant.

For a system of three different particles with only a single resonant interaction, the effective potentials are repulsive in the range $r_0 \ll R \ll |a|$ and can be conveniently parametrized by the coefficients p_0 and p_ν [5]:

$$W_0(R) = \frac{p_0^2 - \frac{1}{4}}{2\mu R^2} \quad \text{and} \quad W_\nu(R) = \frac{p_\nu^2 - \frac{1}{4}}{2\mu R^2}. \quad (2)$$

For $a > 0$, p_0 is associated with the atom-dimer channel and therefore plays an important role in atom-molecule processes. The effective potentials associated with p_ν describe three-body continuum channels. The coefficients p_0 and p_ν in Eq. (2) can be determined by assuming a zero-range model potential for the interatomic interactions (we have confirmed that finite-range two-body interactions give the same results) and writing the three-body wave function in terms of the Faddeev components [5]. By doing this, we have determined that p_0 and p_ν are

$$p_0 = J + 1, \qquad p_\nu = J + 3, J + 5, \ldots, \quad (3)$$

where J is the total orbital angular momentum. Interestingly, in contrast to systems where two or three interactions are resonant, the strengths of the potentials p_0 and p_ν do not depend on the mass ratio or whether the resonant pair is composed of identical bosons or distinguishable particles.

Our derivation of these rates' scaling behavior rests on the observation that the rate limiting step is tunneling through the potential barriers represented by Eq. (2). Estimating the tunneling probability with WKB [5] and using p_0 and p_ν from Eq. (3) gives the scaling laws shown in Table I for systems with one resonant pair of atoms [see Eqs. (14), (18), and (19) of Ref. [5]]. We show the results for XYZ systems (three distinguishable atoms) and for BBX systems (two identical bosons and a third distinguishable atom). For XYZ systems, we have assumed that X-Y is the only resonant pair. For BBX, we assumed that only the intraspecies interaction is resonant. Note that the scaling laws for both XYZ and BBX systems are exactly the same. In Table I, we show the first three partial wave contribu-

TABLE I. Energy and scattering length dependence for three-body collision rates for systems with a single resonant pair of atoms. Boldface indicates the leading contribution at threshold [$k \to 0$, where $k^2 = 2\mu E$ for K_3 and D_3 and $k^2 = 2\mu(E - E_v)$ for $V_{\rm rel}$; $-E_v$ is the molecular binding energy]. The results are valid for all systems with three distinguishable atoms (XYZ) or with two identical bosons (BBX) with a_{BB} resonant.

	$V_{\rm rel}$			$K_3^w(D_3)$, K_3^d	K_3^d
J^π	E	$a > 0$	E	$a > 0$	$a < 0$
0^+	**const**	$\boldsymbol{a^{-1}}$	**const**(k^4)	$\boldsymbol{a^4}$, $\boldsymbol{a^2}$	$\boldsymbol{\lvert a\rvert^2}$
1^-	k^2	a^{-1}	$k^2(k^6)$	a^6, a^2	$\lvert a\rvert^2$
2^+	k^4	a^{-1}	$k^4(k^8)$	a^8, a^2	$\lvert a\rvert^2$

tions to vibrational relaxation of weakly bound molecules $XY^* + Z \to XY + Z$ ($V_{\rm rel}$), to three-body recombination into weakly bound molecules $X + Y + Z \to XY^* + Z$ (K_3^w), to recombination into deeply bound molecules $X + Y + Z \to XY + Z$ (K_3^d), and to dissociation of weakly bound molecules $XY^* + Z \to X + Y + Z$ (D_3).

