New Non-Goldstone Collective Mode of BEC of Magnons in Superfluid ^{3}He-B

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Bose-Einstein condensation of magnons in superfluid ^{3}He-B is experimentally manifested by various states where coherent spin precession is established spontaneously, even in nonhomogeneous magnetic fields. Once such a condensate with coherent spin precession is created, it occupies the state with minimal energy, the ground state. The application of an additional magnetic field to that condensate may cause its deflection from the energy minimum and the condensate responds by creating collective gapless oscillations known as Goldstone modes. This Letter reports the experimental observation of a new (non-)Goldstone mode, which can be viewed as an additional NMR mode of condensed magnons in a rotating frame of reference.

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Bose-Einstein condensation is a fundamental physical phenomenon where a macroscopic number of particles condense into a collective quantum ground state governed by a single wave function. Ultracold atomic gases provide an example of an almost perfect Bose-Einstein condensate (BEC); however, there are many other physical systems exhibiting the properties of Bose-Einstein condensates. In fact, the scenario of Bose-Einstein condensation can also be applicable to systems of long-lived quasiparticle excitations that live a sufficiently long time for the formation of the BEC [1]. Complete Bose-Einstein condensation of magnons in superfluid ^{3}He has been shown recently [2]. BECs of magnons are manifested macroscopically by the existence of various states of coherent spin precession that can be established spontaneously even in nonhomogeneous magnetic field [3–5]. These states exhibit all the properties of magnetic superfluidity: spin supercurrent, the Josephson effect, phase slippage, etc. (for a review, see [6]). The first BEC of magnons in superfluid ^{3}He-B was identified with the formation of the homogeneously precessing domain (HPD) [2]. Once such a coherent spin precession state is generated using NMR, it occupies the ground state. The application of an additional magnetic field to this state deflects it from the energy minimum and the condensate may respond by creating collective oscillations known as Goldstone modes. A few different modes of collective oscillations of the HPD have already been observed as torsional oscillations [7,8] and surface oscillations of the free surface of the HPD state [9,10]. This Letter reports the experimental observation of a new (non-)Goldstone mode of the magnon BEC. We show that this is best described as an additional NMR mode in a rotating frame of reference.

One of the possible magnon BECs in superfluid ^{3}He-B is the HPD. The HPD is created due to the existence of spin supercurrents and the dipole-dipole interaction; both are direct consequences of the orbital p-wave, spin triplet pairing of the ^{3}He quasiparticles (for details see [11]). When the HPD is excited using a standard cw-NMR technique with a high frequency field of amplitude B_{rf}, the volume of superfluid ^{3}He-B in an experimental cell is split into two domains: the precessing domain (HPD), where the spins of the condensate coherently precess at the Larmor frequency ω_L, and a nonprecessing domain (NPD) with the spins being aligned in the direction of an external magnetic field B_0. These two domains are separated by a domain wall of thickness λ_F ($\lambda_F = (c_\parallel^2/\omega_L \nabla\omega)^{1/3}$) localized at the position z_0 for which the frequency of the spin precession ω_L fulfills the Larmor resonance condition (LRC) $\omega_L = \gamma(B_0 - z_0\nabla B)$, where ∇B is the field gradient, γ is the gyromagnetic ratio, $\nabla\omega = \gamma\nabla B$, and $c_\parallel$ is the spin wave velocity in the direction of magnetic field B_0. As the position of the domain wall z_0, at $\omega_L = \mathrm{const}$, can be controlled by the field B_0, it is easy to adjust the length of the precessing domain (L) and to fill the whole volume of the cell with the HPD. Then, all the spins of the condensate precess at the same frequency ω_L. Under these conditions, in a frame of reference rotating at the Larmor frequency ω_L, the magnetic field B_0 in the z direction vanishes and the spins of the condensate interact only with the field B_{rf} which becomes stationary in this frame. Therefore, in this rotating frame of reference, the field B_{rf} can play a similar role as the field B_0 does in the laboratory frame. Accordingly, the application of another oscillating field $\Delta B_\parallel$ at resonance frequency $\Omega_\parallel$, perpendicular to B_{rf}, may excite the spins of the condensate and, as a result, the condensate responds by creating collective oscillations. This manifests itself as NMR in the rotating frame of reference.