Figure 1(a) shows our numerical results for $V_{\rm rel}$ for three distinguishable atoms XYZ for $J^\pi = 0^+$ (π is the total parity). Figure 1(a) shows our numerical results for $V_{\rm rel}$ for three distinguishable atoms XYZ for $J^\pi = 0^+$ (π is the total parity). We have assumed a finite-range model two-body interaction in which the resonant X-Y interaction is adjusted to support one weakly bound s-wave state whose position controls the scattering length, one deeply bound s-wave state, and one deeply bound p-wave state. Our model X-Z interaction supports only one s-wave state, and, for simplicity, we assumed that the Y-Z interaction has no bound state. The figure shows that, when $a/r_0 \gtrsim 10$, our predicted a^{-1} scaling law is verified by the numerical results. Thus, we have clear evidence that vibrational relaxation is indeed suppressed for large a even in the absence of fermionic statistics. By extension, this result also supports our arguments that molecular decay in $BB + X$ collisions are also suppressed as a^{-1} even though bosonic statistics applies. In a two-component gas, however, long lifetimes for BB molecules can be achieved only if there are no free B atoms and with the additional condition of low molecular density, to prevent decay due to molecule-molecule collisions.

Recombination for systems with a single resonant pair of atoms also presents some peculiar properties. As mentioned previously, for $a > 0$, recombination into weakly bound molecules (K_3^w) scales as a^4, while recombination into deeply bound molecules (K_3^d) scales as a^2 (see Table I). Therefore, recombination for large a leads mostly to the formation of weakly bound molecules. This differential scaling with a is in stark contrast to previously studied systems with two or three resonant interactions for which recombination is nonzero at $T \to 0$, as is the case for one- and two-component bosonic systems and boson-fermion mixtures. In these systems, recombination into *all* two-body states scales as a^4. In those systems, recombination into the weakly bound state is typically dominant as well but only via the coefficient of a^4. Systems with only a single resonant pair of atoms share this property in addition to the differential scaling with a. Figure 1(b) shows our numerical calculations of recombination rates for the same system as in Fig. 1(a). The numerical results support all of the above predictions, including not only the smaller magnitude of K_3^d relative to K_3^w but also their scaling with a. Another consequence of this differential scaling is that for $a < 0$, i.e., in the absence of weakly bound molecules, recombination is proportional to $|a|^2$. So recombination at $a < 0$ for systems with a single resonant pair of atoms could be suppressed compared to $a > 0$ for large a.

The results obtained here for three-body processes involving a single resonant pair—summarized in Table I—combined with previous results [5] exhaust all possible three-body processes relevant to three-component atomic gases. The competition between the various collision processes in a multicomponent gas is dictated fundamentally by their energy and scattering length dependence, although it is also crucial to take into account the atomic and molecular densities. In fact, manipulating the atomic and molecular densities can make favorable a particularly desired collisional behavior.

For completeness, we show in Table II the three-body collision rates for *all* possible ultracold three-component atomic gases. For each mixture, we have listed the three-body processes according to which interatomic interaction is resonant by indicating the resonant pair of atoms, and we have included only the dominant contribution at threshold. Processes involving a single resonant pair are indicated by "$\diamond$," while all of the others were obtained in previous work [4,5,7] Further, we have $2 \le p_0 \le 4$ for $V_{\rm rel}$ and $0 \le p_0 \le 2$ for K_3^d, depending on the mass ratio between the collision partners [5], and we have omitted modulation factors that produce the interference minima and resonant peaks related to Efimov physics [5].

We can see from Table II that in all possible three-component mixtures weakly bound molecules can be expected to be long-lived with relaxation scaling as a^{-1}, provided that all free atoms do not interact resonantly with the atoms in the molecule. In fact, for gas mixtures where distinguishable fermions are resonant, weakly bound molecules are expected to be long-lived irrespective of the identity of the free atoms.

Given the suppression possible in atom-molecule collisions, it might be that molecule-molecule collisions are the dominant relaxation mechanism in a three-component gas. For BF molecules, that are themselves fermions, molecule-molecule collisions are suppressed at ultracold temperatures due to their p-wave character. For B_iB_j bosonic molecules, however, long lifetimes can be reached only for low molecular densities, in order to prevent molecule-

TABLE II. All three-body collision rates relevant for three-component atomic gases. For each mixture, the resonant pair of atoms are indicated by parentheses. The symbol $\diamond$ indicates collisions involving a single resonant pair of atoms. Below, $2 \leq p_0 \leq 4$ for V_{rel} and $0 \leq p_0 \leq 2$ for K_3^d, depending on the mass ratio between the atoms resonant in the pair [5].