The experiments were carried out in a cylindrical experimental cell (see Fig. 1) made from Stycast 1266 and mounted on a diffusion welded nuclear stage [12]. The Pt NMR and two vibrating wires (VWs), Ta wire of diameter of 125 μm and NbTi wire of diameter of 13 μm, served as thermometers. The HPD was excited by means of a high frequency field B_{rf} generated by a Helmholtz rf coil at a

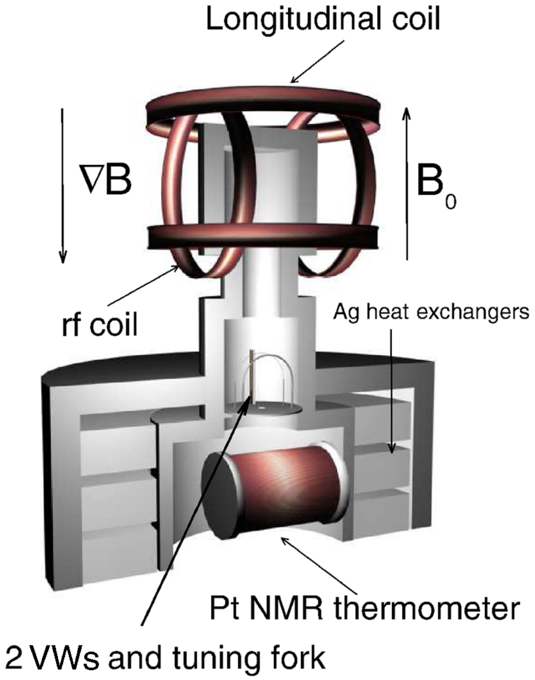

FIG. 1 (color online). A schematic 3D cross section of a double walled experimental cell with the NMR coils setup, including the orientation of applied magnetic fields. The upper part of the cell has diameter and length 6 mm.

resonance frequency of 462 kHz in the upper part of the cell. The additional low frequency magnetic field, with axial symmetry and amplitude $\Delta B_{\parallel}$, was generated by a small longitudinal coil and served as the excitation field for generation of the NMR of the HPD in the rotating frame of reference. The HPD response to the excitation field is an additional low frequency modulation of the precessing spins. It was detected and amplified at the high frequency, after which the high frequency component was removed using a rf detector and a low-frequency filter in combination with a low-frequency lock-in amplifier. The reference signal for the low-frequency lock-in amplifier was supplied from the generator which provided the longitudinal excitation field $\Delta B_{\parallel}$. This allowed measurement of the pure low frequency resonance characteristics.

A fundamental characteristic of the NMR of the magnon BECs in a rotating frame of reference should be the dependence of the resonance frequency $\Omega_{\parallel}$ on the amplitude of $B_{\rm rf}$. For a standard paramagnetic spin system one would expect that the resonance frequency $\Omega_{\parallel}$ should depend linearly on the value of $B_{\rm rf}$. However, the magnon BEC in superfluid ^{3}He-B presents a more complex problem. The condensate stiffness and the dipole torque provide additional "fields" affecting NMR. Applying the Lagrangian equations of spin dynamics provides us with solutions describing various excited modes of the HPD-NPD system [8]. In contrast to [8], where free oscillation modes of two domain structures were considered, here the presence of the field $B_{\rm rf}$ must be taken into account. Then, by solving the Lagrangian equations of spin dynamics with the assumption of small deviations from equilibrium of the Euler angles of the condensate's precessing spins, and including a small spatial inhomogeneity of these spins through a gradient energy term, one can derive the frequency of collective oscillations in the form

$$\Omega_{\parallel}^2 = \frac{\Omega_B^2}{\Omega_B^2 + \omega_L^2}\left(\frac{c_{\parallel}^2\pi^2}{4L^2} + \frac{4}{\sqrt{15}}\gamma B_{\rm rf}\omega_L\right), \qquad (1)$$

where L is the HPD length and $\Omega_B(T)$ is the Leggett frequency. When the HPD only partially fills the cell ($L \to 0$), the frequency $\Omega_{\parallel}$ is determined by the first term which resembles the so-called torsional oscillations of the HPD. This mode was observed as a free oscillation mode of two domain structures generated by pulsed NMR [7]. As the HPD length grows it finally fills the whole cell, the torsional oscillations transform to the long wavelength limit with $1/L \to 0$, and the frequency is determined by the second term. This term determines the frequency of uniform oscillations of the spin condensate which depend on $B_{\rm rf}$, i.e., the frequency of NMR in the rotating frame of reference.