Gas mixture		V_{rel}^{ij+k}	K_3^w, K_3^d		V_{rel}^{ij+k}	K_3^w, K_3^d		V_{rel}^{ij+k}	K_3^w, K_3^d
B_1-B_2-B_3	$(B_1B_1)B_1$	a	a^4, a^4	$(B_1B_2)B_1$	a	a^4, a^4			
	$(B_1B_1)B_2^\diamond$	a^{-1}	a^4, a^2	$(B_1B_2)B_2$	a	a^4, a^4			
	$(B_1B_1)B_3^\diamond$	a^{-1}	a^4, a^2	$(B_1B_2)B_3^\diamond$	a^{-1}	a^4, a^2			
B_1-B_2-F	$(B_1B_1)B_1$	a	a^4, a^4	$(B_1B_2)B_1$	a	a^4, a^4	$(B_1F)B_1$	a	a^4, a^4
	$(B_1B_1)B_2^\diamond$	a^{-1}	a^4, a^2	$(B_1B_2)B_2$	a	a^4, a^4	$(B_1F)B_2^\diamond$	a^{-1}	a^4, a^2
	$(B_1B_1)F^\diamond$	a^{-1}	a^4, a^2	$(B_1B_2)F^\diamond$	a^{-1}	a^4, a^2	B_1FF	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$
B-F_1-F_2	$(BB)B$	a	a^4, a^4	$(BF_1)B$	a	a^4, a^4	$(F_1F_2)B^\diamond$	a^{-1}	a^4, a^2
	$(BB)F_1^\diamond$	a^{-1}	a^4, a^2	$(BF_1)F_1$	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$	$(F_1F_2)F_1$	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$
	$(BB)F_2^\diamond$	a^{-1}	a^4, a^2	$(BF_1)F_2^\diamond$	a^{-1}	a^4, a^2	$(F_1F_2)F_2$	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$
F_1-F_2-F_3	$(F_1F_2)F_1$	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$						
	$(F_1F_2)F_2$	a^{1-2p_0}	k^2a^6, $k^2a^{6-2p_0}$						
	$(F_1F_2)F_3^\diamond$	a^{-1}	a^4, a^2						

molecule collisions. In contrast, for F_iF_j bosonic molecules, relaxation due to molecule-molecule collisions scales as a^{-s}, with $s > 1$ [4], for mass ratios $m_{F_i}/m_{F_j} > 0.11603$ (a condition satisfied for commonly used alkali atoms), and the main molecular collisional decay is due to atom-molecule collisions involving a single resonant pair of atoms, scaling as a^{-1}.

From Table II, we see that the optimal three-component systems utilize molecular formation close to an F_1-F_2 Feshbach resonance. In this case, recombination involving two identical fermions is suppressed at ultracold temperatures, while recombination in both B-F_1-F_2 and F_1-F_2-F_3 mixtures leads mainly to the formation of weakly bound F_1F_2 molecules. The most important property of this system is that, after the weakly bound molecules are formed, molecular loss due to collisions with the remaining atoms is suppressed *at least* as a^{-1}. Similar benefits can also be realized near a B-F resonance under certain circumstances. For instance, in B-F_1-F_2 mixtures, efficient B-F_1 molecular formation could be achieved if the number of nonresonant F_2 atoms is higher than the B and F_1 numbers. If, in the end, all bosons are bound in BF molecules, then collisions with the remaining fermions are suppressed, ensuring the desired molecular stability.

In summary, we have shown that vibrational relaxation of weakly bound molecules can be suppressed near a Feshbach resonance even in the absence of Fermi statistics. Remarkably, the root cause of this suppression can be traced to the same kind of universality that yields the Efimov effect. The systems that show this behavior can also exhibit differential scaling with a of the recombination rate into weakly bound molecules that are finite at zero temperature. These two results combine to make ultracold three-component gases with a single resonant interaction an attractive alternative for efficiently producing stable, ultracold molecules.

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