Figure 2 shows the experimental results. During these measurements the HPD filled the whole cell and thus the long wavelength limit ($1/L \to 0$) was satisfied. The inset of Fig. 2 presents the frequency shifts of the resonance characteristics of NMR signals in the rotating frame of reference due to various values of $B_{\rm rf}$. Figure 2 summarizes the dependencies of the squares of the resonance frequencies as a function of $B_{\rm rf}$ measured for different field gradients and confirms the expected linear dependence on $B_{\rm rf}$. Furthermore it demonstrates the independence of the resonance frequencies on the field gradient ∇B. This also confirms the excitation of the collective oscillation mode from the coherent ground state, i.e., the magnon BEC.

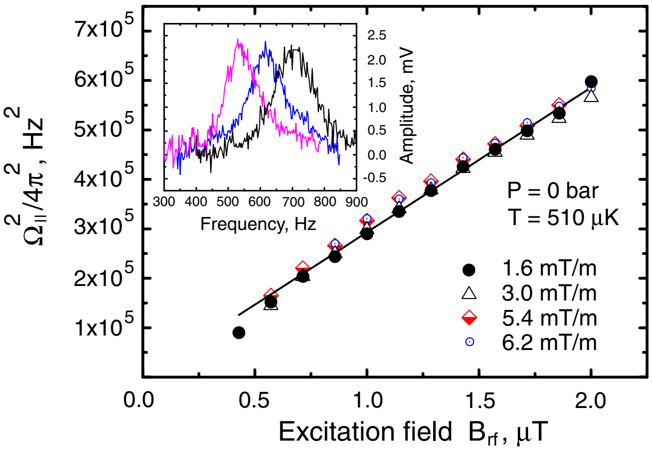

FIG. 2 (color online). Dependence of $\Omega_{\parallel}^2/4\pi^2$ on the amplitude of $B_{\rm rf}$ for various magnetic field gradients. The line represents the fit to the experimental data using expression (1) in the long wavelength limit. The inset shows the frequency shifts of the resonance characteristics of the NMR signals due to different values of $B_{\rm rf}$.

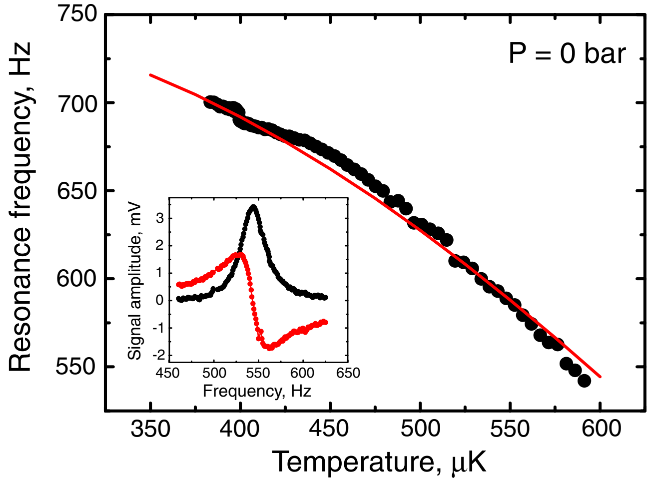

FIG. 3 (color online). Temperature dependence of the resonance frequencies of NMR in the rotating frame of reference. The line represents the theoretical temperature dependence calculated using (1) and (2). The inset demonstrates the measured NMR absorption and dispersion signals in the rotating frame of reference.

The influence of the dipole-dipole interaction [i.e., $\Omega_B(T)$] on the resonance frequency $\Omega_\parallel/2\pi$ measured in the long wavelength limit at $B_{\mathrm{rf}} = \mathrm{const}$ is presented in Fig. 3. The drop of $\Omega_\parallel/2\pi$ with temperature is expected as it follows from expression [13]

$$\Omega_B^2(T) \approx 3\mu_0 \frac{\gamma^2\mu^2}{a^3\chi_B(T)}\left(\frac{\Delta(T)}{E_F}\right)^2 n, \qquad (2)$$

where $\Delta(T)$ is the energy gap, $\chi_B(T)$ is the ^{3}He-B susceptibility, a is the average distance between ^{3}He particles, μ_0 is the vacuum permeability, n is the particle number density, and E_F is the Fermi energy. While the energy gap $\Delta(T)$ decreases, the susceptibility $\chi_B(T)$ rises up as the superfluid ^{3}He-B warms up. The line in Fig. 3 represents a theoretical temperature dependence calculated using (1) and (2) and shows a good agreement with experimental data.

When the HPD only partially filled the cell, in contrast to [14] where it was impossible to excite collective oscillations of the spin condensate by means of the pulsed NMR technique, here we were able to do this by using continuous excitation. However, as follows from Eq. (1), the collective response of the spin condensate is more complex. Figure 4 shows the dependence, as a function of the domain wall position, of the resonance frequencies $\Omega_\parallel/2\pi$ and the widths of the resonance curves Δf_2. The latter represents the energy dissipation associated with NMR in the rotating frame of reference. The dependence of $\Omega_\parallel/2\pi$ reveals the presence of torsional oscillations by the decrease of the resonance frequency as the HPD length grows. The line represents the fit to the experimental data using Eq. (1). The insets confirm the presence of the torsional oscillations. The first by means of the linear dependence of their resonance frequencies [i.e., $\sqrt{\Omega_\parallel^2 - \Omega_\parallel^2(\infty)}/2\pi$] on the reciprocal length of the HPD. The value of the resonance frequency in the long wavelength limit $\Omega_\parallel(\infty)/2\pi$ was taken to be 562 Hz. The second inset shows the dependence on the position of the LRC of the oscillation amplitudes at the resonance frequencies. According to theory [7], the amplitude of the torsional oscillations at the resonance frequency rises with the fourth power of the HPD length ($\sim L^4$). The data show very good agreement with the predicted dependence when the domain wall is localized in the cell. A tendency to saturation starts when the HPD fills the whole cell. Finally, the upper part of Fig. 4 shows a new feature: the gapless torsional oscillations of the HPD representing the Goldstone mode become a non-Goldstone mode as a gap is acquired due to the presence of the field B_{rf} [see Eq. (1)].

The dependence of Δf_2 behaves in a similar way as that for $\Omega_\parallel$; i.e., the dissipation is bigger when the HPD partially occupies the cell and drops down as the HPD volume

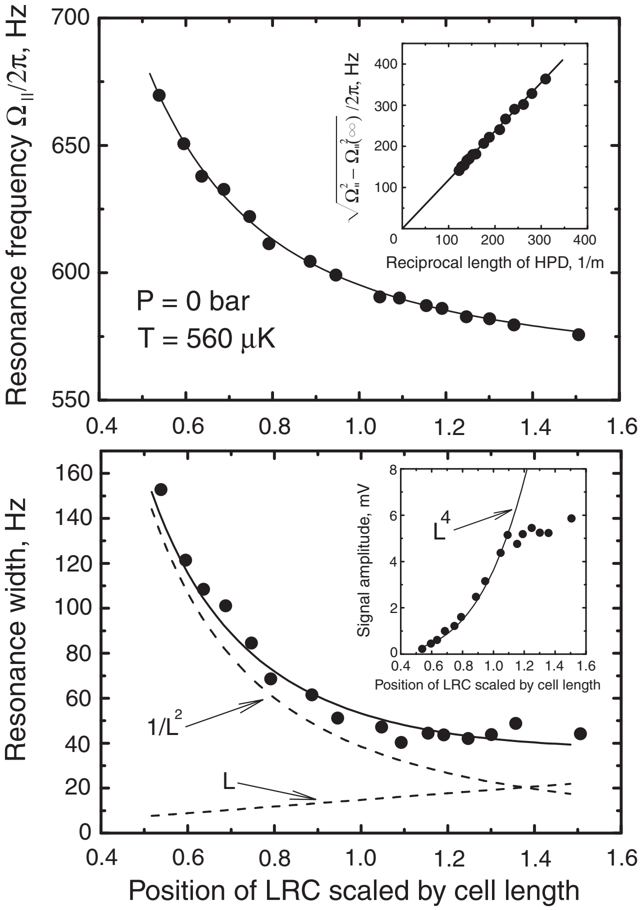

FIG. 4. Dependence of the resonance frequencies $\Omega_\parallel/2\pi$ and the widths Δf_2 as a function of the position of the LRC scaled by the cell length. The insets show the dependence of the torsional oscillation resonance frequencies on $1/L$ (top) and the dependence of the oscillation amplitudes at the resonance frequencies on position of LRC scaled by the cell length (bottom). For details see text.

grows. There are two main processes of energy dissipation in the stationary HPD: spin diffusion and Leggett-Takagi (L-T) relaxation [11]. Both mechanisms are associated with the presence of quasiparticle excitations (normal component) in ^{3}He-B. While the process of spin diffusion acts mostly in the domain wall due to spatially nonuniform motion of the spins there, the L-T process dominates in the HPD volume due to the difference in frequencies of the spin precession between the spin condensate and spins of the normal component. The rate of energy dissipation by these mechanisms along the z axis can be expressed as [11]

$$\dot{Q} = \frac{\chi_B \omega_L^2 S}{\gamma^2}\left(\sigma \frac{D_\perp}{\lambda_F} + \frac{5}{16}\tau_{\mathrm{eff}} \nabla\omega^2 L^3\right), \qquad (3)$$

where $\sigma \sim 1$ characterizes the domain wall shape, $D_\perp$ is the spin diffusion coefficient, S is the cross section of the domain wall, and τ_{eff} is the phenomenological L-T relaxation time constant.

When NMR in the rotating frame of reference is excited, the spin part of the order parameter acquires an additional rotation. The angular velocity of this rotation is connected to the increase of nonequilibrium spin density. The overall increase of spin density, which is proportional to $\Omega_\parallel$, affects both relaxation mechanisms. If the HPD does not fill the whole cell, the dominant process of energy dissipation seems to be spin diffusion. In fact, the calculation of energy dissipation by spin diffusion in the rotating frame leads to a $1/L^2$ dependence. Simultaneously, as the HPD grows the L-T relaxation begins to contribute to the dissipation. An estimation of the energy dissipation rate in the rotating frame leads to a linear dependence on the HPD length. The total energy dissipation rate caused by these processes can be expressed as

$$\dot{Q}^R \sim A\frac{D_\perp}{L^2} + B\tau_{\mathrm{eff}} L, \qquad (4)$$

where the various coefficients are collected in parameters A and B. Figure 4 presents the contribution of the spin diffusion ($1/L^2$) and the L-T relaxation (L) to the total energy dissipation (solid line). As one can see, there is a reasonable qualitative agreement between proposed model and experimental data.

All measurements presented above were made with the rf coil, which detected the induced transverse voltage signal from precessing spins in the x-y plane. To confirm that the oscillation mode of the spin condensate is indeed NMR in the rotating frame of reference, one should also detect an additional induced voltage signal generated by the precessing spins using the longitudinal coil. Figure 5 presents an additional induced signal measured in this coil during excitation. Although the signal is very weak and noisy due to a nonresonance low frequency detection setup, there is evidence of the resonance behavior, confirming the excitation of NMR in the rotating frame of reference.

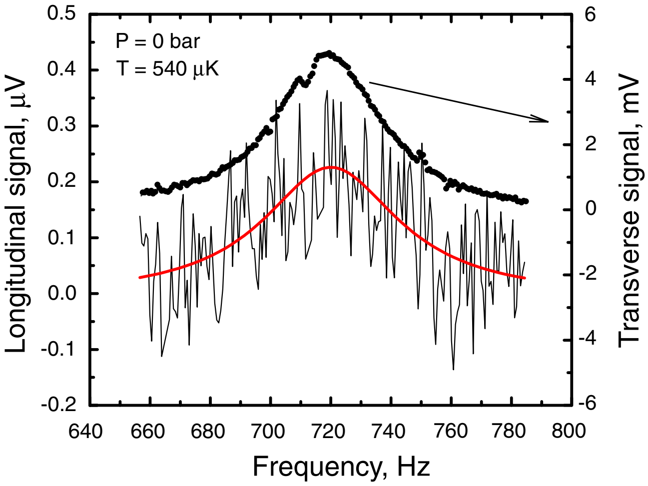

FIG. 5 (color online). Frequency dependence of the induced signal measured with the longitudinal coil, and the induced transverse signal measured in the x-y plane at the same time.

At this point we can summarize that a new collective oscillation mode of the spin condensate in superfluid ^{3}He-B has been experimentally observed. This mode can be represented as the additional NMR of the magnon BEC in the rotating frame of reference. Furthermore, it possesses a new feature: the Goldstone mode becomes a non-Goldstone mode as it acquires a gap due to the presence of the rf field.

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